DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Supplement 2009

Website: www.aimSciences.org

pp. **780–789**

COLLISION DYNAMICS OF CIRCULARLY POLARIZED SOLITONS IN NONINTEGRABLE COUPLED NONLINEAR SCHRÖDINGER SYSTEM

M. D. Todorov

Dept. of Differential Equations, Faculty of Applied Mathematics and Informatics
Technical University of Sofia, 1000 Sofia, Bulgaria

C. I. Christov

Dept. of Mathematics, University of Louisiana at Lafayette
LA 70504-1010, USA

Abstract. The system of Coupled Nonlinear Schrödinger Equations (CNLSEs) is solved by a conservative difference scheme in complex arithmetic developed in earlier author's work. The initial condition represents a superposition of two one-soliton solutions of different circular polarizations. The interaction (collision) of the solitons and their quasi-particle (QP) behavior is examined for different configurations of the initial system of QPs. We found that the polarization angle of a QP can change after a collision with another QP depending on the configuration of the initial phases. The effects found in the present work seem to be novel and enrich the knowledge about the intimate mechanisms of interaction of polarized QPs of CNLSEs.

1. **Introduction.**

Investigation of soliton dynamics in the model presented by the system of Coupled System of Nonlinear Schrödinger Equations (CNLSE) is of great importance from several different perspectives. It has been first formulated for modeling the propagation of light pulses in optical fibers, and is still one of the main tools in theoretical investigation in fiber optics [1, 3]. Another important role the Nonlinear Schrödinger Equations (NLSEs) play is as a mathematical object on which new approaches for finding analytical or approximate solutions are tested (see, e.g., [5, 13]). Still another aspect of this important model is that similarly to the models based on sine-Gordon (sG), Korteweg-de Vries (KdV), and Boussinesq equations (to mention a few), the generic class of NLSEs offers the opportunity to investigate the quasi-particle behavior of solitons. Quasi-particles (QPs) hold the key for the future advances in wave-particle dualism. The NLSE class is perhaps more important for the wave-particle duality than the other famous soliton supporting equations, because its soliton solutions are envelopes (pulses). The carrier frequency of the pulses offers additional dimension for understanding the analogy between waves and particles.

A system of CNLSEs exhibits a rich phenomenology possessing a plethora of different one-soliton solutions. The essential new feature of CNLSEs in comparison with the single NLSE is the polarization, which is related to relative amplitudes of

2000 *Mathematics Subject Classification.* Primary: 65M06, 35Q55; Secondary: 35Q51.
Key words and phrases. Conservative numerical schemes, Quasi-particles, Elliptic polarization, Collision dynamics.

the two components. Adding the fact that each component actually is a complex-valued function, one can appreciate the complexity of the possible soliton interactions. Having two components that satisfy two *nonlinearly* coupled equations means that the cross-excitation of the modes (components) during the soliton collision will give rise of a radically new physics in comparison with the single NLSE. The CNLSE model is even richer when a linear coupling is considered alongside with the main, nonlinear, coupling (see, e.g., [8] and the literature cited therein).

The simplest possible QPs in CNLSE are those with the so-called 'linear polarization', meaning that each QP has only one non-trivial component. Naturally, if the two initial QPs have the same type of linear polarization (say, in both of them the same component is set to zero), then the initially zero second component is never excited, and the behavior is essentially the one of a single NLSE. Yet, it simply takes to invert one of the linear polarizations of the initial QPs, and the coupled nature and excitability of the system show up. The role of the nonlinearity in the interaction of initially linearly polarized solitons was investigated in [11, 12]. It was uncovered that depending on the magnitude of the cross-modulation parameter (presenting the nonlinear coupling), the interaction between the modes during the collision, changes the polarization of the QPs, and/or gives birth to one or more QPs.

