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RAVES, CLUBS AND ECSTASY: THE IMPACT OF PEER PRESSURE

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Abstract. Ecstasy has gained popularity among young adults who frequent raves and nightclubs. The Drug Enforcement Administration reported a 500 percent increase in the use of ecstasy between 1993 and 1998. The number of ecstasy users kept growing until 2002, years after a national public education initiative against ecstasy use was launched. In this study, a system of differential equations is used to model the peer-driven dynamics of ecstasy use. It is found that backward bifurcations describe situations when sufficient peer pressure can cause an epidemic of ecstasy use. Furthermore, factors that have the greatest influence on ecstasy use as predicted by the model are highlighted. The effect of education is also explored, and the results of simulations are shown to illustrate some possible outcomes.

This article is dedicated to Zhien Ma to celebrate his 70$^{\text{th}}$ birthday. It is a small gesture of our appreciation to our personal and professional association to Professor Ma, our teacher, mentor and friend.

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1. **Introduction.** According to John B. Brown III, a former deputy administrator for the Drug Enforcement Administration (DEA), "Ecstasy is the Y Generation's cocaine and is fast becoming the number one drug problem facing America's youth today" [22]. Between 1993 and 1998, the DEA documented a 500 percent increase in the use of ecstasy, an illicit drug best known for its popularity in the rave and night club culture [21]. Figure 1 shows the general trend of ecstasy use in the United States for the past decade. Scientifically termed 3,4–Methylenedioxymethamphetamine (MDMA), ecstasy is a synthetic drug with stimulant and hallucinogenic characteristics similar to methamphetamines and is generally taken in pill or capsule form, though it may also be sniffed, snorted, injected or used in suppository form [14]. First patented in 1912 by a German pharmaceutical company as an appetite suppressant, ecstasy was used by a small number of therapists in the 1970s to enhance communication with patients [13]. Illicit use of the drug did not become popular in the United States until the late 1980s, and since then its abuse has increased dramatically (see Figure 1). In the United States, the DEA initiated emergency designation of MDMA as a controlled substance in June 1985 and in 1988, it was classified as a Schedule I drug under the Controlled Substance Act [19].

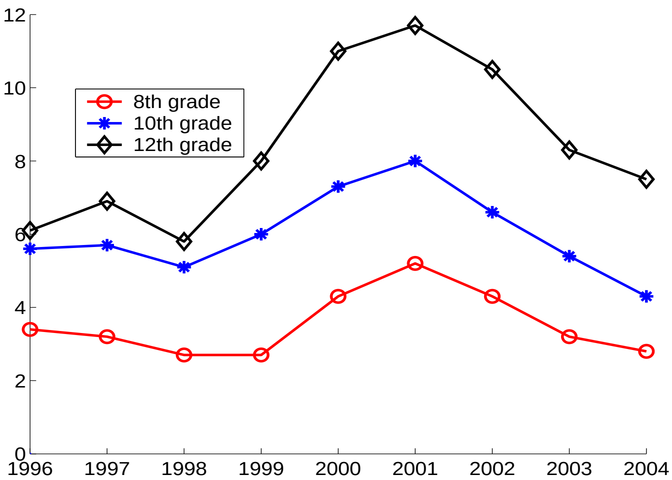

Figure 1. Prevalence of ecstasy users for eighth, tenth and twelfth graders.

Ecstasy is known as the "feel good" drug or "hug drug" because it reduces inhibitions and produces heightened sensuality, the elimination of anxiety and extreme relaxation [22]. Its effects, which can last anywhere from two to six hours, include feelings of well-being, contentment, empathy and love, amplifying the visual and tactile senses [14]. Ecstasy, however, has extremely dangerous side effects on both physical and psychological. It increases heart rate and blood pressure and can cause involuntary teeth clenching, muscle tension, nausea, blurred vision, rapid eye movement, fainting, tremors, heart attacks, sweating, dehydration and even death. According to the Drug Abuse Warning Network's October 2002 report, ecstasy was mentioned over 5,500 times in emergency department visits in 2001, more than a 2,000 percent increase from 1994 [6]. The use of ecstasy has also produced liver and kidney damage that does not appear until days or weeks after the drug is consumed.

Deaths related with ecstasy have been recorded where users' core body temperatures reached as high as 109 degrees. The psychological risks include confusion, depression, sleep problems, drug cravings, severe anxiety and paranoia. Recent research has linked ecstasy use to permanent brain damage which can manifest itself in the form of depression, anxiety, memory loss or some neuro-psychiatric disorder. A message from the director of NIDA (National Institute of Drug Abuse) reads that "over 15 years of research conducted on animals has proven that MDMA damages specific neurons in the brain" [18].

Although in recent years, reports have shown that ecstasy use has expanded to private parties, private residences and college campuses, the primary venue for its abuse was and continues to be rave and nightclub [19]. In agreement, Senator Joseph Bidden (D-Del.) stated, "unfortunately most raves are havens for illicit drug use" [8]. Raves, which are large parties, often with light shows and pyrotechnics, where up to 25,000 attendees dance to loud techno music, are a popular place for young people to experiment with ecstasy. Ecstasy allows rave-goers to dance for hours without food or drink and also heightens the sensory overload caused by the music and lights. The rave scene, which originally emerged in England in the late 1980s, made its move to the United States in the early 1990s, where they were secret "underground" parties advertised primarily by word of mouth. Nowadays, however, raves have gone mainstream, are less secretive and are advertised by various means including radio and the internet [10]. A national survey of American attitudes in 2000 reported that 10 percent of teens say they have been to a rave and that ecstasy was available at 70 percent of these events [17].

In the past decade, ecstasy use has become increasingly popular with teens and young adults, especially those who live in urban areas where nightclubs and raves are commonplace. In a study by the NIDA, young adults (ages 18 to 25 years) were found to be the greatest users of ecstasy, with 13.1 percent reporting that they had used the drug in 2002. According to the study, 11.7 percent of twelfth graders, 8.0 percent of tenth graders, 5.2 percent of eighth graders and 2.0 percent of persons age 26 and over reported using ecstasy in 2002. Although ecstasy use is just beginning to become trendy, its diffusion into the young population is especially alarming because of the speed with which it has gained popularity. The spread of ecstasy is also cause for concern because of the perception among many young adults that ecstasy is as easily available and not as harmful as other drugs [15].

This paper studies the influence of peer pressure on the prevalence of ecstasy use by a mathematical model. By studying individuals in a core population between the ages of 13 and 25 who frequent raves and nightclubs, we show that once an individual is drawn into the core population, preventing ecstasy use is very difficult. It is also shown that elimination of the susceptible core population, individuals who are not yet ecstasy users, is extremely difficult. This model will also provide, through analysis of threshold conditions and estimation of parameters, predictions for the future of ecstasy use in the United States. Finally, using simulations, we are able to recommend a method of combating ecstasy use.

2. **The model.**

This model focuses on a population of individuals between the ages of 13 and 25 who are divided into core and noncore subpopulations. The noncore population, denoted by A, consists of individuals who never use ecstasy and do not frequent raves and nightclubs. The core population, consisting of those who regularly visit nightclubs and raves, is divided into three classes: the susceptible class (S) is composed of those individuals who do not use ecstasy but are likely to become ecstasy users because of their immersion in the rave and nightclub culture; the ecstasy-use class (I) is composed of individuals who are habitual ecstasy users; and the recovered class (V) is composed of those who are no longer using ecstasy.

