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LOCKING-FREE NONCONFORMING FINITE ELEMENTS FOR PLANAR LINEAR ELASTICITY

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Abstract. In this paper we introduce a nonconforming finite element method for a planar linear elasticity problem. We show that this nonconforming method is robust in that error estimates generated by it are uniform with respect to one of the Lamé elasticity constants, λ; i.e., it is locking-free. Applications to nonconforming P_1 and rotated Q_1 finite elements are discussed.

1. **Introduction.** In recent years there have been applications and analysis of the nonconforming rotated Q_1 finite element for the numerical solution of partial differential equations. This nonconforming element was first developed and analyzed in [1, 4, 10]. In [1, 4], it has been derived from a mixed finite element. It has been shown that the nonconforming method using this nonconforming element is equivalent to the mixed method utilizing the lowest-order Raviart-Thomas mixed element on rectangles [11]. In [10], it has been studied for the Stokes problem. This nonconforming element provides the simplest example of discretely divergence-free nonconforming elements on quadrilaterals. There also exist n-dimensional counterparts of this element, with analogous properties [1]. There have been applications of this element to semiconductor devices [4] and liquid crystals [8]. For multigrid and multilevel algorithms for solving linear systems of algebraic equations arising from this element, the reader may refer to [5].

In this paper we apply the nonconforming rotated Q_1 finite element to the numerical solution of a planar linear elasticity problem and carry out an error analysis for this application. It is known [6] that the application of classical conforming finite elements to elasticity generates the phenomenon of locking which arises when the elastic material becomes nearly incompressible. That is, in terms of error estimates, the constant in front of these estimates blows up when one of the Lamé constants, λ, goes to infinity. In this paper we show that this locking phenomenon can be overcome using the nonconforming rotated Q_1 finite element. Namely, the error constant is uniformly bounded with respect to this Lamé constant. The nonconforming finite element method presented is in a general form so that it also includes an application of the usual nonconforming P_1 finite element.

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