DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS SERIES B
Volume **9**, Number **1**, January **2008**

Website: http://aimSciences.org

pp. **145–162**

PHASE-LOCKING AND ARNOLD CODING IN PROTOTYPICAL NETWORK TOPOLOGIES

Stefan Martignoli and Ruedi Stoop

Institute for Neuroinformatics UZH / ETHZ
Winterthurerstrasse 190
8057 Zürich, Switzerland

Abstract. Phase- and frequency-locking phenomena among coupled biological oscillators are a topic of current interest, in particular to neuroscience. In the case of mono-directionally pulse-coupled oscillators, phase-locking is well understood, where the phenomenon is globally described by Arnold tongues. Here, we develop the tools that allow corresponding investigations to be made for more general pulse-coupled networks. For two bi-directionally coupled oscillators, we prove the existence of three-dimensional Arnold tongues that mediate from the mono- to the bi-directional coupling topology. Under this transformation, the coupling strength at which the onset of chaos is observed is invariant. The developed framework also allows us to compare information transfer in feedforward versus recurrent networks. We find that distinct laws govern the propagation of phase-locked spike-time information, indicating a qualitative difference between classical artificial vs. biological computation.

1. **Introduction.**

Stable limit-cycle oscillations arise as the solutions of a variety of autonomous nonlinear differential equations. Primarily, they are generated when systems pass either through saddle-node or through Hopf bifurcations. Correspondingly, stable limit-cycles are widely observed in physics, chemistry and biology. As an example, the most salient neuron models, such as the Hodgkin-Huxley, the Fitzhugh-Nagumo, the Morris-Lecar, or the spatially detailed compartmental cable models of neurons yield stable limit-cycle solutions in biophysically relevant parameter regimes. Whereas for low-dimensional model systems the limit cycle property can be verified from the equations, in the case of the high-dimensional cable model or for biological neurons, the limit-cycle property is exhibited by the phenomenon of phase- and frequency locking.

In biology, the limit-cycles interact by means of sharp voltage pulses, the spikes. For mono-directionally periodically pulse-perturbed limit-cycle oscillators, it is simple to derive one-dimensional maps that describe this interaction. These maps are mathematically well understood: The emerging phenomena of phase- and frequency-locking are globally organized along Arnold tongues. For more general networks, a corresponding theory is still missing. Here, we develop the tools that are applicable, in principle, to arbitrary network topologies. Using this approach, we first generalize mono-directional phase-locking to bi-directional coupling, where a smooth change

---

2000 *Mathematics Subject Classification.* Primary: 37E10; Secondary: 37E45.

Key words and phrases. Arnold tongues, phase-locked oscillators, recurrent topology.

This research was supported by the SNF-grant 65293.