An individual of age 13 automatically enters the population through class A as a member of the noncore. Individuals in the noncore class can become susceptibles (S) because of peer pressure from S and I, and can return to A also by positive peer pressure. Susceptible individuals can become ecstasy users as a result of peer pressure from ecstasy users. An individual who becomes an ecstasy user may quit by moving to the recovered class V. Former ecstasy users can go back to class I or to noncore class A.

Peer pressure results from interactions between core and noncore individuals, assumed to be proportional to the fraction of individuals who exert peer pressure. Furthermore, peer pressure can be either positive (moving individuals out of the core) or negative (moving individuals into the core).

Our mathematical model is given by the following system of differential equations:

$$\frac{dA}{dt} = \mu P + \delta_v A\frac{V}{P} + \delta_s A\frac{S}{P} - \epsilon A\frac{S+I}{P} - \mu A, \tag{1}$$

$$\frac{dS}{dt} = \epsilon\frac{A}{P}(S+I) - \delta_s S\frac{A}{P} - \phi S\frac{I}{P} - \mu S, \tag{2}$$

$$\frac{dI}{dt} = \phi I\frac{S}{P} + \alpha I\frac{V}{P} - \gamma I\frac{A+S+V}{P} - \tau I - \mu I, \tag{3}$$

$$\frac{dV}{dt} = \gamma I\frac{A+S+V}{P} + \tau I - \delta_v V\frac{A}{P} - \alpha V\frac{I}{P} - \mu V, \tag{4}$$

$$P = A+S+I+V. \tag{5}$$

The definitions of parameters can be found in Table 1. It is easy to check that the total population size (P) is constant. That is, the model assumes that the population of individuals of 13 to 25 years does not experience serious fluctuations over the time scale of interest. Note that all the parameters, excluding μ and τ, take into consideration the effect of peer pressure.

TABLE 1. Definition of parameters

Parameter	Definition
μ	The rate of leaving a class as a result of aging or death (also the recruitment rate for the system)
ϵ	Peer pressure rate of the core population on the noncore population
ϕ	Peer pressure rate of ecstasy users on susceptibles
τ	Recovery rate without peer pressure
γ	Recovery rate from peer pressure
α	Relapse rate due to peer pressure
δ_v	Rate at which recovered individuals go back to the noncore as a result of peer pressure
δ_s	Rate at which susceptible individuals go back to the noncore due to peer pressure

We rescale model (1–5) by introducing the nondimensional variables $x = A/P$, $y = S/P$, $z = I/P$, $w = V/P$. The new system is

$$\frac{dx}{dt} = \mu + (\delta_s y - \epsilon(y+z) + \delta_v w - \mu)x, \tag{6}$$

$$\frac{dy}{dt} = \epsilon x(y+z) - (\phi z + \delta_s x + \mu)y, \tag{7}$$

$$\frac{dz}{dt} = (\phi y + \alpha w - (\gamma(1-z) + \tau + \mu))z, \tag{8}$$

$$\frac{dw}{dt} = (\gamma(1-z) + \tau)z - (\delta_v x + \alpha z + \mu)w, \tag{9}$$

$$x + y + z + w = 1.$$

We will focus our study on system (6–9) in the rest of the paper.

3. **Analysis.**

3.1. **Two tipping points.**

The case of ecstasy is not a simple issue. There is no single "tipping point" or "threshold" that determines whether ecstasy use will become an epidemic. Local analysis of equilibria finds multiple thresholds. The first one, R_c, describes the average number of individuals pressured into becoming new core members (rave and nightclub frequenters) by a member of the core population, where

$$R_c = \frac{\epsilon - \delta_s}{\mu}. \tag{10}$$

Whenever $R_c < 1$, the core-free equilibrium $(1, 0, 0, 0)$ is locally asymptotically stable, while unstable if $R_c > 1$. R_c is the product of the net peer pressure felt by the noncore class to begin going to raves and night clubs $(\epsilon - \delta_s)$ and the average residence time $(1/\mu)$ of the system. Naturally, $R_c > 1$ leads to the establishment of a critical mass of susceptibles. Typically, $R_c < 1$ would imply that a core cannot be established. However, if peer pressure is large enough, this is not the case because of the existence of backward bifurcations in the system (this will be discussed further in the next section).

If the pressure to join the core population is not "too" large, the global stability of the core-free equilibrium can be established using the Lyapunov $L(y, z, w) = y + z + w$.

THEOREM 3.1. *If $\epsilon < \mu$ and $R_c < 1$, the core-free equilibrium $(1, 0, 0, 0)$ is globally asymptotically stable.*

If $R_c > 1$, a new equilibrium $(1/R_c, 1 - 1/R_c, 0, 0)$ composed of members of class A and class S comes out–a core population is borne. At this equilibrium, the population is made up of individuals who do not attend raves and nightclubs and individuals who do attend raves and nightclubs but do not use ecstasy. We call this equilibrium the core ecstasy-free equilibrium, and from it we define the second tipping point, the basic reproductive number,

$$R_0 = \frac{\phi(1 - \frac{\mu}{\epsilon - \delta_s})}{\gamma + \tau + \mu}. \tag{11}$$

If $R_0 < 1$, the ecstasy-free equilibrium is locally asymptotically stable. Typically $R_0 < 1$ would imply an ecstasy-free population. This, however, is not entirely correct. Later we will find the existence of multiple endemic equilibrium (non-ecstasy-free) when $R_0 < 1$. The basic reproductive number R_0 is given by the product of core peer pressure (ϕ), the maximum proportion of susceptibles in the core population $(1-\mu/(\epsilon-\delta_s))$ and the average ecstasy conversion period $(1/(\gamma+\tau+\mu))$. Therefore, R_0 describes the conditions that ecstasy must overcome to infect more individuals.

In summary, R_c and R_0 are two local "tipping" points, that is, thresholds based on local conditions. They are very sensitive to peer pressure, and the existence of ecstasy in the system depends on both the power of this peer pressure and initial number of members in the classes. It will be shown that peer pressure destroys the hope of global tipping points and, in the process, enhances the persistence of ecstasy use and an ecstasy favorable environment.

3.2. **Backward bifurcation.** In this subsection, we study the bifurcation problem at $R_0 = 1$ for system (6–9) rigorously and conclude that if $0 < \epsilon - \delta_s - \mu << 1$ and $\delta_s > \gamma + \tau$ or if $\mu << 1$ and $\alpha > (\gamma + \tau)$ the bifurcation is backward (see Figure 2). Our estimation of the parameters (see Section 4) in the model satisfy these conditions resulting in the backward bifurcations which we examine in Section 5.

