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A COMBINATORIAL CLASSIFICATION OF POSTSINGULARLY FINITE COMPLEX EXPONENTIAL MAPS

Dedicated to the memory of Des Sheiham,
our inspiring instructor, valued colleague and great friend

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Abstract. We give a combinatorial classification of postsingularly finite exponential maps in terms of external addresses starting with the entry 0. This extends the classification results for critically preperiodic polynomials [2] to exponential maps. Our proof relies on the topological characterization of postsingularly finite exponential maps given recently in [14]. These results illustrate once again the fruitful interplay between combinatorics, topology and complex structure which has often been successful in complex dynamics.

1. **Introduction.** We study the dynamical systems given by iteration of exponential maps $z \mapsto E_\lambda(z) := \lambda \exp(z)$ for non-zero complex parameters λ. The family of exponential maps is the simplest family of transcendental entire functions and has been investigated by many people (see for example [1, 3, 8]), often in analogy to quadratic polynomials as the simplest family of algebraic entire functions.

The dynamics of iterated holomorphic functions $f\colon \mathbb{C} \to \mathbb{C}$ is determined to a large extent by the dynamics of the singular values: these are values $a \in \mathbb{C}$ which have no neighborhood $U \subset \mathbb{C}$ so that f is an unramified covering over U. For polynomials, singular values are critical values. For transcendental functions, singular values can also be asymptotic values, or limit points of critical or asymptotic values. The exponential family is special because it has only one asymptotic value, just like unicritical polynomials (those conjugate to $z \mapsto z^d + c$) have only one critical value.

In any family of iterated holomorphic functions $f_\lambda\colon \mathbb{C} \to \mathbb{C}$, the easiest maps to understand are usually those for which all singular values have finite orbits, i.e. the singular orbits are periodic or preperiodic; such maps are called *postsingularly finite*

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