NETWORKS AND HETEROGENEOUS MEDIA Website: http://aimSciences.org

Volume **1**, Number **4**, December **2006** pp. **639–673**

HYPERBOLIC-ELLIPTIC MODELS FOR WELL-RESERVOIR FLOW

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Abstract. We formulate a hierarchy of models relevant for studying coupled well-reservoir flows. The starting point is an integral equation representing unsteady single-phase 3-D porous media flow and the 1-D isothermal Euler equations representing unsteady well flow. This 2×2 system of conservation laws is coupled to the integral equation through natural coupling conditions accounting for the flow between well and surrounding reservoir. By imposing simplifying assumptions we obtain various hyperbolic-parabolic and hyperbolic-elliptic systems. In particular, by assuming that the fluid is incompressible we obtain a hyperbolic-elliptic system for which we present existence and uniqueness results. Numerical examples demonstrate formation of steep gradients resulting from a balance between a local nonlinear convective term and a non-local diffusive term. This balance is governed by various well, reservoir, and fluid parameters involved in the non-local diffusion term, and reflects the interaction between well and reservoir.

1. **Introduction.** We are interested in coupled well-reservoir flow modeling. For that purpose we consider a model composed of a hyperbolic system of two conservation laws corresponding to the isothermal Euler equations with source terms, and an integral equation. It results from coupling a transient well flow model with a transient reservoir model and is given on the following form.

$$\begin{aligned} \partial_t(\rho) + \partial_x(\rho u) &= \frac{1}{\eta}\rho q_{\mathrm{V}}, \qquad && \eta > 0, \\ \partial_t(\rho u) + \partial_x(\rho u^2) + \partial_x p(\rho) &= q_{\mathrm{F}}, && q_{\mathrm{F}} = q_{\mathrm{F}}(\rho, u), \\ p_0 - p(x,t) &= \int_0^t \int_0^1 H^r(x, x', t-t') q_{\mathrm{V}}(x', t')\, dx' dt', \end{aligned} \tag{1}$$

for $x \in [0,1]$. Here, ρ, u, and $p(\rho)$ are, respectively, the mass density, fluid velocity, and pressure, whereas q_{V} represents volumetric flow rate accounting for flow between well and reservoir. Thus, the unknown variables are ρ, u, and q_{V}. Moreover, p_0 which we assume to be constant, is initial reservoir pressure whereas η is a small known constant parameter characterizing the well volume relatively the pore volume

2000 *Mathematics Subject Classification.* Primary: 35L65, 35L60; Secondary: 35K40.

Key words and phrases. Non-local conservation law, coupled well-reservoir flow, advanced well, hyperbolic-elliptic model, entropy weak solution, existence, uniqueness.

This research is supported by an Outstanding Young Investigators Award from the Research Council of Norway. Steinar Evje is grateful to Ove Sævareid for helpful discussions.