First, we observe that the hyperplane (manifold) $x + y + z + w = 1$ is invariant. Replacing x by $1-y-z-w$ reduces the model (6-9) to the variables y, z and w. The ecstasy-free equilibrium in the variables y, z and w is $((\epsilon-\delta_s-\mu)/(\epsilon-\delta_s), 0, 0) = (y^*, 0, 0)$, where $y^* = (\epsilon-\delta_s-\mu)/(\epsilon-\delta_s)$. We rewrite R_0 as $R_0 = \phi/\phi^*$ with $\phi^* = (\gamma+\tau+\mu)/y^*$. A variable change is made by letting $x_1 = z$, $x_2 = w$ and $x_3 = y - y^*$. We also make a shift in the parameter ϕ by introducing $\beta = \phi - \phi^*$. Thus, $\beta = 0$ corresponds to $R_0 = 1$. The new system is given by the following set of equations:

$$\frac{dx_1}{dt} = ((\beta+\phi^*)y^* - (\gamma+\tau+\mu))x_1 + (\gamma x_1 + \alpha x_2 + (\beta+\phi^*)x_3)x_1, \tag{12}$$

$$\frac{dx_2}{dt} = (\gamma+\tau)x_1 - \mu\left(\frac{\delta_v}{\epsilon-\delta_s}+1\right)x_2 - \gamma x_1^2 - (\alpha x_1 - \delta_v(x_1+x_2+x_3))x_2, \tag{13}$$

$$\begin{aligned}\frac{dx_3}{dt} &= (\epsilon(1-y^*) - (\beta+\phi^*)y^*)x_1 + (\delta_s+\mu-\epsilon)(x_1+x_2+x_3)\\ &\quad - \epsilon(x_1+x_3)(x_1+x_2+x_3) - x_3((\beta+\phi^*)x_1 - \delta_s(x_1+x_2+x_3)).\end{aligned} \tag{14}$$

The Jacobian matrix of system (12–14) at $(0,0,0)$ when $\beta = 0$ (i.e., $R_0 = 1$) has three eigenvalues, $\lambda_1 = 0$, $\lambda_2 = -\mu(\delta_v/(\epsilon-\delta_s)+1)$ and $\lambda_3 = -(\delta_s+\mu-\epsilon)$. Since $\lambda_1 = 0$ is a simple eigenvalue, any center manifold is one dimensional. A right-dominant eigenvector corresponding to λ_1 is $w = [w_1, w_2, w_3]'$, where

$$w_1 = \mu\left(1+\frac{\delta_v}{\epsilon-\delta_s}\right), \quad w_2 = \gamma+\tau, \quad w_3 = \left(\frac{\epsilon(1-y^*)-\phi^* y^*}{\epsilon-\delta_s-\mu}-1\right)w_1 - w_2,$$

and a left-dominant eigenvector is $v = [1/w_1, 0, 0]$. We choose v so that $v \cdot w = 1$. Let q_i denote a right eigenvector associated with λ_i. Then, $\mathcal{E}_1 = \text{span}\{w\}$, $\mathcal{E}_2 = \text{span}\{q_2\}$ and $\mathcal{E}_3 = \text{span}\{q_3\}$ denote the eigenspaces. Since there are three distinct eigenvalues, $R^3 = \mathcal{E}_1 \oplus \mathcal{E}_2 \oplus \mathcal{E}_3$ and $v \perp \mathcal{E}_i, i = 2, 3$.

We now parameterize the dynamics of the system on an arbitrary center manifold, W^c, near $\beta = 0$. Because W^c has dimension one, it can be characterized by a one-parameter family. If we choose t as the parameter and use the eigenspaces, W^c can be explicitly expressed as

$$W^c = c_1(t)w + c_2(t)q_2 + c_3(t)q_3. \tag{15}$$

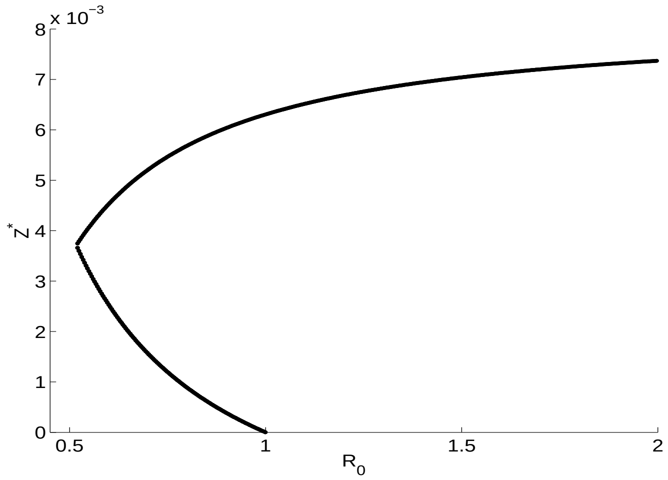

FIGURE 2. Backward bifurcation for the infected class. We choose ϕ as the bifurcation parameter. Other parameter values are $\mu = 0.00174$; $\delta_v = 0.0075$; $\alpha = 0.8$; $\epsilon = 0.241$; $\delta_s = 0.22189$; $\gamma = 0.0065$; and $\tau = 0.0045$.

The fact that W^c is tangent to $\mathcal{E}_1$ (at $t = 0$) and is orthogonal to $\mathcal{E}_2$ and $\mathcal{E}_3$ (at $t = 0$) implies that $c_2(t)q_2 + c_3(t)q_3$ is the higher order term, that is, $c_2(t)q_2 + c_3(t)q_3 = o(c_1)$. Letting $h = c_2(t)q_2 + c_3(t)q_3$, $h \in \mathcal{E}_2 \oplus \mathcal{E}_3$ leads to $v \cdot h = 0$. To keep our notation shorter, let us use the vector form of system (12–14) $dx/dt = f(x, \beta)$, where $x = [x_1, x_2, x_3]'$ and $f = [f_1, f_2, f_3]'$ is the vector filed of system (12–14). The flow on W^c can be characterized by the dynamics of $c_1(t)$. Because the center manifold is invariant, we can obtain

$$\frac{d(c_1 w + h)}{dt} = f(c_1 w + h, \beta). \tag{16}$$

Multiplying by v from the left on both sides of (16) and noticing that $v \perp h$ and $v \cdot w = 1$, we simplify (16) into

$$\frac{dc_1}{dt} = v \cdot f(c_1 w + h, \beta). \tag{17}$$

Substituting the Taylor expansion of $f(c_1 w + h, \beta)$ around $(0, 0, 0, 0)$ into (17), one arrives at the following nonlinear differential equation for $c_1(t)$:

$$\frac{dc_1}{dt} = ac_1^2 + b\beta c_1 + \text{ h.o.t.}, \tag{18}$$

where

$$a = \frac{1}{2}\left(w_1\gamma + 2w_2\alpha + 2\phi^*\left(\left(\frac{\epsilon(1-y^*) - \phi^* y^*}{\epsilon - \delta_s - \mu} - 1\right) w_1 - w_2\right)\right), \tag{19}$$

$$b = 2\frac{\epsilon - \delta_s - \mu}{\epsilon - \delta_s} > 0. \tag{20}$$

When $|\beta| << 1$, equation (18) describes the dynamics on the center manifold. Hence, it follows that if $a > 0$, the bifurcation at $\beta = 0$ is transcritical. That is, when $\beta < 0$ and very small, $(0, 0, 0)$ is locally asymptotically stable, and there is an unstable positive equilibrium when $a > 0$. On the other hand, the equilibrium

$(0,0,0)$ is unstable and there is a locally asymptotically stable positive equilibrium when $a < 0$. Namely, the sign of a determines the direction of the bifurcation at $R_0 = 1$. It follows from (19) that

$$\frac{2a}{w_2} = \frac{\mu(\frac{\delta_v}{\epsilon-\delta_s}+1)}{\gamma+\tau}\left(\gamma - 2\phi^*\left(1 - \frac{\epsilon(1-y^*)-\phi^* y^*}{\epsilon-\delta_s-\mu}\right)\right) + 2\alpha - 2\phi^*. \tag{21}$$

Hence, the following theorem holds:

THEOREM 3.2. *(1) If $\mu << 1$ and $\alpha > (\gamma+\tau)$ or (2) If $0 < \epsilon - \delta_s - \mu << 1$ and $\delta_s > \gamma + \tau$, then $a > 0$. That is, a backward bifurcation occurs.*

Proof.

