

INFIMUM OF THE METRIC ENTROPY OF HYPERBOLIC ATTRACTORS WITH RESPECT TO THE SRB MEASURE

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ABSTRACT. Let M^n be a compact C^∞ Riemannian manifold of dimension $n \geq 2$. Let $\text{Diff}^r(M^n)$ be the space of all C^r diffeomorphisms of M^n , where $1 < r \leq \infty$. For a C^r diffeomorphism f in $\text{Diff}^r(M^n)$ with a hyperbolic attractor Λ_f on which f is topologically transitive, let $U(f)$ be the C^1 open set of $\text{Diff}^r(M^n)$ such that each element in $U(f)$ can be connected to f by finitely many C^1 structural stability balls in $\text{Diff}^r(M^n)$. Then by the structural stability, any element g in $U(f)$ has a hyperbolic attractor Λ_g and $g|_{\Lambda_g}$ is topologically conjugate to $f|_{\Lambda_f}$. Therefore, the topological entropy $h(g|_{\Lambda_g})$ is a constant function when it is restricted to $U(f)$. However, the metric entropy $h_\mu(g)$ with respect to the SRB measure $\mu = \mu_g$ can vary. We prove that the infimum of the metric entropy $h_\mu(g)$ on $U(f)$ is zero.

0. Introduction. Let M^n be a compact C^∞ Riemannian manifold of dimension $n \geq 2$, and $\text{Diff}^r(M^n)$ be the space of all C^r -diffeomorphisms of M^n , $1 < r \leq \infty$. Suppose $f \in \text{Diff}^r(M^n)$ has a hyperbolic attractor Λ_f . Consider an open set $U(f) \subset \text{Diff}^r(M^n)$ in C^1 -topology consisting of C^r diffeomorphisms g that has a hyperbolic attractor Λ_g topologically conjugate to Λ_f . (The precise definition of $U(f)$ is given in Section 1.) Since the hyperbolic attractors of any two elements in $U(f)$ are topologically conjugate, the topological entropy $h(g|_{\Lambda_g})$ is a constant function restricted to $U(f)$. On the other hand, every $g \in U(f)$ has an SRB measure μ_g on Λ_g . The metric entropy $h_{\mu_g}(g)$ can vary in $U(f)$. It has been shown that the dependence of μ_g on the map g is smooth when the maps involved have a higher

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