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A BOUNDED RESONANCE PROBLEM FOR SEMILINEAR ELLIPTIC EQUATIONS

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Abstract. In this paper we study the existence and multiplicity of nontrivial solutions for semilinear elliptic resonance problems with a bounded nonlinearity.

1. **Introduction.** This paper is concerned with the existence and multiplicity of nontrivial solutions for the Dirichlet boundary value problem

$$(P) \qquad \begin{cases} -\triangle u = f(x,u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is a differentiable function. Denoting by $0 < \lambda_1 < \lambda_2 < \cdots < \lambda_j < \cdots$ the distinct eigenvalues of $-\Delta$ in $H_0^1(\Omega)$, we assume f takes the form

$$f(x,t) = \lambda_k t + g(x,t),$$

for some $k \in \mathbb{N}$ and g satisfies the global assumptions

(g_1) There is some $\widehat{C} > 0$ such that $|g(x,t)| \le \widehat{C}$ for $x \in \Omega$, $t \in \mathbb{R}$.
(g_2) $\lim\limits_{|t|\to\infty} g_t'(x,t) = 0$ uniformly in $x \in \Omega$.

The assumption (g_1) characterizes (P) as a resonance problem at infinity with a *bounded* nonlinear perturbed term. Actually, (g_1) and (g_2) do not rule out the possibility of $f(x,t) \equiv \lambda_k t$ for $x \in \Omega$ and $|t|$ large, therefore, *real* resonance for (P) may occur near infinity. It is known that the solutions of (P) are exactly the critical points of the C^2 functional

$$J(u) = \frac{1}{2}\int_\Omega |\nabla u|^2 dx - \int_\Omega F(x,u)dx, \qquad u \in H_0^1(\Omega),$$

where $F(x,t) = \int_0^t f(x,s)ds$. We always assume $f(x,0) \equiv 0$, and therefore (P) has the trivial solution. The current paper will be focused on nontrivial solutions of (P), for which conditions near the origin are usually needed. We assume

$(F_0^{\pm})$ $f_t'(x,0) = \lambda_m$ for some $m \in \mathbb{N}$, and there is $r > 0$ such that

$$\pm(2F(x,t) - \lambda_m t^2) \ge 0, \quad x \in \Omega, \ |t| \le r.$$

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