(1) If $\mu << 1$, then $y^* \approx 1$, $\phi^* \approx \gamma+\tau$. It follows from (21) that a and $\alpha - (\gamma+\tau)$ have the same sign. Therefore, under the conditions that $\mu << 1$ and $\alpha > (\gamma+\tau)$, a is positive.

(2) Rewrite $2a/w_2$ in (21) as

$$\frac{\mu(\frac{\delta_v}{\epsilon-\delta_s}+1)}{\gamma+\tau}\gamma + 2\alpha - 2\phi^*\left(1 + \frac{\mu(\frac{\delta_v}{\epsilon-\delta_s}+1)}{\gamma+\tau}\left(1 - \frac{\epsilon(1-y^*)-(\gamma+\tau+\mu)}{\epsilon-\delta_s-\mu}\right)\right).$$

If $0 < \epsilon - \delta_s - \mu << 1$, then $y^* \approx 0$ and $\phi^* \to +\infty$. Hence, if $\delta_s > \gamma+\tau$, then $\epsilon > \delta_s + \mu > \gamma + \tau + \mu$. The last inequality guarantees that $a > 0$.

We conclude that if $0 < \epsilon - \delta_s - \mu << 1$ and $\delta_s > \gamma+\tau$, or if $\mu << 1$ and $\alpha > (\gamma+\tau)$, the direction of the bifurcation is backward. □

4. **Estimation of parameters.** Some rough estimates of the values of the parameters are required to make qualitative predictions using this model. However, because of the novelty of this topic, most research and statistics regarding ecstasy have been carried out in the past eleven years. For example, *Monitoring the Future*, a survey of drug use administered by the United States Department of Health and Human Services, began including ecstasy in its survey only in 1996 [5]. Additionally, little research has been done specific to ecstasy regarding prevention, treatment and its use as a gateway drug. Thus, all estimations of the parameters are rough approximations, some based on statistical data and some from literature and books that are made up mostly of anecdotal information and generalizations. Our model deals with a specific age group (13–25 years), and consequently the time is measured in months. The estimated values are listed in Table 2. Our arguments for the use of these estimates are addressed.

TABLE 2. Estimated parameter values

Parameter	μ	ϵ	ϕ	τ	γ	α	δ_v	δ_s
Value	0.007	0.0391	0.275	0.016	0.011	0.5	0.05	0.032

4.1. **Estimation of μ:** $\mu \approx 0.007$. The United States Census Bureau estimates that the number of individuals of age between 13 and 25 years is 50 million [20]. The number of non-ecstasy related deaths is around 35,000 per year [1]. Dividing 35,000 by 12 gives the average number of natural deaths per month, and that quotient over 50 million is the rate of death per individual. We add this figure to 1/144 months because our system looks only at 12 years.

4.2. **Estimation of ϕ:** $\phi \approx 0.275$. Using the US Department of Health and Human's service *Monitoring the Future*, we estimated that the initial percentage of habitual ecstasy users is .63 percent and that the number of new infected individuals is approximately 2.1 percent per year [5]. Therefore, each ecstasy user infects approximately 3.3 individuals from the susceptible class each year. This number divided by 12 gives us ϕ. Although this infection rate may seem very high, it is important to recognize that the infection is spread from a susceptible class made up of individuals who are frequent visitors of nightclubs and raves where ecstasy is widely used and easily available [2].

4.3. **Estimation of ϵ:** $\epsilon \approx 0.0391$. In this case there is no data on the numbers or rates of people who frequent raves, nightclubs or both. We assume that the number of new susceptibles, those individuals who begin to frequent nightclubs and raves, is larger than the number of individuals who tried ecstasy in the previous month. By estimating the percent change of ravers or nightclubbers as 0.375, we could approximate that a susceptible recruits .47 individuals per year. This number over 12 yields the value for ϵ. This value of ϵ is much smaller than ϕ because of the different pools from which each is recruiting. Although more people go to nightclubs and raves than become habitual ecstasy users, ϵ is recruiting from a very large noncore population that does not use ecstasy and does not go to these events. On the other hand, research has shown that at events such as raves, a very large proportion of the population will use ecstasy [2].

4.4. **Estimation of δ_s:** $\delta_s \approx 0.032$. To ensure the existence of the ecstasy-free equilibrium, $\epsilon > (\delta_s + \mu)$ and consequently, $\delta_s < 0.0321$ are needed. We assumed that there is a fair amount of movement between the susceptible and noncore classes in selecting δ_s. Again, here we were not able to find any research material on this parameter.

4.5. **Estimation of α:** $\alpha \approx 0.5$. Research has shown that once an individual has used and then stopped using a drug, it is much easier to relapse into drug abuse than those who abuse a drug for the first time [7]. Therefore, α must be larger than ϕ. We approximated that abuse of ecstasy in the second time was at least twice as likely as abuse in the first time and then rounded α down a little to a conservative estimate of 0.5.

4.6. **Estimation of τ and γ:** $\tau \approx 0.016$ and $\gamma \approx 0.011$. The recovery rate τ includes not only the rate of stopping ecstasy use in favor of a drug-free core life, but also the rate of moving onto abuse of substances other than ecstasy. The parameter γ incorporates the effect of peer pressure on both stopping drug use and moving onto other drugs. We conjecture that peer pressure does not have as much effect on removal from drug use or gateway drug usage as other factors. Therefore, τ must be bigger than γ. In any case, whether τ is greater than γ makes no difference in the results because of their position in the expression of R_0. An increase of a certain increment of τ has the same effect as an increase of the equal amount in γ. Although we believe that τ should be greater than γ, in terms of numerical results, only the sum is important. Although there is no data available on recovery from ecstasy use or whether ecstasy is a gateway drug, we used estimates from methamphetamine, a drug used in similar settings with similar effects, but more widely researched, to estimate these recovery parameters. According to the *Monitoring the Future Study*, approximately one-third of methamphetamine users in 1998 did not use methamphetamine in 1999 [7]. This noncontinuation rate divided by 12 gave the value for the sum of τ and γ. The final values for the two parameters were adjusted from simulations, as no data was available regarding these parameters for any illicit drug whatsoever.

4.7. **Estimation of δ_v:** $\delta_v \approx 0.05$. Since the recovered class V is still in the core population, it represents a group of individuals who have stopped using ecstasy, yet still frequent raves and nightclubs. This population is fairly small. We assume that most people who refrain from using ecstasy do so by leaving the ecstasy culture, which embraces long dance parties. Therefore, a large portion of V will leave each year to go to the noncore population. Drug use can still continue in the noncore, so long as the drug of choice is not ecstasy and the setting is not a rave or nightclub. Because once in the susceptible population an individual is much more likely to become infected than to decide to dislike raves and nightclubs, δ_v is larger than δ_s. Individuals leave the surroundings of raves and nightclubs at a higher rate from the recovered class than from the susceptible class because those who recovered have had the time and experience to see the dangers of ecstasy use and decide whether they enjoy the club scene.

