DISCRETE AND CONTINUOUS
DYNAMICAL SYSTEMS
Volume **17**, Number **3**, March **2007**

Website: http://aimSciences.org
pp. **541–552**

REMARKS ON ACCESSIBLE STEADY STATES FOR SOME COAGULATION-FRAGMENTATION SYSTEMS

Miguel A. Herrero

Departamento de Matemática Aplicada
Facultad de Matemáticas
Universidad Complutense
28040 Madrid, Spain

Marianito R. Rodrigo

Departamento Académico de Matemáticas
Instituto Tecnológico Autónomo de México
México D.F., México 01000

(Communicated by Hiroshi Matano)

Abstract. In this paper we consider some systems of ordinary differential equations which are related to coagulation-fragmentation processes. In particular, we obtain explicit solutions $\{c_k(t)\}$ of such systems which involve certain coefficients obtained by solving a suitable algebraic recurrence relation. The coefficients are derived in two relevant cases: the high-functionality limit and the Flory-Stockmayer model. The solutions thus obtained are polydisperse (that is, $c_k(0)$ is different from zero for all $k \geq 1$) and may exhibit monotonically increasing or decreasing total mass. We also solve a monodisperse case (where $c_1(0)$ is different from zero but $c_k(0)$ is equal to zero for all $k \geq 2$) in the high-functionality limit. In contrast to the previous result, the corresponding solution is now shown to display a sol-gel transition when the total initial mass is larger than one, but not when such mass is less than or equal to one.

1. **Introduction.**

This note is concerned with some infinite systems of coupled ordinary differential equations related to classical models of polymer formation. More precisely, we are interested in the way equilibrium distributions may be characterized and arrived at. Before describing our results in detail, it may be worth recalling some relevant background information.

The problem of understanding how large structures form from an initial given distribution of their elementary components has deserved a great deal of attention in several physical contexts. Arguably, the earliest mathematical model proposed to shed light into this question was given by Smoluchowski [24, 5]. His description of coagulation of colloidal particles in a solution can be summarized as follows. Let $c_k(t)$ denote the concentration of aggregates or clusters made up of k ($k \geq 1$) identical individual particles (or monomers) at time $t \geq 0$. Assuming a number of hypotheses on the physical mechanism of coagulation, namely, (a) that monomers

2000 *Mathematics Subject Classification.* Primary: 34A05; Secondary: 34A34.

Key words and phrases. coagulation-fragmentation, exact solutions, gelation, sol-gel transition.

M. A. H. has been partially supported by European Contract MRTN-CT-2004-503661. M. R. R. would like to acknowledge the support of the Asociación Mexicana de Cultura, A. C.