The other important case of analytical one-soliton solution is presented by the so-called 'circularly polarized' solitons when both modes are present. In the cases of both linear and circular polarization, the carrier frequencies of the two components are identical. Another level of complexity is reached when the carrier frequencies are allowed to be different. In this case the solitons are called 'elliptically polarized' (see, e.g., [9], and no analytical solution exists even for the one-soliton solution. Then even the initial shape of the QP has to be found numerically (see [6, 9] and the literature cited therein). As demonstrated numerically in [11, 12], in most of the cases, the initially linearly polarized QPs end up as elliptically polarized after the collision. In this sense, all the discussed polarizations are actually limiting cases of the general elliptic polarization.

In the present paper, we set the goal to expand our work on the QPs collision in CNLSE to include the other class of analytical initial conditions: the superposition of circularly polarized one-soliton solutions, and to outline the limits of the analytical two-soliton solution given by Manakov [5] for one limiting case of the cross-modulation parameter.

Apart from some limiting cases, CNLSE are not fully integrable and require numerical approaches for their treating. As a rule, the model admits two conservation laws: for the 'mass' (better called 'pseudomass' when the QPs are considered) and the energy. In addition, a balance law for the wave momentum (called 'pseudomomentum' for the QPs) is also obeyed by the solution. In the absence of excitations at the boundary of the domain under consideration, the balance law for the pseudomeomentum becomes a conservation law, too.

The conservation laws are to be faithfully represented by the respective numerical scheme, which means that it *has* to be nonlinear. Following the concept of inner iterations (see [2], [8], [11]) we linearize this scheme and solve the discrete problem by using Crank-Nicolson implicit scheme [10]. We implement five-diagonal complex-valued matrices based on generalized Gaussian method with pivoting. The details of the scheme for the complex-valued arithmetic are presented in [11].

2. **Outline of the Model.** CNLSE can be written in different equivalent forms. We use the following one (see, also [12])

$$\mathrm{i}\psi_t + \beta\psi_{xx} + [\alpha_1|\psi|^2 + (\alpha_1 + 2\alpha_2)|\phi|^2]\psi = 0, \tag{1a}$$

$$\mathrm{i}\phi_t + \beta\phi_{xx} + [\alpha_1|\phi|^2 + (\alpha_1 + 2\alpha_2)|\psi|^2]\phi = 0. \tag{1b}$$

Here β is the dispersion parameter, α_1 is called self-focusing, and α_2 is the cross-modulation parameter.

The initial condition for a single QP with circular polarization has the form

$$\psi(x,t;c,n,X,\theta,\vec{\delta}) = A\cos\theta\,\mathrm{sech}[b(x-X-ct)]\exp\{\mathrm{i}[\frac{c}{2\beta}x + nt + \delta_\psi]\}, \tag{2a}$$

$$\phi(x,t;c,n,X,\theta,\vec{\delta}) = A\sin\theta\,\mathrm{sech}[b(x-X-ct)]\exp\{\mathrm{i}[\frac{c}{2\beta}x + nt + \delta_\phi]\}, \tag{2b}$$

where c and n are the phase speed and the carrier frequency of the QP. The localized solution is identified by the presence of a spatial point, $x = ct - X$ ('center') which moves with a given phase speed c and has an internal variable n – the carrier frequency. In our notations given in Eq. (2), the parameters are separated from the spatio-temporal independent variables by a semicolon. Respectively, θ is called 'polarization angle', and $\delta_{\psi,\phi}$ is the phase of each component of the vector solution. In order that an one-soliton solution of the above type exists, the amplitude, A, and the inverse measure of support, b, have to be related to c and n as follows

$$b^2 = \frac{1}{\beta}\left(n + \frac{c^2}{4}\right), \quad A = b\sqrt{\frac{2\beta}{\alpha_1}}, \qquad n > -\frac{c^2}{4}. \tag{2c}$$

The meaning of the above restriction is that the carrier frequency cannot be smaller than a specified negative number, for a QP that propagates with a given phase speed c. In particular, if we are concerned with a non-propagating envelop $c = 0$, then its carrier cannot be a standing wave, but only a propagating wave with $n \neq 0$.

