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THE CAUCHY PROBLEM FOR 1D COMPRESSIBLE FLOWS WITH DENSITY-DEPENDENT VISCOSITY COEFFICIENTS

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Abstract. This paper concerns with Cauchy problems for the one-dimensional compressible Navier-Stokes equations with density-dependent viscosity coefficients. Two cases are considered here, first, the initial density is assumed to be integrable on the whole real line. Second, the deviation of the initial density from a positive constant density is integrable on the whole real line. It is proved that for both cases, weak solutions for the Cauchy problem exist globally in time and the large time asymptotic behavior of such weak solutions are studied. In particular, for the second case, the phenomena of vanishing of vacuum and blow-up of the solutions are presented, and it is also shown that after the vanishing of vacuum states, the globally weak solution becomes a unique strong one. The initial vacuum is permitted and the results apply to the one-dimensional Saint-Venant model for shallow water.

1. **Introduction.** Consider the one-dimensional (1D) compressible Navier-Stokes equations with density-dependent viscosity coefficients

$$\rho_t + (\rho u)_x = 0, \tag{1.1}$$

$$(\rho u)_t + (\rho u^2 + P(\rho))_x = (\mu(\rho)u_x)_x. \tag{1.2}$$

Here $\rho(x,t), u(x,t)$ and $P(\rho) = \rho^\gamma (\gamma \geq 1)$ stand for the fluid density, velocity and pressure respectively. For simplicity, the viscosity coefficient $\mu(\rho)$ is assumed to be $\mu(\rho) = \rho^\alpha$ with $\alpha > \frac{1}{2}$. The initial data is imposed as

$$(\rho, \rho u)|_{t=0} = (\rho_0, m_0). \tag{1.3}$$

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