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CONVERGENCE TO CONVECTION-DIFFUSION WAVES FOR SOLUTIONS TO DISSIPATIVE NONLINEAR EVOLUTION EQUATIONS

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Abstract. In this paper we consider the global existence and the asymptotic behavior of solutions to the Cauchy problem for the following nonlinear evolution equations with ellipticity and damping

$$\begin{cases} \psi_t = -(1-\alpha)\psi - \theta_x + \alpha\psi_{xx} + \psi\psi_x, \\ \theta_t = -(1-\alpha)\theta + \nu\psi_x + 2\psi\theta_x + \alpha\theta_{xx}, \end{cases} \tag{E}$$

with initial data converging to different constant states at infinity

$$(\psi,\theta)(x,0) = (\psi_0(x),\theta_0(x)) \to (\psi_\pm,\theta_\pm) \quad \text{as} \quad x \to \pm\infty, \tag{I}$$

where α and ν are positive constants such that $\alpha < 1$, $\nu < 4\alpha(1-\alpha)$. Under the assumption that $|\psi_+ - \psi_-| + |\theta_+ - \theta_-|$ is sufficiently small, we show that if the initial data is a small perturbation of the convection-diffusion waves defined by (11) which are obtained by the parabolic system (9), solutions to Cauchy problem (E) and (I) tend asymptotically to those convection-diffusion waves with exponential rates. We mainly propose a better asymptotic profile than that in the previous work by [13, 3], and derive its decay rates by weighted energy method instead of considering the linearized structure as in [3].

1. Introduction.

To understand more about systems describing the essential mechanism of the nonlinear interaction between ellipticity and dissipation systems, a set of simplified equations also constructed to yield the Lorenz equations was first proposed by Hsieh [6] (without diffusion terms) as follows:

$$\begin{cases} \psi_t = -(\sigma-\alpha)\psi - \sigma\theta_x + \alpha\psi_{xx}, \\ \theta_t = -(1-\beta)\theta + \nu\psi_x + 2\psi\theta_x + \beta\theta_{xx}, \end{cases} \tag{1}$$

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where α, β, σ and ν are positive constants such that $\alpha < \sigma$ and $\beta < 1$. This model has intrigued great interest and some good results have been obtained (see [6], [11]).

Tang and Zhao in [11] discussed the Cauchy problem for system (1) with $\alpha = \beta$ and $\sigma = 1$. That is,

$$\begin{cases} \psi_t = -(1-\alpha)\psi - \theta_x + \alpha\psi_{xx}, \\ \theta_t = -(1-\alpha)\theta + \nu\psi_x + 2\psi\theta_x + \alpha\theta_{xx}, \end{cases} \tag{2}$$

Under the assumption $\nu < 4\alpha(1-\alpha)$ and the initial data

$$(\psi_0(x), \theta_0(x)) \in L^2 \cap W^{1,\infty}(R, R^2), \tag{3}$$

they proved the global existence of solutions to the Cauchy problem (2)-(3) and obtained the decay rates of the solutions by Fourier analysis and the energy method. However, assumption (3) implied a restriction on the initial data $(\psi_0(x), \theta_0(x))$

$$(\psi_0(x), \theta_0(x)) \to (0, 0), \quad \text{as} \quad x \to \pm\infty. \tag{4}$$

We consider more general initial data here

$$(\psi(x, 0), \theta(x, 0)) = (\psi_0(x), \theta_0(x)) \to (\psi_\pm, \theta_\pm), \quad \text{as} \quad x \to \pm\infty, \tag{5}$$

where $\psi_\pm$, $\theta_\pm$ are constant states and $(\psi_+ - \psi_-, \theta_+ - \theta_-) \neq (0, 0)$.

Also in paper [7], Jian, Wang, and Hsieh studied the evolution equations :

$$\begin{cases} \psi_t = \alpha\psi + \lambda\psi\psi_x + (f(\theta))_x + \varepsilon_1\psi_{xx}, \\ \theta_t = \beta\theta + \nu\psi_x + (\psi\theta)_x + \varepsilon_2\theta_{xx}, \end{cases} \quad 0 < x < L, t > 0, \tag{6}$$

with initial data

$$(\psi(x, 0), \theta(x, 0)) = (\psi_0(x), \theta_0(x)), \tag{7}$$

and boundary condition

$$(\psi, \theta)(x, t) = (0, 0), \quad x \in \{0, L\}, \quad t \geq 0, \tag{8}$$

where L, ε_1, and ε_2 are all constants, while α, β, λ, and ν are given real constants. The nonlinear term $f \in C^\infty_{loc}(R)$ satisfies $|f_z(z)| \leq k, \forall z \in R$. Global smooth solution $(\psi, \theta) \in C([0, \infty), H^1_0([0, L]) \cap C^\infty((0, L)) \times (0, \infty)$ and the global attractor for the nonlinear system is studied in the abstract theory method as in Henry [4].

We observe that the function pairs $(\bar{\psi}, \bar{\theta})$ employed in [13] are technically effective since a faster decay estimates of the singular part will not change the approximation of the normalized system, which help to eliminate the boundary at infinity, but they are improved here since we could not expect the solution decay fast like $e^{-(1-\alpha)t}$ in [13]. Also see [5, 14, 10]. Another improvement is that we propose the weighted energy method to obtain estimates without representing the integral solutions as in [12].Our method could be applied to other nonlinear systems which make Fourier transform difficult, for example, either when the coefficient are spatially dependent in (E), or when spatial domain is a finite interval or half space. A quick comparison of recent work [2], based on the given profile (9), is discussed through the Fourier transform.

Instead, we will study the global existence and decay profile of the solutions to the Cauchy problem (E) and (5) through reformulation of the perturbation problem (15). For this, we discuss in §2 the linear convection-diffusion equations by approximating the system (E). In §3, global existence results are obtained from the local existence and *a priori* estimates. We obtain in §4-5 the decay profile to the solutions to (15) and (5) and estimates of higher order derivatives. Finally we summarize our results about system (E) in §6.