It is important to note here that if the polarization angle θ is arbitrary, the circularly polarized QP from Eq. (2) exists only for $\alpha_2 = 0$ (the Manakov case [5]). Yet, for the particular case of $\theta = 45°$, the general elliptic polarization is reduced to a circular polarization with amplitude given by

$$A_{45^\circ} = b\sqrt{\frac{\beta}{\alpha_1 + \alpha_2}}. \tag{3}$$

For streamlining the notation, it is convenient to introduce the vector notations $\vec{\chi} = (\psi,\phi)^T$ and $\vec{\delta} = (\delta_\psi, \delta_\phi)^T$.

The initial condition is constructed as the superposition of two QPs situated at X_l and X_r, and propagating with phase speeds c_l and c_r, i.e.,

$$\vec{\chi}(x,0) = \vec{\chi}_l(x,0;c_l,n_l,X_l,\theta_l,\vec{\delta}_l) + \vec{\chi}_r(x,0;c_r,n_r,X_r,\theta_r,\vec{\delta}_r). \tag{4}$$

Here $|X_r - X_l|$ has to be large enough so the 'tail' of one QP is fairly well decayed in the place of the other QP.

3. **Difference Scheme.** We use the implicit scheme [2], which is not only convergent (consistent and stable), but also conserves mass and energy

$$\begin{aligned}
\mathrm{i}\frac{\psi_i^{n+1}-\psi_i^n}{\tau} &= \frac{\beta}{2h^2}\left(\psi_{i-1}^{n+1}-2\psi_i^{n+1}+\psi_{i+1}^{n+1}+\psi_{i-1}^n-2\psi_i^n+\psi_{i+1}^n\right)\\
&\quad+\frac{\psi_i^{n+1}+\psi_i^n}{4}\Big[\alpha_1\left(|\psi_i^{n+1}|^2+|\psi_i^n|^2\right)+(\alpha_1+2\alpha_2)\left(|\phi_i^{n+1}|^2+|\phi_i^n|^2\right)\Big],\\
\mathrm{i}\frac{\phi_i^{n+1}-\phi_i^n}{\tau} &= \frac{\beta}{2h^2}\left(\phi_{i-1}^{n+1}-2\phi_i^{n+1}+\phi_{i+1}^{n+1}+\phi_{i-1}^n-2\phi_i^n+\phi_{i+1}^n\right)\\
&\quad+\frac{\phi_i^{n+1}+\phi_i^n}{4}\Big[\alpha_1\left(|\phi_i^{n+1}|^2+|\phi_i^n|^2\right)+(\alpha_1+2\alpha_2)\left(|\psi_i^{n+1}|^2+|\psi_i^n|^2\right)\Big],
\end{aligned}$$

where the superscript refers to the time stage, and the subscript identifies the grid point. The spatial grid is uniform, i.e., $x_i = -L_1 + (i-1)h$, where $h = (L_2 + L_1)/(N-1)$ is the spacing. Here $L_1, L_2 \gg 1$ are the left and right numerical 'infinities'. The time grid is also uniform: $t^n = n\tau$, where τ is the time increment.

Since the above scheme is of Crank-Nicolson type for the linear terms (involving the new time stage), we employ internal iterations to achieve implicit approximation of the nonlinear terms, i.e., we use its linearized implementation (see the details of this approach in [2, 8, 11]).

