

## GLOBAL ATTRACTIVITY, I/O MONOTONE SMALL-GAIN THEOREMS, AND BIOLOGICAL DELAY SYSTEMS

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**Abstract.** This paper further develops a method, originally introduced by Angeli and the second author, for proving global attractivity of steady states in certain classes of dynamical systems. In this approach, one views the given system as a negative feedback loop of a monotone controlled system. An auxiliary discrete system, whose global attractivity implies that of the original system, plays a key role in the theory, which is presented in a general Banach space setting. Applications are given to delay systems, as well as to systems with multiple inputs and outputs, and the question of expressing a given system in the required negative feedback form is addressed.

**1. Introduction.** In their paper, Angeli and Sontag [2] introduced an approach for establishing sufficient conditions under which a dynamical system  $\Phi$ , described by ordinary differential equations, is guaranteed to have a globally stable equilibrium. The method may be applied whenever  $\Phi$  can be decomposed as a negative feedback loop around a monotone controlled system. A discrete system is associated to  $\Phi$ , and its global attractivity toward an equilibrium implies that of  $\Phi$ .

In this paper, we generalize the results of Angeli and Sontag [2] in several directions: (i) we address the stability of the closed loop system, which was not done in [2], (ii) we prove results which are novel even in the finite-dimensional case, in particular allowing the consideration of systems with multiple inputs and outputs, and (iii) we extend considerably the class of systems to which the theory can be applied and the above characterization holds, by formulating our definitions and theorems in an abstract Banach space setting. The extension to Banach space forces us to develop very different proofs, but it permits the treatment of delay-differential and other infinite-dimensional systems. In addition, we work-out a number of interesting examples, exploit a useful necessary and sufficient condition for monotonically decreasing discrete systems to be globally attractive which leads to sufficient tests for stability of our negative feedback loops, and provide a procedure for decomposing a system as the negative feedback closed loop of a monotone controlled system (Appendix 1). We rely on basic results from the theory of monotone systems, but most necessary concepts will be defined in the text. The reader is encouraged to consult Smith [33] for further references on this topic.

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