5. **Numerical approaches.** Mathematically, one can check that $(\epsilon - \delta_s)/\mu \leq 1$ if and only if $\epsilon/(\mu + \delta_s) \leq 1$. As a threshold in determining the qualitative behavior of the core-free equilibrium $(1, 0, 0, 0)$, $(\epsilon - \delta_s)/\mu$ and $\epsilon/(\mu + \delta_s)$ can be considered the same. One can choose either of them as the tipping point. In the rest of this paper, we will use $R_c = \epsilon/(\mu + \delta_s)$ that is always positive.

From Section 3, we have known that there are at least two positive equilibria under certain circumstances and backward bifurcation occurs. Therefore, it would be very hard to do a complete dynamics analysis. However, since our main goal is to explore the effect of peer pressure on ecstasy use, numerical simulations with help of real parameters give some reasonable insights. From a large number of simulations, we picked four relevant cases: $R_c < R_0 < 1$, $1 < R_c < R_0$, $R_c < 1 < R_0$, and finally $R_c > 1 > R_0$. These simulations are done by varying one or two parameters out of the set of estimated values.

5.1. $R_c < R_0 < 1$**.** Results show that when $R_c < R_0 < 1$, the noncore population increases and accounts for the whole population, while the core population of susceptibles, infected and recovered decreases to zero. For this case we used the following parameter values as $\phi = 0.007$, $\mu = 0.007$, $\epsilon = 0.03$, $\delta_s = 0.032$, $\delta_v = 0.05$, $\gamma = 0.011$, $\tau = 0.016$, $\alpha = 0.5$. Hence, $R_c = 0.76923$ and $R_0 = 0.92647$. In other words, as can be shown in Figure 3, all individuals between the ages of 13 and 25 not only stop using ecstasy but stop frequenting raves and nightclubs.

5.2. $1 < R_c < R_0$**.** When $1 < R_c < R_0$ the noncore population declines and the ecstasy class is growing. For this case we used the same parameter values as the example above, except for $\phi = 2.75$ and $\epsilon = 0.0391$. Hence, $R_c = 1.0026$ and $R_0 = 1.1392$. In this situation there is one stable endemic equilibrium $(0.1999, 0.0028, 0.7616, 0.0357)$. These values describe the stabilized conditions that each class will approach over time. This equilibrium predicts that the noncore class will become roughly 20 percent of the population, the susceptibles will make up 0.3 percent, the infected, 76.2 percent, and the recovered, 3.5 percent (see Figure 3).

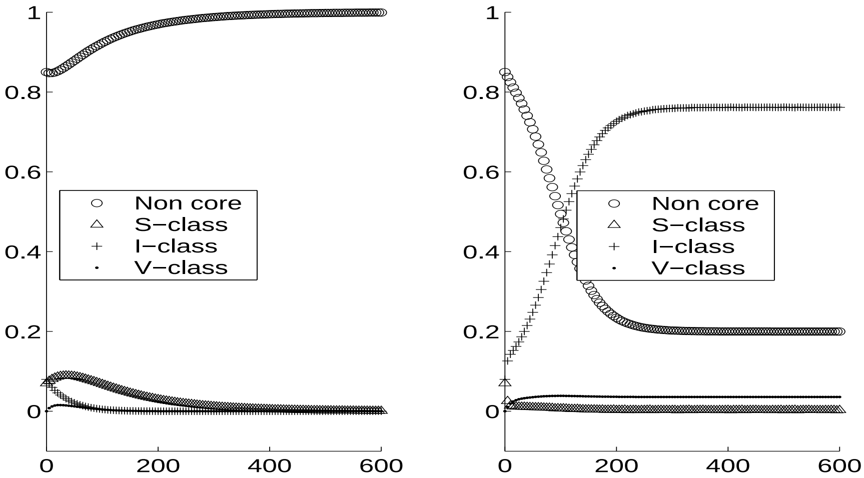

FIGURE 3. The left figure plots the solution for the case $R_c < R_0 < 1$ and the right figure for the case $1 < R_c < R_0$.

5.3. $R_c < 1 < R_0$. Numerically we found a new backward bifurcation when $R_c < 1 < R_0$ (see Figure 4). We will not explore this bifurcation theoretically in this paper. However we are interested in the outcome from this bifurcation. If $R_c < 1 < R_0$, multiple equilibria describe the circumstance of locally stable endemic and core-free equilibrium (see Figure 4). Parameter values within this situation could be the following: $\phi = 0.275$, $\mu = 0.007$, $\epsilon = 0.03$, $\delta_s = 0.032$, $\delta_v = 0.05$, $\gamma = 0.011$, $\tau = 0.016$, $\alpha = 0.5$. In this simulation, we only change ϕ to 0.275 from the last case, and we get $R_c = 0.76923$ and $R_0 = 36.3971$. In this situation, both the core-free equilibrium and the endemic equilibrium are locally asymptotically stable, while the ecstasy-free equilibrium does not exist. For the above-mentioned parameters, the two endemic equilibria are E_1=(0.7906, 0.0676, 0.1134, 0.0283) and E_2 =(0.2860, 0.0299, 0.6468, 0.0373). The endemic equilibrium with more ecstasy users, E_2, is locally asymptotically stable. This bifurcation occurs between two separate boundaries. Let us use ϵ as the bifurcation parameter to elaborate this. First, when $\epsilon < \mu$, the core-free equilibrium is globally asymptotically stable. For example, $\epsilon = 0.006999$ gives $R_c = 0.1795$ and $R_0 = 10.3439$, so this bifurcation no longer happens. On the other spectrum, $\epsilon = 0.039$ leads to $R_c = 1$ and $R_0 = 0$. If ϵ is any larger, R_c becomes greater than 1 and $1 > R_0 > 0$, which is a condition described by the first backward bifurcation discussed in subsection 3.2. Therefore, the second bifurcation occurs when $0.1795 < R_c < 1$ and $R_0 > 10.3439$ for these parameter values. In a backward bifurcation, solutions go to a certain equilibrium depending on the basins of attraction of each equilibrium. In this case, the stable equilibrium exists, where the susceptibles made up approximately 3 percent of the population, the infected made up 64.7 percent of the population; the noncore, 28.6 percent; and the recovered, 3.7 percent (see Figure 5). Hence, if the initial values are in the basin of attraction for the stable endemic equilibrium, a large increase in the infected class can still occur. This backward bifurcation means that even if the rates for leaving the susceptible class are higher than the rates of moving into the susceptible class, the susceptible class can still sustain an epidemic of ecstasy use. If enough people frequently go to raves and nightclubs to begin with, the population using ecstasy can grow despite a declining susceptible class. Our estimate of an initial susceptible population of 8 percent was enough to create an epidemic of ecstasy. Since the population in the susceptible class is too small to create this epidemic, the core classes decrease to zero as described in the core-free equilibrium, and the members of our population stop attending raves and nightclubs (see Figure 4).