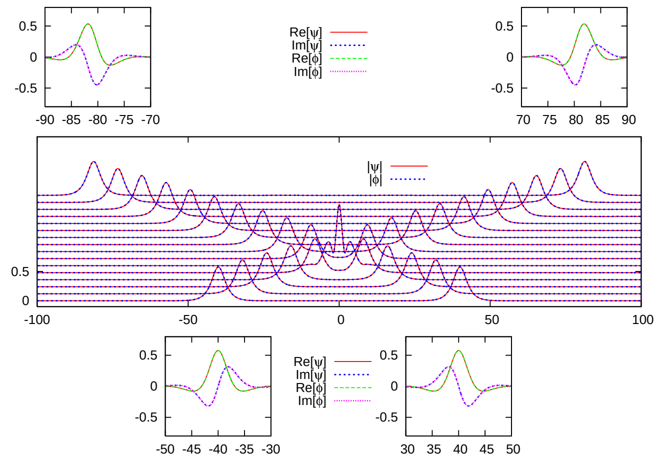
In this paper we use the version of the scheme in complex arithmetic as presented in [11]. Employing complex arithmetic makes the scheme four times faster than the original scheme of [2]. We validate the scheme through standard numerical experiments including halving the time and space steps and estimating the truncation error. For the parameter range of interest for this work, we conducted numerical experiments to identify the adequate grid values. We found that for carrier frequencies $|n| \le 1$ (when the soliton supports are wider) those values are $\tau = 0.04$ and $h = 0.1$. For carrier frequencies $|n| > 1$, the required spacing was $h \in [0.01, 0.05]$ because the support becomes narrower which requires a denser grid.

4. **Interaction of Circularly Polarized Solitons.** The parametric space of the problem is too large to be explored in full, and we choose $n_{l\psi} = n_{r\psi} = n_{l\phi} = n_{r\phi} = -0.5$, $c_l = -c_r = 1$, $\alpha_1 = 0.75$, and focus on the effects of α_2, θ, and $\vec{\delta}$.

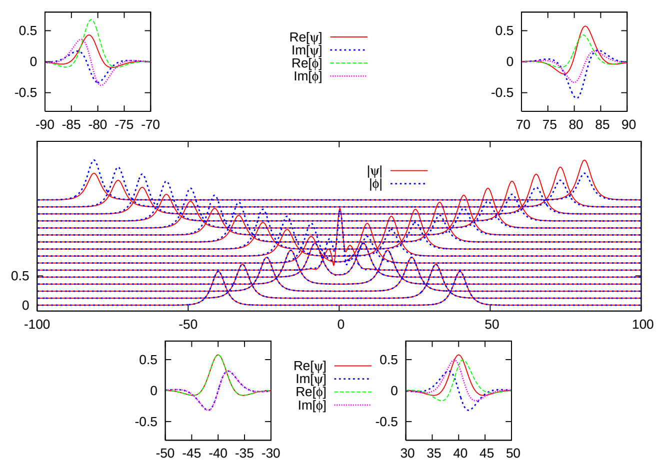
4.1. **The Manakov Case.** The Manakov solution in the form described in Eq. (2) is valid only when θ is a constant that does not depend on x and t. This means that in order to have an analytical two-soliton solution (see [4, 7] for the analytical details), the two initial pulses in Eq. (4) must have the same polarization angles. For this reason we begin our investigation with two pulses of the same polarization. Clearly, $\theta = 0°$ or $\theta = 90°$ need not be considered here because then one of the component is always equal to zero, and the system reduces to a single NLSE: a case thoroughly investigated in the literature.

Concerning the presentation of the results, we show the interaction itself, as well as snapshots of the initial shapes of the QPs. In the lower small subfigures we present the real and imaginary parts of the initial QPs in order to give an idea about the phase. The middle panel shows the actual evolution of the profile, and the upper small boxes show again the real and imaginary parts of the QPs, but from the last time stage shown in the middle panel.

