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MULTISCALE METHODS FOR PARABOLIC EQUATIONS WITH CONTINUUM SPATIAL SCALES

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Abstract. In this paper, we consider multiscale approaches for solving parabolic equations with heterogeneous coefficients. Our interest stems from porous media applications and we assume that there is no scale separation with respect to spatial variables. To compute the solution of these multiscale problems on a coarse grid, we define global fields such that the solution smoothly depends on these fields. We present various finite element discretization techniques and provide analyses of these methods. A few representative numerical examples are presented using heterogeneous fields with strong non-local features. These numerical results demonstrate that the solution can be captured more accurately on the coarse grid when some type of limited global information is used.

1. **Introduction.** The high degree of variability and multiscale nature of formation properties such as permeability pose significant challenges for subsurface flow modeling. Geological characterizations that capture these effects are typically developed at scales that are too fine for direct flow simulation, so techniques are required to enable the solution of flow problems in practice. Upscaling procedures have been commonly applied for this purpose and are effective in many cases (see [29, 26, 18] for reviews and discussion). More recently, a number of multiscale finite element (e.g., [20, 11, 5, 6, 1, 3, 14]) and finite volume [21, 22] approaches have been developed and successfully applied for problems of this type.

Most multiscale methods presented to date have applied local calculations for the determination of basis functions. Though effective in many cases, global effects can be important for some problems. The importance of global information has been illustrated within the context of upscaling procedures in recent investigations [19, 10, 9]. These studies have shown that the use of global information in the calculation of the upscaled parameters can significantly improve the accuracy of

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