

MODELING THE INTRA-VENOUS GLUCOSE TOLERANCE
TEST: A GLOBAL STUDY FOR A
SINGLE-DISTRIBUTED-DELAY MODEL

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ABSTRACT. The Intra Venous Glucose Tolerance Test (IVGTT) is a simple and established experimental procedure in which a challenge bolus of glucose is administered intra-venously and plasma glucose and insulin concentrations are then frequently sampled. The modeling of the measured concentrations has the goal of providing information on the state of the subject's glucose/insulin control system: an open problem is to construct a model representing simultaneously the entire control system with a physiologically believable qualitative behavior. A previously published single-distributed-delay differential model was shown to have desirable properties (positivity, boundedness, global stability of solutions) under the hypothesis of a specific, square-wave delay integral kernel. The present work extends the previous results to a family of models incorporating a generic non-negative, square integrable normalized kernel. Every model in this family describes the rate of glucose concentration variation as due to both insulin-dependent and insulin-independent net glucose tissue uptake, as well as to constant liver glucose production. The rate of variation of plasma insulin concentration depends on insulin catabolism and on pancreatic insulin secretion. Pancreatic insulin secretion at time t is assumed to depend on the earlier effects of glucose concentrations, up to time t (distributed delay). We consider a non-negative, square integrable normalized weight function ω on $R^+ = [0, \infty)$ as the fraction of maximal pancreatic insulin secretion at a given glucose concentration. No change in local asymptotic stability is introduced by the time delay. Considering an appropriate Lyapunov functional, it is found that the system is globally asymptotically stable if the average time delay has a parameter-dependent upper bound. An example of good model fit to experimental data is shown using a specific delay kernel.

1. Introduction. The homeostasis of glucose, involving the secretion of its controlling hormone insulin by the pancreas, has been the object of several mathematical models over the past thirty years ([1]-[13]). One of the goals of this modeling

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