

## MATCHING THE GRADE CORRELATION COEFFICIENT USING A COPULA WITH MAXIMUM DISORDER

JULIA PIANTADOSI, PHIL HOWLETT AND JOHN BOLAND

Centre for Industrial and Applied Mathematics  
Mawson Lakes Campus, University of South Australia  
Mawson Lakes Boulevard, Mawson Lakes, 5095, Australia

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**ABSTRACT.** In this paper we use a doubly stochastic matrix to define a copula that preserves the given marginal distributions and matches a known grade correlation coefficient in such a way that the entropy of the doubly stochastic matrix is maximized. We will describe briefly how this work can be applied to the modelling of daily rainfall.

**1. Introduction.** It is common practice to use a gamma distribution to describe the probability density of non-zero rainfall totals on a particular day. Although recent work by Piantadosi *et al* [9] suggests that daily rainfall totals on successive days are correlated it is not easy to undertake direct construction of a suitable joint probability density. It is possible to use the method of copulas to construct a joint density function that preserves the prescribed marginal distributions and matches key observed statistics but there may be no unique solution to a problem of this type. Hence we seek a method that allows us to match the selected key statistics and avoid unjustified additional assumptions. In this paper we find a family of copulas that matches a known grade correlation coefficient and then select from that family the unique copula with maximum entropy (or maximum disorder). We contend that this copula provides the least prescriptive solution.

**2. A joint distribution with prescribed marginals.** Let  $(X, Y)$  be a pair of real valued random variables on  $\mathbb{R}^2$  and let  $c(x, y)$  be the joint probability density. The corresponding marginal probability densities are

$$p(x) = \int_{\mathbb{R}} c(x, y) dy \quad \text{and} \quad q(y) = \int_{\mathbb{R}} c(x, y) dx$$

for  $x \in \mathbb{R}$  and  $y \in \mathbb{R}$ . In practice we often wish to construct a joint probability distribution where the corresponding marginal distributions are already known. The method of copulas is one possible solution method. Let  $h(u, v)$  be a joint probability density on the unit square with

$$\int_0^1 h(u, v) dv = 1 \quad \text{and} \quad \int_0^1 h(u, v) du = 1$$

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