

ON THE EXISTENCE OF TIME OPTIMAL CONTROLS FOR LINEAR EVOLUTION EQUATIONS

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ABSTRACT. This work concerns with the existence of the time optimal controls for some linear evolution equations without the *a priori* assumption on the existence of admissible controls. Both global and local existence results are presented. Some necessary conditions, sufficient conditions, and necessary and sufficient conditions for the existence of time optimal controls are derived by establishing the relationship between controllability and time optimal control problems.

1. Introduction. Let H be a reflexive Banach space with norm $\|\cdot\|$, U another Banach space with norm $\|\cdot\|_U$, $-A$ the infinitesimal generator of a C_0 -semigroup $\{S(t)\}_{t \geq 0}$ in H , with domain $D(A)$. Let $B(\cdot) \in L^\infty(\mathbf{R}^+; L(H))$ and $D \in L(U, H)$ (Here, $\mathbf{R}^+ = (0, \infty)$, $L(H)$ and $L(U, H)$ denote respectively the space of all linear bounded operators on H and that of all linear bounded operators from U into H). In the sequel, we shall fix a $\rho > 0$ and an $\varepsilon \geq 0$.

One of the most important infinite dimensional control problems is the time optimal driving of the solution $y \equiv y(t; y_0, u)$ of the following controlled system

$$y_t + Ay + B(t)y = Du(t), \quad t > 0 \tag{1}$$

from an initial point $y_0 \in H$ into a target set $Q \subset H$, i.e.,

$$y(0; y_0, u) = y_0 \text{ and } y(T; y_0, u) \in Q, \tag{2}$$

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