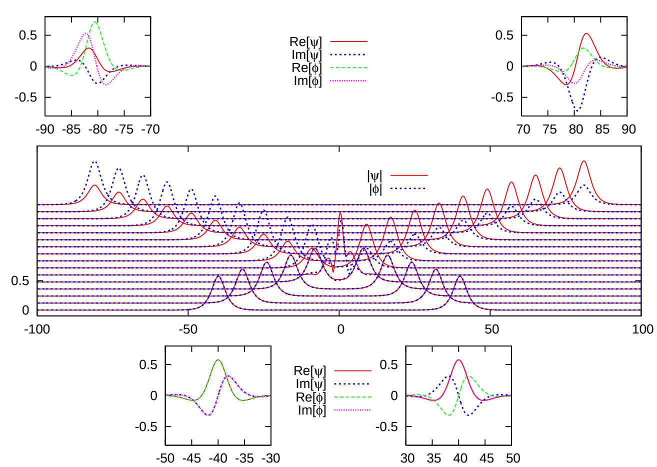
First we examine the case $\theta_l = \theta_r = 45°$, because it has relevance both for the systems with and without cross-modulation. In the succession of drawings in Fig. 1 we present the interaction of two pulses (QPs) of the same initial polarization, but



(a) $\delta_l = 0^\circ,\ \delta_r = 0^\circ$



(b) $\delta_l = 0^\circ,\ \delta_r = 45^\circ$



(c) $\delta_l = 0^\circ,\ \delta_r = 90^\circ$

FIGURE 1. Interaction of two QPs with initially equal polarizations $\theta_l = \theta_r = 45^\circ$

whose phases differ. In Fig. 1-(a) is the case when both of QPs have zero phases. The interaction follows exactly the analytical Manakov two-soliton solution.

A note is due here about the initial phases shown in the lower small panels of the figures where the real and imaginary parts are depicted. Although in some cases the two QPs have zero initial phases they appear reflected to each other, because the spatial wave number of the right QP is negative. The analytical pulses exist only for $k_r = \frac{1}{2}c_r < 0$.

The surprise comes in Fig. 1-(b) where the interaction of two QPs is presented: the right one having a nonzero phase $\delta_r = 45°$. One can see that after the interaction, the two QPs become again Manakov solitons, but different than the original two that entered the collision. The outgoing QPs have polarizations $33°48'$ and $56°12'$. Something that can be called a 'polarization shock' takes place. Note that all the solutions are perfectly smooth, but because the property called polarization cannot be defined in the cross-section of interaction and for this reason, it appears as undergoing a shock. There is some indication in the analytical works (see [4]) that this kind of exchange of polarization can take place, but the direct comparison is rather technical and goes beyond the scope of this paper.

Fig. 1-(c) presents a case in which the phases of the original QPs differ by 90°. The 'shock' effect on the polarization is even more pronounced than in Fig. 1-(b) (see Table 1 for the numbers). Here is to be mentioned that the moduli of ψ and ϕ from Fig. 1-(c) perfectly match each other when rescaled, which means that the resulting solitons have circular polarization (see Fig. 6-(a) below).

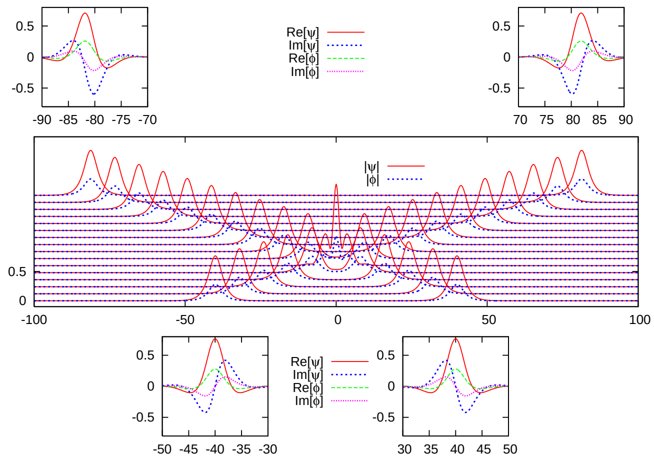
An important quantitative feature of the result for non-zero phase of one of the QPs is that the sum of the outgoing polarizations is equal to 90° which is the sum of the two in-going polarizations, each being 45°. In other words, our results show that a kind of 'conservation of polarization' law is obeyed (see Table 1). In most of