5.4. $R_c > 1$ **and** $R_0 < 1$**.** If $R_c > 1$ and $R_0 < 1$ two situations are possible because of the backward bifurcation occurring in this system. In this illustration, we used $\phi = 0.275$, $\mu = 0.007$, $\epsilon = 0.0391$, $\delta_s = 0.032$, $\delta_v = 0.05$, $\gamma = 0.011$, $\tau = 0.016$, $\alpha = 0.5$. Note that only ϵ was changed from the last case. Here, $R_c = 1.0026$ and $R_0 = 0.11392$. Let R_e denote the point where the backward bifurcation turns around. We call R_e the turning point [3, 9]. It is also the value of $R_0 < 1$ when the backward bifurcation no longer occurs and the endemic states cease to exist. By fixing all the parameters except for ϕ and then varying this last parameter, we can approximate the condition of R_0 where the backward bifurcation turns around at $R_0 = R_e = 0.015$. When $R_e < R_0 < 1$, the model exhibits this bifurcation, and if $R_0 < R_e$, the classes will stabilize at the ecstasy-free equilibrium. In the latter case, the infected and recovered classes will decrease to zero, and the susceptible and noncore classes will stabilize to a certain proportion of the total population based on the parameter values μ, ϵ and δ_s, which are in this equilibrium. For these parameters, since the ecstasy-free equilibrium is $(0.9858, 0.0142, 0, 0)$, almost the entire population stops attending raves and nightclubs (98.58 percent), while the remainder of the population (1.42 percent) continues to go to raves and nightclubs but does not use ecstasy. In this case, there are no ecstasy users left in the population, taking away the peer pressure to experiment with the drug at these events. On the other hand, if $R_e < R_0 < 1$, there are two endemic equilibria. The equilibrium, E_2=(0.2066, 0.0286, 0.7285, 0.0362) with the larger infection value, is locally asymptotically stable, while the other equilibrium, E_1=(0.8642, 0.1064,

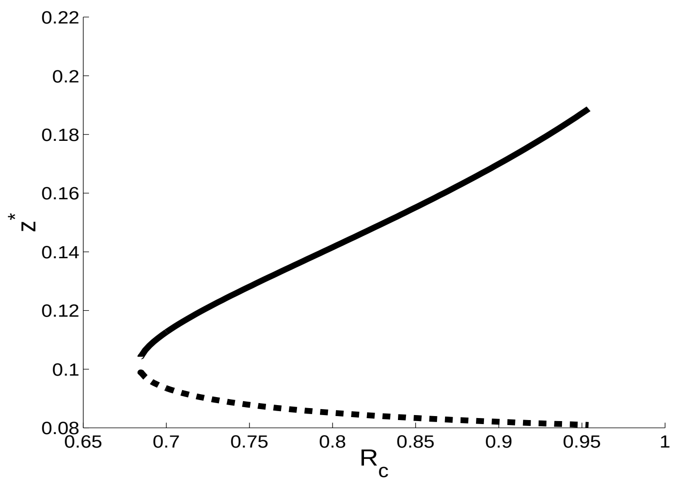

FIGURE 4. Bifurcation diagram when $R_0 > 1$. The bifurcation parameter is ϕ, and other parameters are $\mu = 0.0174$; $\delta_v = 0.0075$; $\alpha = 0.8$; $\epsilon = 0.25$; $\delta_s = 0.22189$; $\gamma = 0.0065$; and $\tau = 0.0045$.

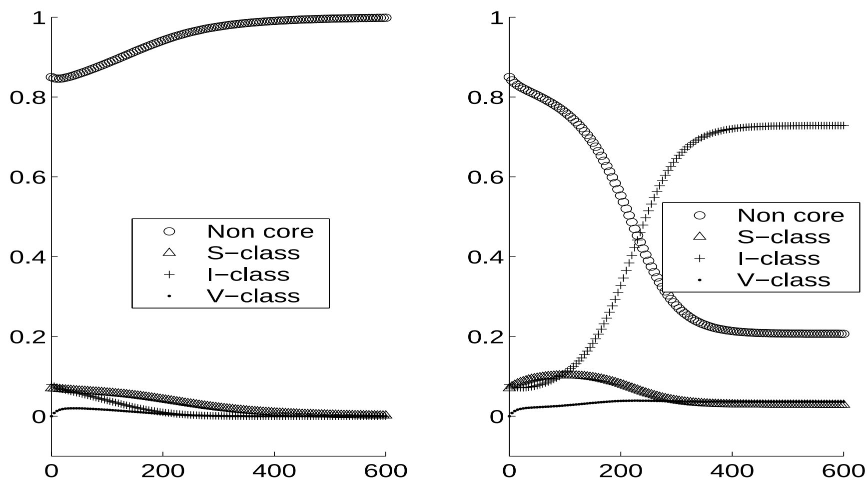

FIGURE 5. When $R_c < 1 < R_0$ there is no outbreak (see the left figure), while when $1 < R_c$ and $R_0 < 1$, outbreak occurs (see the right figure).

0.0203, 0.0090), is unstable. The ecstasy-free equilibrium is also locally asymptotically stable. Therefore, for these parameters, if ecstasy is at an epidemic level, R_0 must be reduced below $R_e = 0.015$ to get rid of ecstasy use. The results from these parameters then suggest that ecstasy use is extremely difficult to get rid of.

When $R_0 < 1$, an ecstasy epidemic may arise very suddenly and produce a large change in the number of infected during a short period of time. Ecstasy use then increases steadily up to and past R_0. For these parameters, it can also be concluded that the basin of attraction for the ecstasy-free equilibrium is minute, and therefore a small number of ecstasy users may lead to an epidemic. This possibility for an epidemic when $R_0 < 1$ is significant because it implies that peer pressure can cause a large, sudden increase in ecstasy use, even when rates describing infection are low. A substantial initial population of ecstasy users can cause an epidemic even if the "infectious period" $1/(\gamma + \tau + \mu)$ and the"infection rate" ϕ are very small. Furthermore, since $R_e = .015$, the initial population of ecstasy users needed to cause this epidemic is not considerable. This phenomenon, described by the backward bifurcation, is exactly what have occurred in the last decade, whereby ecstasy use has gone from virtually nonexistence to an epidemic where 1 in 9 high school seniors have tried the drug [22]. Additionally, this bifurcation shows the influence of the rave culture. Again, this is what we have witnessed among today's youth. As raves go more mainstream, ecstasy becomes more popular [10].

As $R_c > 1$, the noncore population will decline. At the same time, the number of ecstasy users will grow, since $R_0 > R_e$. A simulation was carried out by setting 85 percent of the population in the noncore class, 8 percent in susceptible, 7 percent in infected and 0 individuals in the recovered class. Therefore, if 7 percent of our population goes to raves and nightclubs and uses ecstasy, this is enough to cause an epidemic of ecstasy use when $R_0 < 1$.

It should be recognized that the parameters values which caused this situation are the exact set that we calculated from research. Although our parameters predict an eventual equilibrium point where approximately 73 percent of the population are ecstasy users, we still believe they are a good indicator of the trend that ecstasy use will follow for the next decade unless serious prevention and education programs are implemented. From these results, we predict that if ecstasy abuse is left to itself, abuse of the drug will continue its gradual increase among young people. In addition, these parameters show that if raves go even more mainstream, ecstasy use could skyrocket.

Also note that from the previous case of $R_c < 1$ and $R_0 > 1$, all the parameters remain the same while ϵ is increased by .001. This tiny change, however, is enough to jump the results from one bifurcation to another. A small augmentation in the recruitment rate creates a base for widespread ecstasy use. Then ϵ is a very important parameter to watch.

6. **The effect of education.** On March 21, 2001, in front of the Senate Caucus on International Narcotics Control, Donnie R. Marshall gave a testimony entitled, "MDMA and the 'Rave' Scene: A Rapidly Growing Threat". Marshall outlined a list of demand reduction strategies, which have been institutionalized by the DEA. The strategies included providing accurate, complete and current information on the scientific findings and medical effects of ecstasy on the human body through Internet web sites and publications; purchasing Internet "keywords" to ensure these anti-drug messages are seen first; working with local, state, and other federal agencies and other nonprofit organizations in an effort to advance drug education; enhancing parental knowledge of raves and ecstasy use and engaging their active participation in education and prevention of drug abuse; and educating high school and college students on the realities of raves and the effects of ecstasy use on the body. Here, it is clear that the DEA believes that the best approach to halt ecstasy popularity is through knowledge [21]. Our model strongly supports this viewpoint.

