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CRITICAL POINT, ANTI-MAXIMUM PRINCIPLE AND SEMIPOSITONE P-LAPLACIAN PROBLEMS

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Abstract. In this paper, we use Nehari manifold to extend the anti-maximum principle of Laplacian operator to an existence theorem for p-Laplacian ($p \neq 2$), then consider the existence of nonnegative solutions to semipositone quasilinear elliptic problems $-\Delta_p u = \lambda f(u), x \in \Omega; u > 0, x \in \Omega; u = 0, x \in \partial\Omega$.

1. **Introduction.** In this paper, we use Nehari manifold to extend the anti - maximum principle to an existence theorem for p-Laplacian, and use it to consider the existence of nonnegative solutions to semipositone quasilinear elliptic equations.

Let $\Omega \subset R^n$ be a bounded smooth domain ($\partial\Omega$ is of class C^2). Let L denote the differential operator:

$$Lu = -\sum_{i,j=1}^{n} a_{i,j} \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{i=1}^{n} a_i \frac{\partial u}{\partial x_i} + au, \tag{1}$$

where $a_{i,j} \in C(\bar{\Omega}), a_{i,j} = a_{j,i}, a \geq 0$, and $\sum\limits_{i,j=1}^{n} a_{i,j}\xi_i\xi_j > 0$ for $x \in \bar{\Omega}$ and $\xi = (\xi_i) \in R^n \backslash \{0\}$, and $a_i, a \in L^\infty(\Omega)$. The following Dirichlet boundary value problem was considered by several authors to get the anti-maximum principle, which was first proved by using a Lyapunov-Schmidt reduction by Clément and Peletier [11] and Hess [15], several extensions and refinements of anti-maximum principle have been proved in [2][5][12],[13],[18],[20] etc. Let consider

$$Lu - \lambda m u = f(x), x \in \Omega, u = 0, x \in \partial\Omega, \tag{2}$$

where $m \in L^\infty(\Omega), m(x_0) > 0$ for some $x_0 \in \Omega$.

Let $r > n$, and let $X = \{u \in W^{2,r}(\Omega) : u = 0 \text{ on } \partial\Omega\}$ and let $Y = L^r(\Omega)$. Let the operator $A : X \to Y$ be defined by $Au = Lu$. Then it is well known [16] that A has a unique principal eigenvalue $\lambda_1(m)$ ($\lambda_1(m)$ is the principal eigenvalue of A with weight m) having a strict positive eigenfunction e_1 such that $e_1(x) > 0, x \in \Omega, \frac{\partial e_1}{\partial n} < 0, x \in \partial\Omega$. The anti-maximum principle for (2) can be stated along with classical maximum principle as follows:

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