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ZERO CORRELATION ZONE SEQUENCE SET WITH INTER-GROUP ORTHOGONAL AND INTER-SUBGROUP COMPLEMENTARY PROPERTIES

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Abstract. In this paper, a novel method for constructing complementary sequence set with zero correlation zone (ZCZ) is presented by interleaving and combining three orthogonal matrices. The constructed set can be divided into multiple sequence groups and each sequence group can be further divided into multiple sequence subgroups. In addition to ZCZ properties of sequences from the same sequence subgroup, sequences from different sequence groups are orthogonal to each other while sequences from different sequence subgroups within the same sequence group possess ideal cross-correlation properties, that is, the proposed ZCZ sequence set has inter-group orthogonal (IGO) and inter-subgroup complementary (ISC) properties. Compared with previous methods, the new construction can provide flexible choice for ZCZ width and set size, and the resultant sequences which are called IGO-ISC sequences in this paper can achieve the theoretical bound on the set size for the ZCZ width and sequence length.

1. Introduction

Complementary sequence (CS) set with zero correlation zone (ZCZ), seen as a tradeoff between traditional CSs [7]-[15] and ZCZ sequences [2], possesses larger set size than traditional CS set and better correlation properties than ZCZ sequence set. It is well known that the set size of CS set is smaller than or equal to its flock size in spite of ideal auto-correlation function (ACF) and cross-correlation function (CCF), where each CS includes a flock of element sequences. By employing the idea of ZCZ to the construction of CSs, the generated CS set with ZCZ can efficiently solve the drawback of traditional CS set. For the construction of ZCZ sequences, a lot of methods have been presented and these constructions are mainly based on

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traditional CSs [2], perfect sequences [28]-[31], Gray mapping [14] and Hadamard matrix [8]-[19].

ZCZ sequence set is defined in terms of the minimum ZCZ width between any two sequences and its ZCZ distribution is not provided. In order to analyze ZCZ distribution condition in detail, one can divide a ZCZ sequence set into multiple ZCZ sequence groups, and generally intra-group cross-correlation property is different from inter-group one. In terms of the set-dividing idea, a lot of constructions for ZCZ sequences with each set containing multiple sequence groups were generated [25]-[29]. Some studies were interested in the ZCZ sequence set with inter-group zero cross-correlation zone (ZCCZ) larger than or equal to intra-group ZCCZ. According to the difference of sequence element, such sequence sets contain binary sequence sets [25]-[24], ternary sequence sets [11]-[10] and polyphase sequence sets which were called asymmetric ZCZ (A-ZCZ) sequence sets [27]-[20]. Also, Hayashi et al proposed a generalized construction of a ZCZ sequence set with a wide inter-group ZCZ from a given ZCZ sequence set [9], which did not limit the values of sequence element and can approach the theoretical bound. By employing the advantage of inter-group ZCCZ larger than intra-group ZCCZ, these sequence sets can be used to decrease inter-cell interference when different sequence groups are assigned to different cells. However, it is difficult for such sequence sets to achieve the corresponding theoretical bound of ZCZ sequences even if several constructions are quasi-optimal. In addition to the ZCZ sequence set with inter-group ZCCZ larger than or equal to intra-group ZCCZ, the other studies paid more attention to ZCZ sequence set with intra-group ZCCZ larger than inter-group ZCCZ. Based on mutually orthogonal complementary sequence sets, Rathinakumar et al proposed a novel binary sequence set including two mutually orthogonal ZCZ (MO-ZCZ) sequence groups [21]. In order to generate MO-ZCZ sequence set containing more sequence groups, several new constructions were presented [22]-[18], where the presented constructions generated polyphase MO-ZCZ sequences and the sequence set generated by Construction 2 in [18] can achieve the theoretical bound. Different from MO-ZCZ sequence set whose inter-group ZCCZ width is equal to zero, Torii et al proposed generalized MO-ZCZ (GMO-ZCZ) sequence set, where the inter-group ZCCZ width can be larger than or equal to zero [30]-[29].

Compared with traditional ZCZ sequence set, ZCZ sequence set containing multiple sequence groups are more suitable for reducing inter-channel and inter-cell interference since it posesses better correlation properties than traditional ZCZ sequence set. However, as a kind of unitary sequences working on a one-sequence-per-user basis, these constructed ZCZ sequence sets with each set containing multiple sequence groups can not possess both of ideal periodic and aperiodic correlation properties although their out-of-phase ACFs and CCFs are equal to zero within ZCZ. In contrast, the CS set with ZCZ working on a one-flock-per-user basis can obtain not only ZCZ properties for the whole sequence set but also ideal correlation properties among some special sequences.

