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A NOTE ON EPIDEMIC MODELS WITH INFECTIVE IMMIGRANTS AND VACCINATION

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Abstract. The roles of immigration and vaccination on disease dynamics are explored in a simple setting that considers the possibility of conferred immunity. We focus on SIR and SIS models with a vaccinated class. Conditions for the existence of multiple endemic steady states and a fold bifurcation are discussed.

1. **Introduction.** It is well known that immigrants play a critical role in disease dynamics, and yet there is hardly any theoretical work [1], [2], [3]. Epidemics "ignited" or "enhanced" by the immigration of infectious cases include HIV [4], severe acute respiratory syndrome(SARS) [5], [6], avian influenza and measles [5]. The effect of vaccination on simple epidemic models with immigration is the subject of this note, which is motivated by the research in [1], as expanded in [2]. Extensions of the work in [1] and [2] to an SIRS model with vaccination have recently been published [3].

Some diseases, such as gonorrhea, do not confer immunity and can be modeled using a Susceptible-Infective-Susceptible (SIS) framework. The SIS model is the core of any disease transmission model and serves as a basic template to be expanded for special cases. Thus a full understanding of a simple SIS model is essential regardless of how well any particular disease can be forced into its framework. The use of control measures, such as the use of condoms (for example, in the case of gonorrhea), can be interpreted as a partially effective "vaccine." Hence, a vaccinated class is added to the classic SIS model in a population where the impact of infective immigrants is considered. "Vaccine" efficacy is a function of the disease. Our models (SVIS and SVIR) consider various levels of efficacy and waning effect (e.g., influenza vaccines have a 70% to 90% efficacy rate among healthy young adults but only 30% to 40% among the elderly [3]).

Models that include a constant flow of infective immigrants cannot eliminate a disease through vaccination or "standard" control measures. These standard policies are based on the *basic reproduction number*, which cannot be applied to models with a constant flow of infective individuals [7], [8]. Here, the value of a critical ratio, computed as the fraction of infective immigrants approaches zero, is used as a "limiting" threshold for the study of disease invasion and control. In addition, the result of partially effective vaccines is evaluated within appropriate submodels [9]. The possibilities of subthreshold coexistence and bistability are

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