2. **Analysis of the Linear Convection-Diffusion Waves.** We observe that the initial value system (E) with different ending states (I) has infinite L^2 mass. Some renormalization is needed to derive the corresponding solution with finite energy. Precisely, motivated by Nishihara [9] for the case of $(\psi_\pm, \theta_\pm) = (0, 0)$, we expect the solutions of (1) time-asymptotically behave as those of the following linear system

$$\begin{cases} \bar\psi_t \ = -(1-\alpha)\bar\psi - \bar\theta_x + \alpha\bar\psi_{xx}, \\ \bar\theta_t = -(1-\alpha)\bar\theta + \nu\bar\psi_x + \alpha\bar\theta_{xx}, \end{cases} \tag{9}$$

with initial data

$$(\overline{\psi}(0,x), \overline{\theta}(0,x)) = (\overline{\psi}_0(x), \overline{\theta}_0(x)).$$

This becomes by taking the Fourier transform

$$\begin{pmatrix} \widehat\psi \\ \widehat\theta \end{pmatrix}_t = -\begin{pmatrix} 1-\alpha+\alpha\xi^2 & i\xi \\ -i\nu\xi & 1-\alpha+\alpha\xi^2 \end{pmatrix}\begin{pmatrix} \widehat\psi \\ \widehat\theta \end{pmatrix},$$

where $\widehat\psi$ and $\widehat\theta$ are the variables in the wave space, and ξ the frequency. The solution to this set of ordinary differential equations is

$$\begin{cases} \widehat\psi = \dfrac{e^{-(1-\alpha+\alpha\xi^2)t}}{2i\sqrt{\nu}}\left[e^{\sqrt{\nu}\xi t}\left(\widehat\theta_0 + i\sqrt{\nu}\widehat\psi_0\right) - e^{-\sqrt{\nu}\xi t}\left(\widehat\theta_0 - i\sqrt{\nu}\widehat\psi_0\right)\right], \\ \widehat\theta \ = \dfrac{e^{-(1-\alpha+\alpha\xi^2)t}}{2}\left[e^{-\sqrt{\nu}\xi t}\left(\widehat\theta_0 - i\sqrt{\nu}\widehat\psi_0\right) + e^{\sqrt{\nu}\xi t}\left(\widehat\theta_0 + i\sqrt{\nu}\widehat\psi_0\right)\right], \end{cases}$$

with $(\widehat\psi_0, \widehat\theta_0)$ the corresponding initial data in the wave space. Interpreted back to the physical space, the solution is

$$\begin{cases} \bar\psi(x,t) = \dfrac{1}{2i\sqrt{\nu}}\left[K_1(x,t) * (\bar\theta_0 + i\sqrt{\nu}\bar\psi_0) - K_2(x,t) * (\bar\theta_0 - i\sqrt{\nu}\bar\psi_0)\right], \\ \bar\theta(x,t) \ = \frac{1}{2}\left[K_2(x,t) * (\bar\theta_0 - i\sqrt{\nu}\bar\psi_0) + K_1(x,t) * (\bar\theta_0 + i\sqrt{\nu}\bar\psi_0)\right]. \end{cases}$$

Here the kernel functions are

$$\begin{cases} K_1(x,t) = F^{-1}\left[e^{-(1-\alpha+\alpha\xi^2+\sqrt{\nu}\xi)t}\right] \\ \qquad\qquad = \dfrac{1}{\sqrt{4\pi\alpha t}}\exp\left(-(1-\alpha-\frac{\nu}{4\alpha})t\right)\exp\left(-\frac{i\sqrt{\nu}}{2\alpha}x - \frac{x^2}{4\alpha t}\right), \\ K_2(x,t) = F^{-1}\left[e^{-(1-\alpha+\alpha\xi^2-\sqrt{\nu}\xi)t}\right] \\ \qquad\qquad = \dfrac{1}{\sqrt{4\pi\alpha t}}\exp\left(-(1-\alpha-\frac{\nu}{4\alpha})t\right)\exp\left(\frac{i\sqrt{\nu}}{2\alpha}x - \frac{x^2}{4\alpha t}\right). \end{cases} \tag{10}$$

Considering the initial data

$$(\bar\psi, \bar\theta)(x,0) = (\bar\psi_0(x), \bar\theta_0(x)) \to (\psi_\pm, \theta_\pm) \ \text{ as } \ x \to \pm\infty,$$

we may construct the solutions of (9) as

$$\begin{cases} \bar\psi(x,t) = \frac{1}{2i\sqrt{\nu}}\left((\theta_+ - \theta_-)\int_{-\infty}^{x}(K_1(y,t+1) - K_2(y,t+1))dy\right) \\ \qquad +\frac{1}{2}\left((\psi_+ - \psi_-)\int_{-\infty}^{x}(K_1(y,t+1) + K_2(y,t+1))dy + 2e^{-(1-\alpha)t}\psi_-\right), \\ \bar\theta(x,t) = \frac{1}{2}\left((\theta_+ - \theta_-)\int_{-\infty}^{x}(K_1(y,t+1) + K_2(y,t+1))dy + 2e^{-(1-\alpha)t}\theta_-\right) \\ \qquad +\frac{i\sqrt{\nu}}{2}\left((\psi_+ - \psi_-)\int_{-\infty}^{x}(K_1(y,t+1) - K_2(y,t+1))dy\right). \end{cases} \tag{11}$$

We briefly justify the observation that (11) is a solution to system (9): Using $H(x)$ as the unit step function: $H(x) = 1$ if $x > 0$ and $H(x) = 0$ if $x < 0$, we have

$$K_i(x,t) * H(x) = \int_{-\infty}^{\infty} H(y)\cdot K_i(x-y,t)dy = \int_{0}^{\infty} K_i(x-y,t)dy.$$

