

GENERIC POINTS IN SYSTEMS OF SPECIFICATION AND BANACH VALUED BIRKHOFF ERGODIC AVERAGE

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(Communicated by Lan Wen)

ABSTRACT. We prove that systems satisfying the specification property are saturated in the sense that the topological entropy of the set of generic points of any invariant measure is equal to the measure-theoretic entropy of the measure. We study Banach valued Birkhoff ergodic averages and obtain a variational principle for its topological entropy spectrum. As application, we examine a particular example concerning with the set of real numbers for which the frequencies of occurrences in their dyadic expansions of infinitely many words are prescribed. This relies on our explicit determination of a maximal entropy measure.

1. Introduction. By dynamical system (X, T) , we mean a continuous transformation $T : X \rightarrow X$ on a compact metric space X with metric d . We shall adopt the notion of topological entropy introduced by Bowen ([9], recalled in the section 2), denoted by h_{top} , to describe the sizes of sets in X . We denote by \mathcal{M}_{inv} the set of all T -invariant probability Borel measures on X and by \mathcal{M}_{erg} its subset of all ergodic measures. The measure-theoretic entropy of μ in \mathcal{M}_{inv} is denoted by h_μ .

Let us first recall some notions like generic points, saturated property and the specification property which are quite known in dynamical systems nowadays.

For $\mu \in \mathcal{M}_{\text{inv}}$, the set G_μ of μ -generic points is defined by

$$G_\mu := \left\{ x \in X : \frac{1}{n} \sum_{j=0}^{n-1} \delta_{T^j x} \xrightarrow{w^*} \mu \right\},$$

2000 *Mathematics Subject Classification.* Primary: 37B40, 37B45; Secondary: 37A05.

Key words and phrases. Generic points, Systems of specification, Topological entropy, Variational principle.