Recently, CSs with ZCZ have received wide attentions and studies [3]-[34]. Thus sequences may be generally called as Z-complementary (ZC) sequences [3]. In addition to binary ZC sequences in [3], Li et al further studied quadriphase ZC sequences [17]. When the flock size of CSs with ZCZ is limited to be 2, there exist the generalized pairwise complementary (GPC) sequences [1] and the generalized pairwise Z-complementary (GPZ) sequences [4]. For more detail analyses of ZCZ distribution properties, CSs with multiple ZCZs or multiple different ZCZ widths were discussed, such as three ZCZs (T-ZCZ) sequences [35], inter-group complementary (IGC) sequences [16]-[6], ZC sequences with two-width ZCCZ [36] and multi-width ZCCZ complementary (MWZC) sequences [37]. In addition to CSs with aperiodic ZCZ, CSs with periodic ZCZ, namely Z-periodic complementary (ZPC) sequences, were also studied in [32]-[34].

In this paper, a new construction of CS set with ZCZ is presented on the basis of orthogonal matrices. One of motivations of this paper results from the fact that for ZCZ sequence set and CS set with ZCZ, the set-dividing method will possess better performance [25] [11] [21] [16]. Based on the set-dividing idea, this paper not only divides the constructed sequence set into multiple sequence groups, but also divides each sequence group into multiple sequence subgroups, which is different from previous constructions and will obtain more flexible choice for ZCZ width and set size. The properties of the generated sequence set can be listed as follows:

1) The sequence set consists of multiple sequence groups and further each sequence group consists of multiple sequence subgroups;

2) The sequences from different sequence groups are mutually orthogonal, namely inter-group orthogonal (IGO) property;

3) The sequences from different sequence subgroups in one sequence group have ideal periodic and aperiodic cross-correlation properties, namely inter-subgroup complementary (ISC) property;

4) Each sequence subgroup possesses T-ZCZ properties;

5) The presented IGO-ISC sequence set and its each sequence group can achieve the theoretical bound on the set size for the ZCZ width and sequence length, that is, they are optimal ones.

After presenting preliminaries in Section 2, the construction algorithm and properties of the proposed IGO-ISC sequence set are provided in Section 3 and 4, respectively.

2. Preliminary knowledge

Given a sequence set $\{\mathbf{a}_k, 0 \le k \le K-1\}$ with the k-th sequence $\mathbf{a}_k = (a_k(0), a_k(1), \cdots, a_k(L-1))$ of length L, the aperiodic CCF $\psi_{\mathbf{a}_k,\mathbf{a}_{k'}}(\tau)$ and the periodic CCF $\phi_{\mathbf{a}_k,\mathbf{a}_{k'}}(\tau)$ for a phase shift τ are respectively given as

$$\psi_{\mathbf{a}_k,\mathbf{a}_{k'}}(\tau)=\begin{cases}\sum_{l=0}^{L-1-\tau} a_k(l)\cdot a_{k'}^*(l+\tau), & 0\le\tau\le L-1;\\ \sum_{l=0}^{L-1+\tau} a_k(l-\tau)\cdot a_{k'}^*(l), & 1-L\le\tau<0;\\ 0, & |\tau|\ge L,\end{cases} \tag{1}$$

and

$$\phi_{\mathbf{a}_k,\mathbf{a}_{k'}}(\tau)=\sum_{l=0}^{L-1} a_k(l)\cdot a_{k'}^*(l+\tau)_L, \qquad 0\le\tau\le L-1, \tag{2}$$

where the symbol * denotes a complex conjugate and the notation $(\cdot)_L$ in (2) denotes a modulo L operation. When $k=k'$, the Eqs. (1) and (2) become aperiodic and periodic ACFs, respectively.

The interleaving operation among K sequences $\{\mathbf{a}_k, 0 \le k \le K-1\}$ can be given as

$$\begin{aligned}\mathbf{a}_0 \odot \mathbf{a}_1 \odot \cdots \odot \mathbf{a}_{K-1} = (&a_0(0), a_1(0), \cdots, a_{K-1}(0), a_0(1), a_1(1), \\ &\cdots, a_{K-1}(1), \cdots, a_0(L-1), a_1(L-1), \cdots, a_{K-1}(L-1)) \, . \end{aligned} \tag{3}$$

Let $\mathbf{B} = \{\mathbf{B}_m, 0 \le m \le M-1\}$ with $\mathbf{B}_m = \{\mathbf{B}_{m,n}, 0 \le n \le N-1\}$ and $\mathbf{B}_{m,n} = \{B_{m,n}(0), B_{m,n}(1), \cdots, B_{m,n}(L-1)\}$ be a sequence set with set size M, flock size N and element sequence length L. The sequence $\mathbf{B}_m$ is called a ZC sequence if

$$\sum_{n=0}^{N-1} \psi_{\mathbf{B}_{m,n},\mathbf{B}_{m,n}}(\tau) = \begin{cases} \sum\limits_{n=0}^{N-1} E_{\mathbf{B}_{m,n}}, & \tau = 0; \\ 0, & 1 \le |\tau| \le Z_a - 1, \end{cases} \tag{4}$$

where the notations Z_a and $E_{\mathbf{B}_{m,n}}$ denote the width of zero autocorrelation zone (ZACZ) and the energy of element sequence $\mathbf{B}_{m,n}$, respectively.