From the explicit expression of the solution, we find that the system (9) is stable if and only if $0 < \nu < 4\alpha(1-\alpha)$. Moreover, the decay rate of the solution is determined by that of $K_1(x,t)$ and $K_2(x,t)$. In fact, for $i = 1,2$, $p \in [1,+\infty]$, and $t \to +\infty$,

$$\begin{aligned} \|K_i(x,t)\|_{L^p(R)} &= \left\| \frac{1}{\sqrt{4\pi\alpha t}} \exp\left(-(1-\alpha-\tfrac{\nu}{4\alpha})t\right)\exp\left(-\tfrac{x^2}{4\alpha t}\right)\right\|_{L^p(R)} \\ &= O(1)t^{-\frac{1}{2}}e^{-(1-\alpha-\frac{\nu}{4\alpha})t}\left\|e^{-\frac{x^2}{4\alpha t}}\right\|_{L^p(R)} \\ &= O(1)t^{-\frac{1}{2}+\frac{1}{2p}}e^{-(1-\alpha-\frac{\nu}{4\alpha})t}. \end{aligned} \tag{12}$$

Correspondingly, for the higher order derivatives, we have

$$\left\|\partial_x^k K_i\right\|_{L^p(R)} \le Ct^{-\frac{1}{2}+\frac{1}{2p}}e^{-(1-\alpha-\frac{\nu}{4\alpha})t}. \tag{13}$$

Thus, the same optimal decay rate hold for derivatives $\partial_x^k K_i(x,t)$, and so for the derivatives of $(\bar\psi(x,t), \bar\theta(x,t))$.

From the above Lemma 2.1, we have

Lemma 2.2. *The solutions $\bar\psi(x,t)$ and $\bar\theta(x,t)$ to (9) satisfy the properties*

(i) $||\partial_t^l \bar\psi(t)||_{L^\infty} \le Ce^{-(1-\alpha-\frac{\nu}{4\alpha})t}$, $||\partial_t^l \bar\theta(t)||_{L^\infty} \le Ce^{-(1-\alpha-\frac{\nu}{4\alpha})t}$, $l = 0,1,2,\cdots$;

(ii) *for any p with $1 \le p \le +\infty$, it holds that*
$||\partial_t^l\partial_x^k \bar\psi(t)||_{L^p} \le C|\psi_+ - \psi_-|e^{-(1-\alpha-\frac{\nu}{4\alpha})t}(1+t)^{\frac{1}{2p}-\frac{1}{2}}$, $k = 1,2,\cdots$, $l = 0,1,2,\cdots$,
$||\partial_t^l\partial_x^k \bar\theta(t)||_{L^p} \le C|\theta_+ - \theta_-|e^{-(1-\alpha-\frac{\nu}{4\alpha})t}(1+t)^{\frac{1}{2p}-\frac{1}{2}}$, $k = 1,2,\cdots$, $l = 0,1,2,\cdots$.

3. Global Existence of Solutions.

3.1. Reformulation of the problem. Let

$$\begin{cases} u(x,t) = \psi(x,t) - \bar\psi(x,t), \\ v(x,t) = \theta(x,t) - \bar\theta(x,t). \end{cases} \tag{14}$$

Then from (9), we can rewrite problem (1) and (5) as follows,

$$\begin{cases} u_t = -(1-\alpha)u - v_x + \alpha u_{xx} + uu_x + u\bar{\psi}_x + u_x\bar{\psi} + \bar{\psi}\bar{\psi}_x, \\ v_t = -(1-\alpha)v + \nu u_x + \alpha v_{xx} + 2uv_x + 2\bar{\psi}v_x + 2u\bar{\theta}_x + 2\bar{\psi}\bar{\theta}_x, \end{cases} \tag{15}$$

with initial data

$$\begin{cases} u(x,0) = u_0(x) = \psi_0(x) - \bar{\psi}(x,0) \to 0, \quad x \to \pm\infty, \\ v(x,0) = v_0(x) = \theta_0(x) - \bar{\theta}(x,0) \ \to 0, \quad x \to \pm\infty, \end{cases} \tag{16}$$

We seek the solutions of (15), (16) in the set of function $X(0,T)$ defined by

$$X(0,T) = \left\{(u,v)|u,v \in L^\infty(0,T;H^2) \cap L^2(0,T;H^3)\right\}.$$

Now we state our first main results as follows.

Theorem 3.1. *Let $(u_0(x), v_0(x)) \in H^2(R,R^2)$. Furthermore, suppose that both $\delta = |\psi_+ - \psi_-| + |\theta_+ - \theta_-|$ and $\delta_0 = ||u_0||_2^2 + ||v_0||_2^2$ are sufficiently small. Then for any $0 < \alpha < 1$, $\nu < 4\alpha(1-\alpha)$, the Cauchy problem (15), (16) admits a unique global solution $(u(x,t), v(x,t)) \in X(0,T)$ satisfying*

$$||u(t)||_2^2 + ||v(t)||_2^2 + \int_0^t (||u(\tau)||_3^2 + ||v(\tau)||_3^2)d\tau \le C(\delta + \delta_0) \tag{17}$$

and

$$\sup_{x\in R}(|(u,v)(x,t)| + |(u_x,v_x)(x,t)|) \to 0, \quad \text{as} \quad t \to \infty. \tag{18}$$

3.2. Local existence of solutions. In this subsection, we can construct the approximate solution sequences by iteration and obtain the local existence based upon the standard arguments with fixed point principle.

