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ANALYSIS OF A MODEL OF NUTRIENT DRIVEN SELF-CYCLING FERMENTATION ALLOWING UNIMODAL RESPONSE FUNCTIONS

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Abstract. A system of impulsive ordinary differential equations is used to model the growth of microorganisms in a self-cycling fermentor. The microorganisms are being used to remove a non-reproducing contaminant that is limiting to growth at both high and low concentrations. Hence it is the concentration of the contaminant that triggers the emptying and refilling process. This model predicts that either the process fails or the process cycles indefinitely with one impulse per cycle. Success or failure can depend on the choice of microorganisms, the initial concentration of the microorganisms and contaminant, as well as the choice for the emptying/refilling fraction. Either there is no choice of this fraction that works or there is an interval of possible choices with an optimal choice within the interval. If more than one strain is available, it does not seem to be the strains that have the highest specific growth rate over the largest range of the concentrations of the contaminant, but rather the ones that have the highest specific growth rate over very low concentrations of the contaminant, just above the threshold that initiates recycling that appear to be the most efficient, i.e., processing the highest volume of medium over a specified time period.

1. **Introduction.** Self-cycling fermentation (SCF) is a technique used to culture microorganisms. There are various potential applications, including water purification, waste decomposition, and production of antibiotics [9, 10, 13, 19, 21, 23, 26, 29].

SCF has been extensively described in [20, 21, 23], and thus only a brief review is needed here. The process is a computer-controlled semi-batch fermentation. A well-stirred tank containing fresh medium is initially inoculated with microorganisms. The microorganisms consume the nutrient until some criterion, sensed by the computer is met, (in the case studied in this paper, a particular concentration of the nutrient). The computer then initiates a rapid emptying and refilling process.

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