If any two sequences $\mathbf{B}_m$ and $\mathbf{B}_{m'}$ of $\mathbf{B}$ satisfy the following equation, then the set $\mathbf{B}$ becomes a ZC sequence set $(M, Z) - \mathrm{CS}_N^L$,

$$\sum_{n=0}^{N-1} \psi_{\mathbf{B}_{m,n},\mathbf{B}_{m',n}}(\tau) = 0, \qquad |\tau| \le Z_c - 1 \ \text{ and } \ m \ne m', \tag{5}$$

where Z_c denotes the width of ZCCZ and $Z = \min\{Z_a, Z_c\}$.

In terms of [3], the ZC sequence set $(M, Z) - \mathrm{CS}_N^L$ satisfies

$$M \le N \cdot \lfloor L/Z \rfloor, \tag{6}$$

where $\lfloor L/Z \rfloor$ denotes the largest integer smaller than or equal to L/Z. When $M = N \cdot \lfloor L/Z \rfloor$, the ZC sequence set is optimal. To evaluate the performance of a ZC sequence set, the performance parameter can be defined as $\eta = M/(N \cdot \lfloor L/Z \rfloor)$. It is obvious that $\eta \le 1$ and a ZC sequence set is optimal when $\eta = 1$.

3. Construction of IGO-ISC sequence set

We present a construction method of ZC sequence set with IGO and ISC properties in this section. The constructed IGO-ISC sequence set can be divided into multiple sequence groups with each sequence group further including multiple sequence subgroups.

Let $\mathbf{S} = [S_{k,p}]_{K \times P}$ be a $K \times P$ orthogonal matrix which can be expressed as

$$\mathbf{S} = [S_{k,p}]_{K \times P} = \begin{bmatrix} \mathbf{S}_0 \\ \mathbf{S}_1 \\ \vdots \\ \mathbf{S}_{K-1} \end{bmatrix} = \begin{bmatrix} S_{0,0} & S_{0,1} & \cdots & S_{0,P-1} \\ S_{1,0} & S_{1,1} & \cdots & S_{1,P-1} \\ \vdots & \vdots & \ddots & \vdots \\ S_{K-1,0} & S_{K-1,1} & \cdots & S_{K-1,P-1} \end{bmatrix}, \tag{7}$$

where $0 \le k \le K-1$, $0 \le p \le P-1$, the k-th row $\mathbf{S}_k = (S_{k,0}, S_{k,1}, \cdots, S_{k,P-1})$ of the matrix $\mathbf{S}$ can be seen as a sequence with length P and the absolute value of each element is equal to 1, namely $|S_{k,p}| = 1$.

The matrix $\mathbf{S}$ satisfies $\phi_{\mathbf{S}_k,\mathbf{S}_{k'}}(0) = 0$ for $k \ne k'$ since it is an orthogonal one. Let $\mathbf{U} = [U_{n_1,n_2}]_{N \times N}$ and $\mathbf{V} = [V_{g,n_3}]_{G \times N}$ be another two orthogonal matrices, where $0 \le n_1, n_2, n_3 \le N-1$, $0 \le g \le G-1$, $|U_{n_1,n_2}| = 1$ and $|V_{g,n_3}| = 1$. In addition to orthogonal property, the matrix $\mathbf{U}$ satisfies that it is a symmetrical one and different row sequences have ideal periodic cross-correlation properties, namely

$$\begin{cases} U_{n_1,n_2} = U_{n_2,n_1}; \\ \phi_{\mathbf{U}_{n_1},\mathbf{U}_{n_1'}}(\tau) = 0, \quad \forall\tau \text{ and } n_1 \neq n_1'. \end{cases} \tag{8}$$