Lemma 3.2 (Local existence). *If $(u_0(x), v_0(x)) \in H^2(R,R^2)$, then there exists t_0 depending only on $||(u_0(x), v_0(x)||_{H^2(R,R^2)}$, such that the Cauchy problem (15), (16) admits a unique smooth solution $(u(x,t), v(x,t)) \in X(0,t_0)$ satisfying*

$$||(u(x,t), v(x,t))||_{H^2(R,R^2)} \le 2||(u_0(x), v_0(x))||_{H^2(R,R^2)}. \tag{19}$$

3.3. Global existence of solutions. Given the local existence result, in order to get the global existence of Cauchy problem (15) and (16), it is sufficient to obtain *a priori* estimates. Specifically, we need to prove that there exists a constant C depending only on $||(u_0(x), v_0(x))||_{H^2(R,R^2)}$ such that any solution $(u,v)(x,t)$ in $X(0,T)$ satisfies $||u_0||_2 + ||v_0||_2 \le C$ for any t in $[0,T]$. Next, we devote ourselves to the estimate of the solution $(u(x,t), v(x,t))$ of (15), (16) under the *a priori* assumption

$$N(T) = \sup_{0<t<T}\left\{\sum_{k=0}^{2} ||\partial_x^k u(t)||^2 + \sum_{k=0}^{2} ||\partial_x^k v(t)||^2\right\} \le \delta_1^2, \tag{20}$$

where $0 < \delta_1 \ll 1$.

By the Sobolev inequality $||f||_{L^\infty} \le ||f||^{\frac{1}{2}}||f_x||^{\frac{1}{2}}$, we have the inequality

$$||(u, u_x, v, v_x)||_{L^\infty} \le \delta_1, \tag{21}$$

which will be used later.

Moreover, for any $\bar\nu$, if $\nu < \bar\nu < 4\alpha(1-\alpha)$, we can find $c_0 > 0$ such that

$$\begin{cases} c_0 - \dfrac{1}{\bar\nu} > 0, \\ 1 - \dfrac{c_0\nu^2}{\bar\nu} > 0. \end{cases} \tag{22}$$

Choosing $c_0 = \frac{1}{2}\left(\frac{1}{\bar\nu} + \frac{\bar\nu}{\nu^2}\right)$, one can easily verify that c_0 satisfy (22).

What follows will be a series of Lemmas leading to our desired estimates.

Lemma 3.3. *Suppose that the assumptions in Theorem 3.1 hold and $(u(x,t), v(x,t))$ is a solution to (15), (16) obtained in Lemma 3.2, then it holds that for any $\nu < 4\alpha(1-\alpha)$*

$$\int_R (u^2+v^2)dx + \int_0^t\int_R (u^2+v^2)dxd\tau + \int_0^t\int_R (u_x^2+v_x^2)dxd\tau \le C(\delta+\delta_0), \tag{23}$$

provided δ and δ_1 are sufficiently small.

Proof. Multiplying the first equation of (15) by $2u$ and the second equation of (15) by $2c_0v$ and integrating the resulting identity over $R\times(0,t)$, we obtain by Cauchy-Schwartz inequality

$$\begin{aligned}
&\int_R (u^2+c_0v^2)dx + 2(1-\alpha)\int_0^t\int_R (u^2+c_0v^2)dxd\tau + 2\alpha\int_0^t\int_R (u_x^2+c_0v_x^2)dxd\tau \\
&= ||u_0||^2 + c_0||v_0||^2 - 2\int_0^t\int_R uv_x dxd\tau + 2\nu c_0\int_0^t\int_R vu_x dxd\tau + 2\int_0^t\int_R u^2u_x dxd\tau \\
&\quad -2c_0\int_0^t\int_R u_xv^2 dxd\tau - 2c_0\int_0^t\int_R \bar\psi_x v^2 dxd\tau + 4c_0\int_0^t\int_R \bar\theta_x uv dxd\tau \\
&\quad +2c_0\int_0^t\int_R v\bar\psi\bar\theta_x dxd\tau + 2\int_0^t\int_R u\bar\psi\bar\psi_x dxd\tau + \int_0^t\int_R \bar\psi_x u^2 dxd\tau \\
&\le (1+c_0)\delta_0 + \frac{\bar\nu}{2\alpha}\int_0^t\int_R u^2 dxd\tau + \frac{2\alpha}{\bar\nu}\int_0^t\int_R v_x^2 dxd\tau + \frac{2c_0\nu^2\alpha}{\bar\nu}\int_0^t\int_R u_x^2 dxd\tau \\
&\quad + \frac{c_0\bar\nu}{2\alpha}\int_0^t\int_R v^2 dxd\tau + (2||u_x||_{L^\infty} + ||\bar\psi_x||_{L^\infty})\int_0^t\int_R u^2 dxd\tau \\
&\quad +2c_0(||u_x||_{L^\infty} + ||\bar\psi_x||_{L^\infty})\int_0^t\int_R v^2 dxd\tau + 2c_0||\bar\theta_x||_{L^\infty}\int_0^t\int_R (u^2+v^2)dxd\tau \\
&\quad +c_0\delta\int_0^t\int_R v^2 dxd\tau + \frac{c_0}{\delta}\int_0^t\int_R (\bar\psi\bar\theta_x)^2(x,\tau)dxd\tau \\
&\quad +\delta\int_0^t\int_R u^2 dxd\tau + \frac{1}{\delta}\int_0^t\int_R (\bar\psi\bar\psi_x)^2(x,\tau)dxd\tau.
\end{aligned} \tag{24}$$

Employing Lemma 2.3 and (21), we have from the above inequality

$$\begin{aligned}
&\int_R (u^2+c_0v^2)dx + \Big\{2(1-\alpha-\frac{\bar\nu}{4\alpha}) - C\delta - C\delta_1\Big\}\int_0^t\int_R u^2dxd\tau \\
&+\Big\{2\alpha-\frac{2c_0\nu^2\alpha}{\bar\nu}\Big\}\int_0^t\int_R u_x^2dxd\tau + \Big\{2(1-\alpha-\frac{\bar\nu}{4\alpha}) - C(\delta_1+\delta)\Big\}\int_0^t\int_R c_0v^2dxd\tau \\
&+\Big\{2c_0\alpha-\frac{2\alpha}{\bar\nu}\Big\}\int_0^t\int_R v_x^2dxd\tau \\
&\le C\delta_0 + \frac{c_0}{\delta}\int_0^t\int_R(\bar\psi\bar\theta_x)^2(x,\tau)dxd\tau + \frac{1}{\delta}\int_0^t\int_R(\bar\psi\bar\psi_x)^2(x,\tau)dxd\tau,
\end{aligned} \tag{25}$$

