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UNIQUENESS OF LARGE INVARIANT MEASURES FOR $\mathbb{Z}^k$ ACTIONS WITH CARTAN HOMOTOPY DATA

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ABSTRACT. Every C^2 action α of $\mathbb{Z}^k$, $k \geq 2$, on the $(k+1)$-dimensional torus whose elements are homotopic to the corresponding elements of an action α_0 by hyperbolic linear maps has exactly one invariant measure that projects to Lebesgue measure under the semiconjugacy between α and α_0. This measure is absolutely continuous and the semiconjugacy provides a measure-theoretic isomorphism. The semiconjugacy has certain monotonicity properties and preimages of all points are connected. There are many periodic points for α for which the eigenvalues for α and α_0 coincide. We describe some nontrivial examples of actions of this kind.

1. PRELIMINARIES

1.1. **Introduction.** This paper constitutes a direct continuation of [3]. We shall use the terminology and results from [3] with only occasional references.

Let α_0 be a $\mathbb{Z}^k$ Cartan action on $\mathbb{T}^{k+1}$ and let α be a smooth $\mathbb{Z}^k$ action whose elements are homotopic to the corresponding elements of α_0.[1] "Smooth" in our context means C^2 although most of the arguments hold for $C^{1+\varepsilon}$ actions for any positive ε and on the other hand for C^∞ actions various geometric structures we describe are also C^∞. Those include leaves of various stable and unstable foliations, affine structures on those leaves and regularity of the semiconjugacy on leaves of good points.

We will say that such an action α has *Cartan homotopy data* or simply call it *homotopically Cartan*. The action α_0 will be referred to as the *linear model* for α.

Let $h\colon \mathbb{T}^{k+1} \to \mathbb{T}^{k+1}$ be the semiconjugacy between α and α_0, i.e., a unique continuous map $h\colon \mathbb{T}^{k+1} \to \mathbb{T}^{k+1}$ homotopic to identity such that

$$h \circ \alpha = \alpha_0 \circ h \tag{1.1}$$

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[1] In general one should not expect such actions to be actually homotopic in the space of $\mathbb{Z}^k$-*actions*, whether differentiable or continuous. See Section 4 for a discussion of some negative results in this direction