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ON THE GEOMETRY AND TOPOLOGY OF SINGULAR OPTIMAL CONTROL PROBLEMS AND THEIR SOLUTIONS

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Abstract. The existence of singular arcs for optimal control problems is studied by using a geometric recursive algorithm inspired in Dirac's theory of constraints. It is shown that singular arcs must lie in the singular locus of a projection map into the coestate space. After applying the geometrical recursive constraints algorithm, we arrive to a reduced set of hamiltonian equations that replace Pontriaguine's maximum principle. Finally, a global singular perturbation theory is used to obtain nearly optimal solutions.

1. **Introduction.** Optimal control problems are often singular [1] and the equations obtained using Pontriaguine's maximum principle become algebraic-differential equations whose analysis is much more involved than ordinary differential equations in normal form. Neither existence nor uniqueness of solutions are guaranteed anymore.

Diverse methods have been developed to study the existence and uniqueness of solutions in problems leading to implicit differential equations, some of them are inspired in Dirac's theory of constraints [7], a geometric algorithm invented by P.A.M. Dirac to study the existence and uniqueness of solutions of Euler-Lagrange equations for singular first order Lagrangians, i.e., Lagrangians such that their Euler-Lagrange equations cannot be put in normal form. Such algorithm was refined until it got its final geometrical form, the Presymplectic Constraint Algorithm, Gotay *et al* [12]. In parallel, other approaches to the theory of implicit differential equations were taking place. Just to mention a few we can refer to Takens [25], Campbell [3], Rheinboldt [22], Chua [4] and Gracia and Pons [9]. More recently, and almost simultaneously, an unifying version of the various constraint algorithms was proposed by Rabier and Rheinboldt [23] and Mendella, Marmo and Tulczyjew [19]. This algorithm has a neat geometrical formulation.

In this paper we will analyze the existence and uniqueness of solutions for singular optimal control problems by using the geometric recursive constraints algorithm for implicit differential equations mentioned above. We will review first the geometric formalism of singular optimal control problems following López and Martínez [17], Cortés [5] and Bloch and Crouch [2] which is closer to the work by Jurdjevic [14] and Sussmann [24]. After that we will review the geometric recursive constraint algorithm for implicit differential equations in general, and for quasilinear equations and presymplectic systems in particular. The application of the algorithm to

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