In our model, almost all parameters can take into consideration the effect of education on ecstasy use. We assumed that education would decrease rates leading into the susceptibles (ϵ) and the infected (ϕ and α) and increase rates moving out from the susceptibles (δ_s), infected (γ and τ) and recovered (δ_v). We found that the most influential parameters in reducing the use of ecstasy are ϕ, ϵ and δ_s.

One intended effect of education is to keep people from using ecstasy. To investigate this effect, we set all the parameters at the estimated values, and then lowered ϕ by 0.02. Initial conditions are set at 85 percent noncore population, 8 percent susceptibles, 7 percent infected and 0 percent recovered. These estimates are based on the 2001 *Monitoring the Future* reported by the NIDA[16]. As ϕ is lowered, the infection rate and the value of R_0 decrease. Therefore, if ϕ is small enough, the number of those who become habitual ecstasy users will decline. From these simulations, however, it is clear that this infection rate must be lowered by a substantial portion. At these initial values, ϕ must be decreased from 0.275 to 0.195 before ecstasy use stabilizes and then must be lowered even further to 0.175 before there is any clear decline in ecstasy's popularity. It is highly unlikely that education would reduce this rate that much, as can be seen from education campaigns directed at other illicit drugs. Therefore, focusing education on the infection rate, is not a good solution to this country's ecstasy problem. On the other hand, a reduction of the rate of recruitment into the population that frequents raves and nightclubs (ϵ) has a much greater effect on the number of infected individuals than the infection rate (ϕ). Starting from the estimated value of 0.0391, if the value for ϵ is cut to

0.0331, the number of infected individuals begins a clear decline. This decrease in ϵ is about 1/16 the reduction of ϕ needed to have any impact on ecstasy abuse. Additionally, when the value for ϵ is decreased, the value of R_c also decreases. Since R_c is the threshold at which a change occurs between the susceptible and noncore population, a decrease in ϵ leads to a decline in the number of individuals in the susceptible class and an increase in the population of the noncore class. At the estimated parameters, R_c is barely greater than 1, meaning a small change in ϵ will have a large effect on the system. Therefore, education should be focused on keeping young adults from going to raves and nightclubs. Increasing δ_s has a similar effect as decreasing ϵ. This makes sense because δ_s is just the recruitment rate out of the susceptible class into the noncore population. Here, R_c becomes less than 1 and the number of individuals in the susceptible and infected classes decrease, while the noncore population increases. Again, this result is another ideal solution to the ecstasy problem. On the other hand, decreasing ϵ seems to be more efficient because the increase of δ_s necessary to achieve the same effect as ϵ also depends on the μ value since both μ and δ_s appears in the denominator of R_c.

Of all the parameters, ϵ, the recruitment rate from the noncore population into the susceptible class, requires the smallest percent change in increment to decrease ecstasy use, and ϵ is followed most closely by δ_s. Therefore, any education efforts should be focused not on ecstasy use itself, but instead on the behaviors that lead to taking that first ecstasy pill: the surrounding of a rave or nightclub. Attempts to decrease ecstasy use would be most successful if education programs were aimed at keeping young adults from seeking the entertainment of raves and nightclubs.

7. **Results and conclusions.** To obtain a representation of the effects of peer pressure on a population between the ages of 13 and 25, we crafted a deterministic model. We studied this model analytically and then illustrated our model numerically using the estimated parameters. In the numerical analysis, we focused on the parameters ϵ and ϕ to compare the importance of peer-driven recruitment into the susceptible population to peer-driven ecstasy use. Finally, we varied all the parameters so as to predict the most efficient manner of decreasing ecstasy use by means of education.

From the simulations we find four basic situations that can take place. One situation might be that all people between the ages of 13 to 25 stop attending nightclubs and raves so that the noncore class becomes 100 percent of the population. This state arises when the core-free equilibrium is globally asymptotically stable, implying $R_c < R_0 < 1$ and $\epsilon < \mu$. Here, peer pressure from the core population has little effect on the noncore.

Second, the core population could also become zero when $R_0 > 1$, $R_c < 1$ and $\epsilon > \mu$, if R_c and R_0 are within the boundaries of a backward bifurcation. These boundaries are different for every set of parameter values and are just the values of R_c and R_0 where multiple endemic equilibria exist. The core-free equilibrium is locally asymptotically stable under this condition, indicating that if there is a small enough initial population of susceptibles, the core classes will cease to exist. On the other hand, if the number of people in the susceptible class is large enough to be within the basin of attraction of the stable endemic equilibrium, the number of people habitually using ecstasy can grow despite decreasing numbers coming into the susceptible class. Here, peer pressure to use ecstasy in the raves and nightclubs is very strong, but the pressure to go to raves is not. This is enough to maintain high levels of ecstasy prevalence in the population.

Third, when $1 < R_c < R_0$, there is one stable endemic equilibrium. Given that $R_0 > 1$, the noncore class will decline and the ecstasy class will grow. In this case the pressure to use ecstasy is very strong and is felt by all the classes. However, if R_0 is not too large, then the prevalence of ecstasy is not huge.

Finally, another backward bifurcation describes what happens when $R_0 < 1$ and $R_c > 1$. Two positive endemic equilibria exist and the endemic equilibrium with the larger infection value along with the ecstasy-free equilibrium are both locally asymptotically stable. Depending on the location of the initial population of infected within the basins of attraction of each equilibrium, ecstasy use will either decrease to zero or reach an epidemic state. The existence of a large population of ecstasy users, therefore, creates a lot of pressure on young adults, especially in the susceptible class, to begin using ecstasy. On the other hand, a small population of ecstasy users may exert little influence on the other classes.

Simulations were used to test the effectiveness of education on decreasing the popularity of ecstasy use. In these simulations, we used a set of parameter values, estimated from published research, to describe the current situation of ecstasy use in the United States. These estimated parameters project a slow increase in the population of habitual ecstasy users over the next 12 years. Next, to study possible strategies of education to combat ecstasy use, we varied all the values of parameters (excluding μ, the death and aging rate). From these simulations, we observed that ϵ, the recruitment rate from the noncore to the susceptible class, was the most crucial value in causing an ecstasy epidemic. As ϵ is in both R_c and R_0, it plays a vital role in both the movement into the rave and nightclub culture and the start of ecstasy abuse. Furthermore, ϵ (the peer pressure based recruitment rate) is the parameter for which the least decrease in value has the most impact in halting the spread of ecstasy growth.

On June 3, 1996, the *New Yorker* published an article by Malcolm Gladwell entitled "The Tipping Point" [4]. In this commentary, Gladwell describes how epidemic theory is being applied to social problems. He also mentions the concept of a "tipping point" or a threshold at which a stable phenomenon can turn into a social crisis. "Every epidemic has its tipping point, and to fight an epidemic you need to understand what that point is," writes Gladwell [4]. Unfortunately, from modeling ecstasy we have shown that the situation is not as simple as Gladwell descriptions. There is often more than one tipping point, each with complicated conditions, all of which must be understood to effectively battle a problem such as ecstasy use. Furthermore, abrupt changes can occur from slight variations in initial conditions below the tipping point.

