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DISTANCE ENTROPY OF DYNAMICAL SYSTEMS ON NONCOMPACT-PHASE SPACES

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Abstract. Let X be a separable metric space not necessarily compact, and let $f\colon X \to X$ be a continuous transformation. From the viewpoint of Hausdorff dimension, the authors improve Bowen's method to introduce a dynamical quantity distance entropy, written as $ent_{\mathtt{H}}(f;Y)$, for f restricted on any given subset Y of X; but it is essentially different from Bowen's entropy(1973). This quantity has some basic properties similar to Hausdorff dimension and is beneficial to estimating Hausdorff dimension of the dynamical system. The authors show that if f is a local lipschitzian map with a lipschitzian constant ℓ then $ent_{\mathtt{H}}(f;Y) \le \max\{0, \mathtt{HD}(Y)\log\ell\}$ for all $Y \subset X$; if f is locally expanding with skewness λ then $ent_{\mathtt{H}}(f;Y) \ge \mathtt{HD}(Y)\log\lambda$ for any $Y \subset X$. Here $\mathtt{HD}(-)$ denotes the Hausdorff dimension. The countable stability of the distance entropy $ent_{\mathtt{H}}$ proved in this paper, which generalizes the finite stability of Bowen's h-entropy (1971), implies that a continuous pointwise periodic map has the distance entropy zero. In addition, the authors show examples which demonstrate that this entropy describes the real complexity for dynamical systems over noncompact-phase space better than that of various other entropies.

1. **Introduction.** Rudolf Clausius created the thermodynamical concept of entropy in 1854; Shannon carried it over to information theory in 1948 [32], to describe the complexity of information. In 1958 Kolmogorov [23] introduced the concept of measure-theoretic entropy to ergodic theory. Kolmogorov's definition was improved by Sinai in 1959 [33]. In 1960's Adler, Konheim, and McAndrew [1] introduced the concept of topological entropy, written as $ent(f)$ in this paper, as an analogue of measure-theoretic entropy but for a continuous map $f\colon X \to X$ of a compact Hausdorff topological space X. In each setting entropy is a measure of uncertainty

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