DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS–SERIES B
Volume **8**, Number **1**, July **2007**

Website: http://AIMsciences.org

pp. **45–60**

AN AGE AND SPATIALLY STRUCTURED MODEL OF TUMOR INVASION WITH HAPTOTAXIS

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Abstract. A model of tumor growth into surrounding tissue is analyzed. The model consists of a system of nonlinear partial differential equations for the populations of tumor cells, extracellular matrix macromolecules, oxygen concentration, and extracellular matrix degradative enzyme concentration. The spatial growth of the tumor involves the directed movement of tumor cells toward the extracellular matrix through haptotaxis. Cell age is used to track progression of cells through the cell cycle. The existence of unique global solutions is proved using the theory of fractional powers of analytic semigroup generators.

1. **Introduction.** Tumor growth involves a hierarchy of population levels - molecular, cellular, tissue, and organism. Mathematical models which combine these levels must transcend their very different scales. We analyze a multi-scale model of tumor invasion into surrounding tissue using continuum densities to bridge individual tumor cell and total tumor cell population behavior. The model is continuous in all variables, and the individual processes of cells are modeled according to cell age and spatial position. The model is derived from the hybrid discrete-continuous model of tumor invasion in [6], which uses a discrete cellular automata formulation to account for individual cell processes. A numerical treatment of a continuum age-space structured model of tumor growth similar to ours is given in [9]. Our model is based on the density of proliferating tumor cells, the density of surrounding extracellular matrix, the concentration of matrix degradative enzyme, and the concentration of oxygen. The tumor is contained in a region $\Omega \subset \mathbb{R}^3$, which also

2000 *Mathematics Subject Classification.* Primary:92B05,92D25,47D03,47H20; Secondary: 35M10.

Key words and phrases. Semigroups of operators, diffusion, haptotaxis, tumor model.

Glenn Webb is supported by PHS-NIH grant 1P50CA113007-01. The authors acknowledge the assistance of Christoph Walker for Lemma 1 in the Preliminaries Section.