Despite the complexity of our system of equations, we can still learn a lot about ecstasy use from this model. First, peer pressure can drive a sudden increase in ecstasy use, even when threshold conditions seem to predict against this growth. A small group of ecstasy users can also cause an epidemic of ecstasy use if there are enough individuals going to raves and nightclubs. Recruitment into the susceptible class, therefore, is the most important factor in determining the extent of ecstasy use. A small increase in ϵ can jump a solution from few infected individuals to an epidemic. Conversely, a small decrease in this term can also solve the problem of ecstasy use in entirety. For this reason, we conclude that most education efforts should be focused at keeping young adults from seeking the excitement of raves and nightclubs. Finally, this model shows once a considerable number of people begin to use ecstasy, decreasing this number is extremely difficult. In other words, a peer-driven drug epidemic should be avoided at all costs.

Donald Vereen, Jr., the deputy director of the office of National Drug Control Policy, stated that "ecstasy is one of the most problematic drugs that has emerged in recent years", and is clearly the most widely abused club drug. Fortunately, recent actions by Congress and organizations such as the DEA seem to indicate that the US government has awakened to the serious problem of ecstasy use among our nation's youth and the importance of raves and nightclubs in creating an ecstasy-friendly environment. Senator Bob Graham (D-Florida), who has said that "ecstasy use and trafficking continue to grow at *epidemic* proportions", has led the fight in Congress with the introduction and passage of two bills, the Ecstasy Anti-Proliferation Act of 2000 and the Ecstasy Prevention Act of 2001. These acts have led to longer jail sentences for ecstasy traffickers, ten million dollars in funding to specifically combat the use and abuse of ecstasy and assistance to communities, law enforcement and research facilities. Moreover, the Ecstasy Prevention Act of 2001 grants "shall give priority to communities that have taken measures to combat club drug use, including passing ordinances restricting rave clubs" [11]. Similarly, DEA divisions in New Orleans and Idaho have shown that taking initiatives against rave promotors has a significant impact on ecstasy-related overdoses and ecstasy abuse in general [12]. Although data on the rates of ecstasy abuse are contradictory, depending on the study (the Monitoring the Future study for 2002 showed decreases in use while the 2002 Partnership Attitude Tracking study found a 20 percent increase in use since 2001), there seem to be some signs that at least the spread of ecstasy use my be slowing [11, 12]. These advances against the spread of ecstasy along with the predictions from our model on the impact of education programs demonstrate that the combination of knowledge and the control of raves and nightclubs would win the fight against an ecstasy epidemic.

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REFERENCES

[1] R. N. Anderson, K. D. Kochanek, and B. L. Smith, *Deaths: Preliminary Data for 1999,* Division of Vital Statistics, vol. 49, number 3. http://www.cdc.gov/nchs/ data/nvsr/nvsr49/ nvsr4903.pdf (accessed on July 26, 2003).

[2] J. Beck and M. Rosenbaum, *Pursuit of Ecstasy*, State University of New York Press, Albany, 1994.

[3] C. Castillo-Chavez and B. Song, *Models for the Transmission Dynamics of Fanatic Behaviors*, in: *Bioterrorism: Mathematical Modeling Applications in Homeland Security (SIAM Frontier Series in Applied Mathematics)*, pp.155-172, H. T. Banks and C. Castillo-Chavez (eds.), SIAM, Philadelphia, PA, 2003.

[4] M. Gladwell, *The Tipping Point. New Yorker.* June 3, 1996.

[5] L. D. Johnston, P. M. O'Malley, and J. G. Bachman, *Monitoring the Future: National Survey Results On Drug Use*, 1975-1999, Vol 1-2, US Department of Health and Human Services, 2000.

[6] W. Kissin and J. Ball, Drug Abuse Warning Network: The DAWN Report. *Club Drugs, 2001 Update*, October 2002.

[7] A. I. Leshner (ed.), *Principles of Drug Addiction Treatment: A Research Based Guide*, National Institute on Drug Abuse, http://www.nida.nih. gov/PODAT/PODATIndex.html. August 1, 2001 (accessed on April 15, 2003).
[8] A. Schafter, National Clearinghouse for Alcohol and Drug Abuse, *Congress' RAVE Act Facing Opposition from ACLU and Ravers*, July 25, 2002, http://www.health.org /newsroom/rep/195.aspx (accessed on April 15, 2003).
[9] B. Song, *Dynamical Epidemical Models and Their Applications*, Phd Dissertation, August 2002, Cornell University.
[10] C. Vaughn, National Clearinghouse for Alcohol and Drug Information Reporter, *MDMA: Part II in a Series*, October 19, 2001, http://www.health.org/newsroom/rep/175.aspx. (accessed on April 15, 2003).
[11] Congress Acts, *US Senator Bob Graham: Anti Ecstasy Initative*, http://www.senate.gov /˜graham /ecstasy.html #legislation (accessed on April 29, 2003).
[12] DEA Congressional Testimony, *Statement of Asa Hutchinson, Administrator, Drug Enforcement Administration, Before the House Government Reform Subcommittee on Criminal Justice, Drug Policy, and Human Resources*, September 19, 2002 (accessed on April 15, 2003).
[13] Drug Enforcement Administration, *3,4-Methylenedioxymethamphetamin* (MDMA, XTC, X, Ecstasy, Adam), July 15, 2001. http://www.usdoj.gov/dea/concern/mdma/mdma020700.htm (accessed on December 20, 2004).
[14] National Clearinghouse for Alcohol and Drug Abuse, *Prevention Alert*, July 28, 2000. http://www.health.org/govpubs/prevalert/v3i25.aspx (accessed on May 10, 2003).
[15] National Institute on Drug Abuse, *US Senate Caucus on International Narcotics Control*, July 25, 2000, http://www.drugabuse.gov/testimony17-25-00testimony.html(accessed on April 15, 2003).
[16] National Institute of Drug Abuse, *Monitoring the Future: Trends in Prevalence of Various Drugs for 8th-Graders, 10th-Graders, and High School Seniors 1999-2001*, http://www.nida.nih.gov/Infofax/HSYouthtrends.html (accessed on December 20, 2004).
[17] National Center on Addiction and Substance Abuse at Columbia University, *2000 CASA National Survey of American Attitudes on Substance Abuse VI: Teens*, http://www.casacolumbia.org/newsletter1457/newsletter (accessed on December 20, 2004).
[18] Message From the Director of NIDA, http://www.nida.nih.gov/about/welcome/message MDMA404.html (accessed on December 31, 2004).
[19] Strategic Europe/Asia Unit of the Office of International Intelligence, Drug Enforcement Administration, *Ecstasy: Rolling Across Europe*, August 2001. http://www.usdoj.gov/dea/pubs/intel/01008/ (accessed on April 15, 2003).
[20] United States Census Bureau, *Population by Age*, July 29, 2001, http://www.census.gov /population/estimates/ nation/intfile2-1.txt(accessed on December 20, 2004).
[21] United States Department of Justice, *DEA Congressional Testimony*, March 21, 2001, http://www/usdoj.gov/dea/pubs/cngrtest/ct032101/htm (accessed on May 20, 2003).
[22] U.S. Department of Justice Drug Enforcement Administration, *Ecstasy and Predatory Drugs*, Febuary 2003, http://www.dea.gov/concern/clubdrugs.html (accessed on May 10, 2003).

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