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# A COUPLED MAP LATTICE MODEL OF TREE DISPERSION

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Abstract. We study the coupled map lattice model of tree dispersion. Under quite general conditions on the nonlinearity of the local growth function and the dispersion (coupling) function, we show that when the maximal dispersal distance is finite and the spatial redistribution pattern remains unchanged in time, the moving front will always converge in the strongest sense to an asymptotic state: a traveling wave with finite length of the wavefront. We also show that when the climate becomes more favorable to growth or germination, the front at any nonzero density level will have a positive acceleration. An estimation of the magnitude of the acceleration is given.

1. **Introduction: Mathematical models of tree dispersion.** The main types of mathematical models of plant dispersion or invasion of single species in literature include the deterministic model in the form of a reaction-diffusion equation [10, 23], where space and time are both continuous and the stochastic diffusion or contact process model where the space variables are often continuous but the time variable can be either continuous or discrete (e.g., the integrodifference equation model [14]). For these models, the existence and stability of traveling wave solutions representing the moving front of the plant are proven in many interesting cases [2, 11, 14, 20, 25]. In this article, under some basic assumptions on the dispersal pattern of seeds, we model the change of mature plant density with a deterministic model called couple map lattices where both space and time variables are discrete. Kot mentioned this model briefly in [13] in comparison with the integrodifference model. Weinberger gave a population genetics example of this model in [25]. Recently, White and White studied dispersal-driven instabilities in a version of this model [26]. In this paper, we provide a detailed analysis of this model concerning the existence and global stability of traveling wave solutions. Our study is motivated by a series of articles on tree migration in paleorecords (Reid's paradox) [5, 6, 14] as well as our biologist colleagues' interests in the dynamics of a band of forest along the slope of Andes in Peru in the warming scenario [22]. We expect our model to provide at

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