Table 1. Conservation of the total polarization

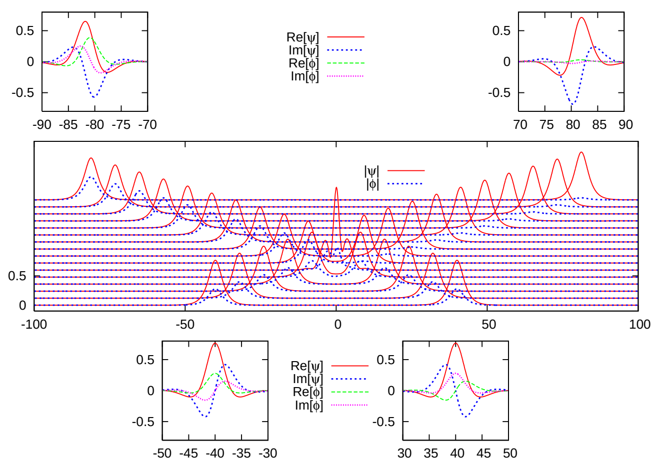
$\delta_r - \delta_l$	θ_l^i	θ_r^i	$\theta_l^i + \theta_r^i$	θ_l^f	θ_r^f	$\theta_l^f + \theta_r^f$
45°	45°	45°	90°	33°48′	56°12′	90°
90°	45°	45°	90°	24°06′	65°54′	90°
0°	20°	20°	40°	20°00′	20°00′	40°
90°	20°	20°	40°	28°48′	2°02′	30°50′
0°	36°	36°	72°	36°00′	36°00′	72°
90°	36°	36°	72°	53°00′	13°20′	66°20′
0°	10°	80°	90°	21°05′	68°54′	89°59′
90°	10°	80°	90°	9°27′	80°30′	89°57′
0°	20°	70°	90°	35°46′	54°11′	89°58′
90°	20°	70°	90°	16°07′	73°49′	89°56′
0°	30°	60°	90°	42°58′	46°59′	89°57′
90°	30°	60°	90°	19°41′	70°14′	89°55′

the cases presented in this paper, the total polarization is conserved with accuracy better than $4'$. The superscripts i and f in the table refer to the initial and final values of the polarization angles.

We would like to add here that we did a numerical experiment with $\delta_l = 0°$ and $\delta_r = 180°$ and it was identical to the case $\delta_l = \delta_r = 0°$. We can say that the most pronounced change of the polarization is observed for difference between the two phases around 90°. In order to verify that the above described behavior is authentic



(a) $\delta_l = \delta_r = 0^\circ$



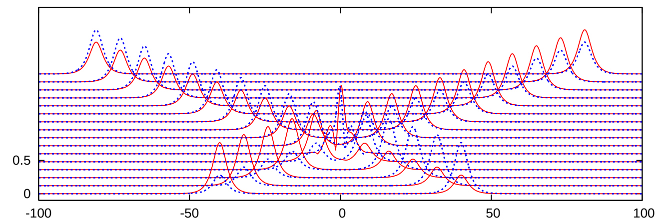
(b) $\delta_l = 0^\circ$, $\delta_r = 90^\circ$

FIGURE 2. Interaction of two QPs with initially equal polarizations $\theta_l = \theta_r = 20^\circ$

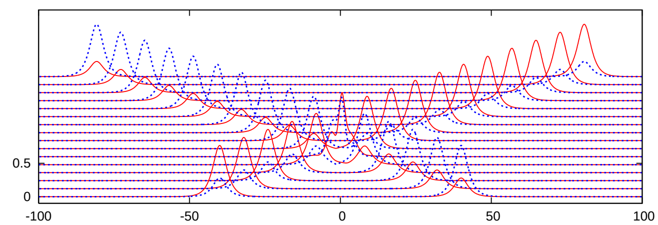
to the system when $\alpha_2 = 0$, we also consider the case $\theta_l = \theta_r = 20^\circ$ shown in Fig. 2 for $\delta_l = 0^\circ$, $\delta_r = 90^\circ$. One sees that once again the desynchronization of the phases leads to the break up of the Manakov solution (transforming it to a superposition of two Manakov one-soliton solutions).

