

TRAVELLING WAVES IN A NONLINEAR DEGENERATE DIFFUSION MODEL FOR BACTERIAL PATTERN FORMATION

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ABSTRACT. We study a reaction diffusion model recently proposed in [5] to describe the spatiotemporal evolution of the bacterium *Bacillus subtilis* on agar plates containing nutrient. An interesting mathematical feature of the model, which is a coupled pair of partial differential equations, is that the bacterial density satisfies a degenerate nonlinear diffusion equation. It was shown numerically that this model can exhibit quasi-one-dimensional constant speed travelling wave solutions. We present an analytic study of the existence and uniqueness problem for constant speed travelling wave solutions. We find that such solutions exist only for speeds greater than some threshold speed giving minimum speed waves which have a sharp profile. For speeds greater than this minimum speed the waves are smooth. We also characterise the dependence of the wave profile on the decay of the front of the initial perturbation in bacterial density. An investigation of the partial differential equation problem establishes, via a global existence and uniqueness argument, that these waves are the only long time solutions supported by the problem. Numerical solutions of the partial differential equation problem are presented and they confirm the results of the analysis.

1. Introduction. Many biological systems exhibit the phenomenon of self organisation, whereby complex spatiotemporal patterns emerge from the biochemical processes occurring within the system (for review see, for example, Murray [15]). One very widely studied system is the spatiotemporal patterning behaviour in bacterial colonies. Not only does this serve as a model paradigm for patterning in higher organisms, its study also has important implications in biotechnology and related areas.

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