Now we estimate the last two terms on the right side of inequality (25). From Lemma 2.3, we have by the Cauchy-Schwartz inequality

$$\begin{aligned}
\frac{1}{\delta}\int_0^t\int_R(\bar\psi\bar\psi_x)^2(x,\tau)dxd\tau + \frac{c_0}{\delta}\int_0^t\int_R(\bar\psi\bar\theta_x)^2(x,\tau)dxd\tau &\le \frac{C}{\delta}\int_0^t\int_R\left(\bar\psi_x^2+\bar\theta_x^2\right)dxd\tau \\
&\le C\delta.
\end{aligned} \tag{26}$$

Thus, (25) and (26) give

$$\begin{aligned}
&\int_R (u^2+c_0v^2)dx + \Big\{2(1-\alpha-\frac{\bar\nu}{4\alpha}) - C\delta - C\delta_1\Big\}\int_0^t\int_R u^2dxd\tau \\
&\quad+\Big\{2\alpha-\frac{2c_0\nu^2\alpha}{\bar\nu}\Big\}\int_0^t\int_R u_x^2dxd\tau \\
&\quad+\Big\{2(1-\alpha-\frac{\bar\nu}{4\alpha}) - C(\delta_1+\delta)\Big\}\int_0^t\int_R c_0v^2dxd\tau \\
&\quad+\Big\{2c_0\alpha-\frac{2\alpha}{\bar\nu}\Big\}\int_0^t\int_R v_x^2dxd\tau \\
&\le C(\delta_0+\delta),
\end{aligned} \tag{27}$$

which implies with the help of (22)

$$\int_R(u^2+v^2)dx + \int_0^t\int_R(u^2+v^2)dxd\tau + \int_0^t\int_R(u_x^2+v_x^2)dxd\tau \le C(\delta_0+\delta), \tag{28}$$

provided δ and δ_1 are sufficiently small. This proves Lemma 3.3. □

We multiply the first equation of (15) by $(-2u_{xx})$ and the second equation of (15) by $(-2c_0v_{xx})$ respectively, and add the resulting equations together. Similarly, we obtain

Lemma 3.4. *Let the assumptions of Theorem 3.1 hold. Then the solution* $(u(x,t),v(x,t))$ *of (15), (16) obtained in Lemma 3.2 satisfies for any* $\nu < 4\alpha(1-\alpha)$

$$\int_R(u_x^2+v_x^2)dx + \int_0^t\int_R(u_{xx}^2+v_{xx}^2)dxd\tau \le C(\delta+\delta_0), \tag{29}$$

provided δ *and* δ_1 *are sufficiently small.*

Lemma 3.5. *If* $(u(x,t),v(x,t))$ *is a solution to (15), (16) obtained in Lemma 3.2 under the assumptions in Theorem 3.1, then for any* $\nu < 4\alpha(1-\alpha)$*, we have*

$$\int_R(u_{xx}^2+v_{xx}^2)dx + \int_0^t\int_R(u_{xxx}^2+v_{xxx}^2)dxd\tau \le C(\delta+\delta_0), \tag{30}$$

provided δ *and* δ_1 *are sufficiently small.*

Remark. Differentiating (15) twice with respect to x, multiplying the results by $2u_{xx}$ and $2c_0v_{xx}$, respectively, integrating the resulting equation with respect to (x,t) over $R\times(0,t)$, we obtain (30) by inequality technics applied in Lemma 3.3.

Thus the combination of Lemma 3.3, Lemma 3.4 and Lemma 3.5 implies (17).

Finally, we have to show that the *a priori* assumption (20) can be closed. Since, under the *a priori* assumption (20), we deduced that (17) holds provided δ and δ_1 are sufficiently small, assumption (20) is always true provided δ and δ_0 are sufficiently small.

Next, we turn to show that (18) is true. To do this, we introduce the following Lemma.

Lemma 3.6. *If $g(t)\geq 0,\ g(t)\in L^1(0,\infty)$ and $g'(t)\in L^1(0,\infty)$, then $g(t)\to 0$ as $t\to\infty$.*

We can derive (18) from Lemma 3.6. Details are may be referred to [13].
Thus the proof of Theorem 3.1 is completed.

4. **Decay Rates of Solutions.**

4.1. Basic L^2 decay estimates. In this section, we will study the decay rates of solutions to the Cauchy problem (15), (16) under the *a priori* assumption

$$\sum_{k=0}^{2}||\partial_x^k u(t)||^2+\sum_{k=0}^{2}||\partial_x^k v(t)||^2\leq e^{-At},\quad 0<t<T. \tag{31}$$

Where

$$A:=2(1-\alpha-\frac{\bar\nu}{4\alpha}), \tag{32}$$

here $\bar\nu$ is any fixed constant ($\nu<\bar\nu<4\alpha(1-\alpha)$), and related parameters are defined by (22).

By Sobolev's inequality, we have from (31)

$$||(u,u_x,v,v_x)||_{L^\infty}\leq e^{-\frac{A}{2}t}, \tag{33}$$

which will be used later.

Moreover, we recall the following Gronwall's inequality which will be used in the sequel.

Now we can state the main results of decay rates of solutions.

Theorem 4.2. *Suppose that $(u(x,t),v(x,t))$ is a solution to problem (15), (16) under the assumptions imposed in Theorem 3.1, then for $\nu<4\alpha(1-\alpha)$, we have for any $t\in[0,T]$*

$$||\partial_x^k u(t)||^2+||\partial_x^k v(t)||^2\leq C(\delta+\delta_0)^{\frac14}e^{-At},\ k=0,1,2, \tag{34}$$

provided δ and δ_0 are sufficiently small, where A is defined by (32).

Proof. The parts of the proof are divided into three steps.