The conclusion is that although the system is integrable for $\alpha_2 = 0$, the analytical solution of Manakov is not unique. Yet, its relevance is obvious, because after the interaction, the two QPs that leave the site of the collision are once again Manakov solitons but with polarizations that are different from the initial polarization.

To complete the case $\alpha_2 = 0$, we investigate the collision of two QPs whose initial polarizations are not equal. Clearly, one cannot expect in this case to get something like the Manakov two-soliton solution, because of the differences in the polarizations. Fig. 3 shows the result for two different initial phases. Contrary to the case $\alpha_2 = 0$, a zero phase leads to larger disparity of the outgoing polarizations, while a phase of 90° preserves the polarizations. Clearly, the nonlinearity embodied in the cross-modulation terms can change qualitatively the character of the soliton



(a) $\delta_l = \delta_r = 0°$



(b) $\delta_l = 0°$, $\delta_r = 90°$

FIGURE 3. Non-Manakov Initial condition: two solitons of different polarization in the initial moment: $\theta_l = 20°$, $\theta_r = 70°$

interactions. The last entries of Table 1 demonstrate the good conservation of the total polarization in this case.

4.2. **Case of Nontrivial Cross-Modulation $\alpha_2 \neq 0$.** As we have shown in the precedence, an analytical solution with circular polarization is possible even for nontrivial values of cross-modulation parameter, $\alpha_2 \neq 0$, but only for one particular polarization $\theta = 45°$. In the previous subsection we have found that the phases of the components play an essential role when the system chooses one or another of polarization of the one-soliton solutions that break away from the site of collision. It seems important to understand how the circularly polarized initial QPs evolve in the case of nontrivial cross-modulation parameter. We have computed the collision of two initial QPs with circular polarization $\theta = 45°$ for different α_2 and we have discovered that the interaction obeys the two-soliton solution with polarization $\theta = 45°$ for all of the values of cross-modulation that we have considered. In Fig. 4 we present the case $\alpha_2 = 0.5$ as a featuring example of the above statement.

The result shown in the figure is qualitatively similar to Fig. 1-(a), save the different amplitudes of the QPs stemming from Eq. (2c), rather than Eq. (2b). This means that the circularly polarized two-soliton solutions can be accessed via direct numerical simulation even when $\alpha_2 \neq 0$, which is an important new information about the CLNSE systems.

Now, the last question that remains to be answered is what will happen when there is a difference between the initial phases of the QPs. To answer this question we perform a similar experiment with different phases for $\alpha_2 = 0.25$. Fig. 5 shows the result which appears similar to the result with $\alpha_2 = 0$ in Fig. 1-(c).

When $\alpha_2 \neq 0$, it is not clear in advance that the resulting soliton has circular polarization. In order to judge what is the polarization of the QPs on the final stage of Fig. 5, we show the comparison of the scaled profiles of ψ and ϕ in Fig. 6-(b). These profiles do not coincide, which means that they are not *sech*-es, and the polarization is not circular, but is generally elliptic.

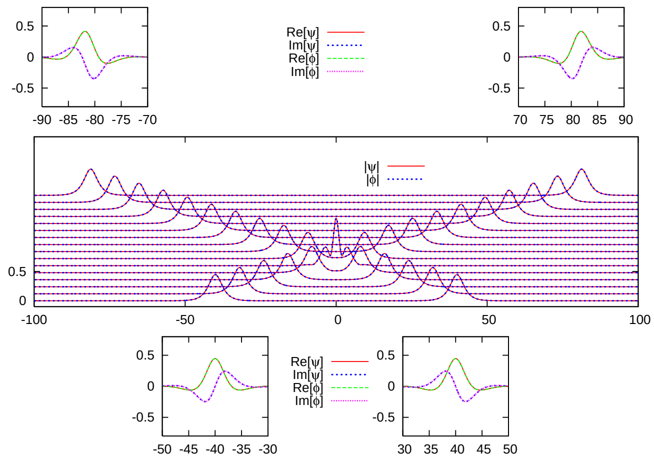


FIGURE 4. Interaction of two QPs of equal polarizations of 45° for $\alpha_2 = 0.5$ and no phase difference $\delta_l = \delta_r = 0°$

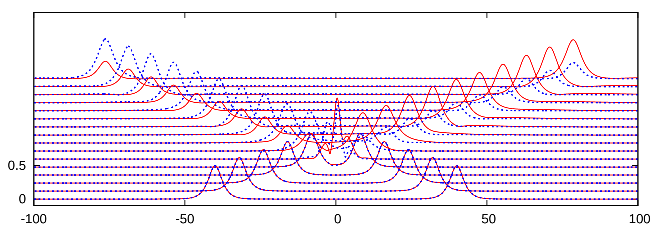
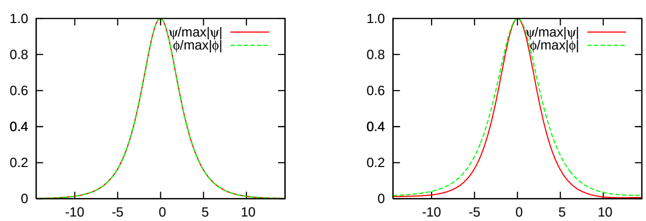


FIGURE 5. Interaction of two QPs of equal polarizations of 45° for $\alpha_2 = 0.25$ and initial phases $\delta_l = 0°, \delta_r = 90°$



(a) from Fig. 1-(c) (b) from Fig. 5

FIGURE 6. Test for the polarization of the outgoing QPs

5. **Conclusions.** In the present paper we have investigated numerically the collisions of circularly polarized solitons in Coupled Nonlinear Schrödinger Equations (CNLSEs). We compose the initial conditions as a superposition of two one-soliton solutions and investigate their collision as quasi-particles (QPs).

For the case of trivial value of the cross-modulation parameter $\alpha_2 = 0$, we have investigated different combination of initial QPs. If the two initial QPs have the same polarization angle, we are able to recover the analytical two-soliton Manakov's solution only when the two real and imaginary parts of the QPs are in phase. When there is a difference between the phases of the initial QPs, the two QPs that re-emerge from the collision, have polarizations different from the initial one.

We have also investigated cases when the initial polarizations of the two QPs are different. Such a case does not fall in the category of Manakov's two-soliton solution. The polarization of each QP changes after the collision.

For the case nontrivial cross-modulation parameter, we have shown that a solution of Manakov type can still be obtained analytically for one specific polarization: $\theta = 45°$. After the collision, two QPs with this polarization which have the same phases remain QPs with the same polarization. Contrary to the case $\alpha_2 = 0$, a zero phase leads to larger disparity of the outgoing polarizations, while a phase of 90° preserves the polarizations. As it should have been expected for a problem with nontrivial cross-modulation, the resulting QPs are not longer circularly polarized, but have a general elliptic polarization.

In all considered case we found a conservation of the total polarization.

Acknowledgments. MDT would like to thank the Department of Mathematics of the University of Texas at Arlington for the creative conditions for work during his visit in the Spring of 2008 when part of this investigation has been completed.

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Received August 2008; revised March 2009.

E-mail address: `mtod@tu-sofia.bg`
E-mail address: `christov@louisiana.edu`