First, from (15)$_1 \times 2u$ + (15)$_2 \times 2c_0 v$ and by integrating the resulting identities over $x \in R$, we reach by the Cauchy-Schwartz inequality from Lemma 2.3

$$\begin{aligned}
&\frac{d}{dt}\int_R (u^2+c_0v^2)dx + 2(1-\alpha)\int_R (u^2+c_0v^2)dx + 2\alpha\int_R (u_x^2+c_0v_x^2)dx \\
&= -2\int_R uv_x dx + 2\nu c_0\int_R vu_x dx - 2c_0\int_R (u_x+\bar{\psi}_x)v^2 dx + \int_R (u_x+\bar{\psi}_x)u^2 dx \\
&\quad +4c_0\int_R \bar{\theta}_x uv dx + 2c_0\int_R v\bar{\psi}\bar{\theta}_x dx + \int_R (\bar{\psi}\bar{\psi}_x)u dx \\
&\le \frac{\bar{\nu}}{2\alpha}\int_R u^2 dx + \frac{2\alpha}{\bar{\nu}}\int_R v_x^2 dx + \frac{2c_0\nu^2\alpha}{\bar{\nu}}\int_R u_x^2 dx + \frac{c_0\bar{\nu}}{2\alpha}\int_R v^2 dx \\
&\quad +2c_0||\bar{\theta}_x||_{L^\infty}\int_R u^2 dx + 2c_0(||\bar{\psi}_x||_{L^\infty} + ||\bar{\theta}_x||_{L^\infty})\int_R v^2 dx \\
&\quad -2c_0\int_R u_x v^2 dx + 2c_0\int_R v\bar{\psi}\bar{\theta}_x dx + ||\bar{\psi}_x||_{L^\infty}\int_R u^2 dx \\
&\quad +\int_R (\bar{\psi}\bar{\psi}_x)u dx + \int_R u_x u^2 dx.
\end{aligned} \tag{35}$$

After rearranging the terms, we have by Lemma 2.2

$$\begin{aligned}
&\frac{d}{dt}\int_R (u^2+c_0v^2)dx + 2(1-\alpha-\frac{\bar{\nu}}{4\alpha})\int_R u^2 dx + 2(1-\alpha-\frac{\bar{\nu}}{4\alpha})\int_R c_0v^2 dx \\
&\quad +\left\{2\alpha - \frac{2c_0\nu^2\alpha}{\bar{\nu}}\right\}\int_R u_x^2 dx + \left\{2c_0\alpha - \frac{2\alpha}{\bar{\nu}}\right\}\int_R v_x^2 dx \\
&\le C\delta e^{-(1-\alpha-\frac{\nu}{4\alpha})t}\int_R (u^2+c_0v^2)dx - 2c_0\int_R u_x v^2 dx + 2c_0\int_R v\bar{\psi}\bar{\theta}_x dx \\
&\quad +\int_R (\bar{\psi}\bar{\psi}_x)u dx.
\end{aligned} \tag{36}$$

Next, we will estimate the terms on the right side of (41).

We get from (17) by Sobolev's inequality

$$||(u,u_x,v,v_x)||_{L^\infty} \le C(\delta+\delta_0)^{\frac{1}{2}}. \tag{37}$$

Thus we derive by the Cauchy-Schwartz inequality from (31), (33) and (37)

$$\begin{aligned}
-2c_0\int_R u_x v^2 dx &\le c_0\left(\int_R u_x^2|v|dx + \int_R |v|^3 dx\right) \\
&\le c_0||v||_{L^\infty}\int_R (u_x^2+v^2)dx \\
&= c_0||v||_{L^\infty}^{\frac{1}{2}}||v||_{L^\infty}^{\frac{1}{2}}\int_R (u_x^2+v^2)dx \\
&\le C(\delta+\delta_0)^{\frac{1}{4}}e^{-\frac{5A}{4}t}.
\end{aligned} \tag{38}$$

Also

$$\int_R u_x u^2 dx \le (\delta+\delta_0)^{\frac{1}{4}}e^{-\frac{5A}{4}t}. \tag{39}$$

In addition, we deduce from Lemma 2.3 and the Cauchy-Schwartz inequality

$$
\begin{aligned}
2c_0\int_R v\bar{\psi}\bar{\theta}_x dx &\le \delta\|\bar{\psi}\|_\infty(\int_R v^2dx+\int_R \bar{\theta}_x^2dx)\\
&\le \delta e^{-(1-\alpha-\frac{\nu}{4\alpha})t}e^{-At}+\frac{C}{\delta}e^{-(1-\alpha-\frac{\nu}{4\alpha})t}\int_R\bar{\theta}_x^2dx\\
&\le \delta e^{-(1-\alpha-\frac{\nu}{4\alpha})t}e^{-At}+\frac{C}{\delta}e^{-(1-\alpha-\frac{\nu}{4\alpha})t}\delta^2e^{-2(1-\alpha-\frac{\nu}{4\alpha})t}\\
&\le C\delta e^{\{-A+(1-\alpha-\frac{\nu}{4\alpha})\}t}. \qquad (40)
\end{aligned}
$$

Also

$$
\int_R(\bar{\psi}\bar{\psi}_x)udx\le C\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t} \qquad (41)
$$

Thus, we get from (41)-(40) and (35)

$$
\begin{aligned}
&\frac{d}{dt}\int_R(u^2+c_0v^2)dx+A\int_R(u^2+c_0v^2)dx\\
&\le C\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t}+C\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t}+C(\delta+\delta_0)^{\frac14}e^{-\frac{5A}{4}t}\\
&\le C(\delta+\delta_0)^{\frac14}e^{-\frac{5A}{4}t}+C\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t}. \qquad (42)
\end{aligned}
$$

Recalling the definition (32) of A, we have from (42)

$$
\begin{aligned}
&\frac{d}{dt}\int_R(u^2+c_0v^2)dx+A\int_R(u^2+c_0v^2)dx\\
&\le C(\delta+\delta_0)^{\frac14}e^{-\frac{5A}{4}t}+C\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t}. \qquad (43)
\end{aligned}
$$

Noticing that $\frac{A}{2}<1-\alpha-\frac{\nu}{4\alpha}$, we obtain from Lemma 4.1

$$
\begin{aligned}
&\int_R(u^2+c_0v^2)dx\\
&\le\left\{C\delta_0+C(\delta+\delta_0)^{\frac14}\int_0^t e^{-\frac{5A}{4}\tau}e^{A\tau}d\tau+C\delta\int_0^t e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}\tau}e^{A\tau}d\tau\right\}e^{-At}\\
&\le C(\delta+\delta_0)^{\frac14}e^{-At}, \qquad (44)
\end{aligned}
$$

which implies (4.4) for $k=0$.

Similarly, for $k=1$, if we multiply $(15)_1$ by $(-2u_{xx})$ and $(15)_2$ by $(-2c_0v_{xx})$, add and integrate the resulting identities over $x\in R$. We get by applying the same procedure to the proofs of inequality (44)

$$
\int_R(u_x^2+c_0v_x^2)dx\le C(\delta+\delta_0)^{\frac14}e^{-At}, \qquad (45)
$$

which proves (57) for $k=1$.

Finally, we show that (57) is true for $k=2$. In fact, by differentiating (15) twice with respect to x, multiplying the results by $2u_{xx}$ and $2c_0v_{xx}$, respectively, integrating the resulting equation over $x\in R$, we derive (57).

The proof of Theorem 4.2 is completed. □

Finally, we verify that the *a priori* assumption (31)is reasonable. Indeed, under this *a priori* assumption, we showed (57) holds. Therefore, the assumption (31) is always true provided δ and δ_0 are sufficiently small.

4.2. Improved L^2 decay estimates. Next, we may further sharpen the decay estimates by energy integral again.

First, $(15)_1 \times 2u(1+t)^{\frac{1}{2}} + (15)_2 \times 2c_0 v(1+t)^{\frac{1}{2}}$ and integrating the resulting identities over $x \in R$, adding the term $\frac{1}{2}(1+t)^{-\frac{1}{2}}\int_R (u_x^2 + c_0 v_x^2)dx$, we reach by Cauchy-Schwartz inequality from Lemma 2.3 and (45),

$$\begin{aligned}
&\frac{d}{dt}\{(1+t)^{\frac{1}{2}}\int_R (u^2+c_0v^2)dx\} + 2(1-\alpha)(1+t)^{\frac{1}{2}}\int_R (u^2+c_0v^2)dx \\
&+2\alpha(1+t)^{\frac{1}{2}}\int_R (u_x^2+c_0v_x^2)dx \\
=& (1+t)^{\frac{1}{2}}\{2\nu c_0\int_R vu_x dx - 2\int_R uv_x dx - 2c_0\int_R (u_x+\bar\psi_x)v^2dx \\
&+\int_R (u_x+\bar\psi_x)u^2dx + 4c_0\int_R \bar\theta_x uvdx + 2c_0\int_R v\bar\psi\bar\theta_x dx + 2\int_R u\bar\psi\bar\psi_x dx\} \\
\le& (1+t)^{\frac{1}{2}}\{\frac{\bar\nu}{2\alpha}\int_R u^2dx + \frac{2\alpha}{\bar\nu}\int_R v_x^2dx + \frac{2c_0\nu^2\alpha}{\bar\nu}\int_R u_x^2dx + \frac{c_0\bar\nu}{2\alpha}\int_R v^2dx \\
&+2c_0||\bar\theta_x||_{L^\infty}\int_R u^2dx + 2c_0(||\bar\psi_x||_{L^\infty}+||\bar\theta_x||_{L^\infty})\int_R v^2dx - 2c_0\int_R u_xv^2dx \\
&+(||\bar\psi_x||_{L^\infty}+||u_x||_{L^\infty})\int_R u^2dx + 2c_0\int_R v\bar\psi\bar\theta_x dx\} + C(1+t)^{-\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-At}.
\end{aligned} \tag{46}$$

After shuffling the terms like (41)-(42), we have by Lemma 2.2

$$\begin{aligned}
&\frac{d}{dt}\{(1+t)^{\frac{1}{2}}\int_R (u^2+c_0v^2)dx\} + A(1+t)^{\frac{1}{2}}\int_R (u^2+c_0v^2)dx \\
\le& C(1+t)^{\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-\frac{5A}{4}t} + C(1+t)^{\frac{1}{2}}\delta e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}t} \\
&+ C(1+t)^{-\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-At}.
\end{aligned} \tag{47}$$

Noticing that $\frac{A}{2} < 1-\alpha-\frac{\nu}{4\alpha}$, we obtain from Lemma 4.1

$$\begin{aligned}
&(1+t)^{\frac{1}{2}}\int_R (u^2+c_0v^2)dx \\
\le& \{C\delta_0 + C(\delta+\delta_0)^{\frac{1}{4}}\int_0^t (1+\tau)^{\frac{1}{2}}e^{-\frac{5A}{4}\tau}e^{A\tau}d\tau \\
&+C\delta\textstyle\int_0^t(1+\tau)^{\frac{1}{2}}e^{-\{A+(1-\alpha-\frac{\nu}{4\alpha})\}\tau}e^{A\tau}d\tau + C\displaystyle\int_0^t (1+\tau)^{\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-A\tau}d\tau\}e^{-At} \\
\le& C(\delta+\delta_0)^{\frac{1}{4}}e^{-At},
\end{aligned} \tag{48}$$

which implies

$$\int_R (u^2+c_0v^2)dx \le C(1+t)^{-\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-At}, \tag{49}$$

We may repeat the above argument, easily we obtain the following Theorem

Theorem 4.3. *Let the assumptions imposed in Theorem 4.3 hold, then we have A defined by (32), for any $t \in R$*

$$||\partial_x^k u(t)||^2 + ||\partial_x^k v(t)||^2 \le C(1+t)^{-\frac{1}{2}}(\delta+\delta_0)^{\frac{1}{4}}e^{-At},\ k=0,1,2, \tag{50}$$

.

Compared with the kernel estimates in (12), we would like to mention in this paper the initial data $(u_0(x), v_0(x)) \in H^2(R, R^2)$. Correspondingly, we may establish the optimal L^p estimates as follows by using the kernel functions (10). A easy application of Sobolev inequality for L^∞

$$||\partial_x^k u(t)||_{L^\infty} + ||\partial_x^k v(t)||_{L^\infty} \le C(1+t)^{-\frac{1}{4}}(\delta+\delta_0)^{\frac{1}{8}} e^{-\frac{At}{2}},\ k = 0, 1, \tag{51}$$

Furthermore, we observe the Soboev interpolation interpolation for L^p space: Let $p \in [2, \infty)$ then $\|f\|_p \le \|f\|_2^{\frac{2}{p}} \|f\|_\infty^{\frac{p-2}{p}}$. Applying this inequality in our case yields

Theorem 4.3. *Let conditions in Theorem 3.1 hold, the Cauchy problem (1) admits a unique global solution $(u(x,t), v(x,t)) \in X(0,T)$, for any $p \ge 2$, satisfying*

$$||\frac{\partial^k}{\partial x^k}(\psi(t,x), \theta(t,x))||_{L^p(R,R^2)} \le C(\epsilon, k) t^{-\frac{1}{4}+\frac{1}{2p}} e^{-\frac{At}{2}}. \tag{52}$$

Similar results obtained through Kernel function (10) may be referred to [1, 8].

5. **Estimates of Higher Order derivatives.** Differentiating (15) m times with respect to x, multiplying the results by $2\frac{\partial^m u(t,x)}{\partial x^m}$ and $2c_0\frac{\partial^m v(t,x)}{\partial x^m}$, respectively, integrating the equation over $x \in R$, and using the Cauchy-Schwartz inequality with proper coefficients as was chosen in the derivation of (35). For example $m = 3$, we integrate the resulting inequality with respect to t over (ϵ, t),

$$\left[\frac{\partial^m u(t,x)}{\partial x^m}\right]^2 + \left[\frac{\partial^m v(t,x)}{\partial x^m}\right]^2 \le Ce^{-At}, \tag{53}$$

By performing mathematical induction from, also further sharpening the estimates as done in (46-49) to obtain the decay factor $(1+t)^{-\frac{1}{2}}$, we extend Theorem 4.3 as follows,

Theorem 5.1. *Suppose that $(u(x,t), v(x,t))$ is a solution to problem (15), (16) under the assumptions imposed in Theorem 3.1. Then when $\nu < 4\alpha(1-\alpha)$, we have*

$$||\partial_x^k u(t)||^2 + ||\partial_x^k v(t)||^2 \le C(1+t)^{-\frac{1}{2}} e^{-\bar{A}t},\ k \ge 3 \tag{54}$$

provided δ and δ_0 are sufficiently small. Here

$$\bar{A} := 2(1 - \alpha - \frac{\nu}{4\alpha}). \tag{55}$$

As above, for any $\bar{\nu}$, if $\nu < \bar{\nu} < 4\alpha(1-\alpha)$, with $A := 2(1 - \alpha - \frac{\bar{\nu}}{4\alpha})$ defined in (32), we obtain Theorem 4.2 and Theorem 5.1, i.e.

$$||\partial_x^k u(t)||^2 + ||\partial_x^k v(t)||^2 \le C(1+t)^{-\frac{1}{2}} e^{-At},\ k \ge 0. \tag{56}$$

Observing the constant C may be chosen independent of $\bar{\nu}$, then we may sharpen the result

$$||\partial_x^k u(t)||^2 + ||\partial_x^k v(t)||^2 \le C(1+t)^{-\frac{1}{2}} e^{-\bar{A}t},\ k \ge 0. \tag{57}$$

We thus obtain an improvement of the results in Theorem 5.1, i.e, we replace A by $\bar{A}$, which leads to an optimal decay estimates in the L^2 sense as compared to [11] for the system with small L^2 initial data.

6. **Concluding remarks.** Theorem 3.1 and Theorem 4.2 on (u, v) are expressed back to a theorem in terms of the original variables (ψ, θ) by (14). Directly from Theorem 5.1 and Lemma 2.3, we have our main theorem:

Theorem 6.1. *Let $(u_0(x), v_0(x)) \in H^2(R, R^2)$. Furthermore, Suppose that both $\delta = |\psi_+ - \psi_-| + |\theta_+ - \theta_-|$ and $\delta_0 = ||u_0||_2^2 + ||v_0||_2^2$ are sufficiently small. Then for any $0 < \alpha < 1$, $\nu < 4\alpha(1-\alpha)$, the Cauchy problem (E), (I) admits a unique global solution $(\psi(x,t), \theta(x,t)) \in X(0,T)$ satisfying*

$$||\partial_x^k \psi(t)||_2^2 + ||\partial_x^k \theta(t)||_2^2 \leq C(1+t)^{-\frac{1}{2}} e^{-2(1-\alpha-\frac{\nu}{4\alpha})t}, \ k \geq 1, \tag{58}$$

and

$$||\partial_t^l \psi(t)||_{L^\infty} + ||\partial_t^l \theta(t)||_{L^\infty} \leq C e^{-(1-\alpha-\frac{\nu}{4\alpha})t} \quad l = 0, 1, 2, \cdots \tag{59}$$

To conclude, we obtain the global existence and its decay estimates for system (E) by splitting the solution of (E) as its decay profile (9), which is a better asymptotic profile, plus the perturbation system (15). Even the analysis is based on energy integral without considering the linearized structure of system (E), we still obtain the optimal decay rates in the L^2 sense, which is also an improvement result in [13].

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