Combining $\mathbf{U}$ with $\mathbf{V}$, we can obtain a $(G \cdot N) \times (N \cdot N)$ coefficient matrix $\mathbf{E} = [\mathbf{E}^{(0)} \quad \mathbf{E}^{(1)} \quad \cdots \quad \mathbf{E}^{(N-1)}]$. The matrix $\mathbf{E}$ includes N sub-matrices with each sub-matrix being a $(G \cdot N) \times N$ one, and the n-th sub-matrix $\mathbf{E}^{(n)}$ can be given as

$$\begin{aligned} \mathbf{E}^{(n)} &= \left[E^{(n)}_{g,r,s}\right]_{(G\cdot N)\times N} \\ &= \begin{bmatrix} E^{(n)}_{0,0,0} & E^{(n)}_{0,0,1} & \cdots & E^{(n)}_{0,0,N-1} \\ E^{(n)}_{0,1,0} & E^{(n)}_{0,1,1} & \cdots & E^{(n)}_{0,1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ E^{(n)}_{0,N-1,0} & E^{(n)}_{0,N-1,1} & \cdots & E^{(n)}_{0,N-1,N-1} \\ E^{(n)}_{1,0,0} & E^{(n)}_{1,0,1} & \cdots & E^{(n)}_{1,0,N-1} \\ E^{(n)}_{1,1,0} & E^{(n)}_{1,1,1} & \cdots & E^{(n)}_{1,1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ E^{(n)}_{1,N-1,0} & E^{(n)}_{1,N-1,1} & \cdots & E^{(n)}_{1,N-1,N-1} \\ \vdots & \vdots & \vdots & \vdots \\ E^{(n)}_{G-1,0,0} & E^{(n)}_{G-1,0,1} & \cdots & E^{(n)}_{G-1,0,N-1} \\ E^{(n)}_{G-1,1,0} & E^{(n)}_{G-1,1,1} & \cdots & E^{(n)}_{G-1,1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ E^{(n)}_{G-1,N-1,0} & E^{(n)}_{G-1,N-1,1} & \cdots & E^{(n)}_{G-1,N-1,N-1} \end{bmatrix}. \end{aligned} \tag{9}$$

The matrix element $E^{(n)}_{g,r,s}$ in the $(g \cdot N + r)$-th row and the s-th line of the sub-matrix $\mathbf{E}^{(n)}$ satisfies

$$E^{(n)}_{g,r,s} = V_{g,n} \cdot U_{(r+n)_N,s}, \tag{10}$$

where $0 \leq r, s, n \leq N - 1$.

Based on the orthogonal matrix $\mathbf{S}$ and the coefficient matrix $\mathbf{E}$, we can construct a novel IGO-ISC sequence set $\mathbf{\Gamma} = \{\mathbf{\Gamma}_g, 0 \leq g \leq G-1\}$ which includes G sequence groups and the g-th sequence group $\mathbf{\Gamma}_g = \{\mathbf{\Gamma}_{g,r}, 0 \leq r \leq N-1\}$ includes N sequence subgroups. In the form of matrix, the r-th sequence subgroup $\mathbf{\Gamma}_{g,r}$ in $\mathbf{\Gamma}_g$ can be expressed as

$$\mathbf{\Gamma}_{g,r} = \begin{bmatrix} \mathbf{\Gamma}_{g,r,0} \\ \mathbf{\Gamma}_{g,r,1} \\ \vdots \\ \mathbf{\Gamma}_{g,r,K-1} \end{bmatrix} = \begin{bmatrix} \mathbf{\Gamma}_{g,r,0,0} & \mathbf{\Gamma}_{g,r,0,1} & \cdots & \mathbf{\Gamma}_{g,r,0,N-1} \\ \mathbf{\Gamma}_{g,r,1,0} & \mathbf{\Gamma}_{g,r,1,1} & \cdots & \mathbf{\Gamma}_{g,r,1,N-1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{\Gamma}_{g,r,K-1,0} & \mathbf{\Gamma}_{g,r,K-1,1} & \cdots & \mathbf{\Gamma}_{g,r,K-1,N-1} \end{bmatrix}, \tag{11}$$

where the n-th element sequence $\mathbf{\Gamma}_{g,r,k,n}$ of the k-th sequence $\mathbf{\Gamma}_{g,r,k}$ in $\mathbf{\Gamma}_{g,r}$ satisfies

$$\mathbf{\Gamma}_{g,r,k,n} = \left(E^{(n)}_{g,r,0} \cdot \mathbf{S}_k\right) \odot \left(E^{(n)}_{g,r,1} \cdot \mathbf{S}_k\right) \odot \cdots \odot \left(E^{(n)}_{g,r,N-1} \cdot \mathbf{S}_k\right). \tag{12}$$

From Eq. (12), it is obvious that each element sequence of the constructed IGO-ISC sequence set $\mathbf{\Gamma}$ is generated by interleaving the products of the orthogonal sequences and their corresponding coefficients. Then, the length of each element sequence is equal to $N \cdot P$.

Let IGO-ISC$_{c_2}^{c_1}(c_3, \{c_4; c_5\}, c_6)$ denote a IGO-ISC sequence set whose element sequence length, flock size, set size, the number of sequence groups, the number of sequence subgroups and the ZCZ width of each sequence subgroup are equal to c_1, c_2, c_3, c_4, c_5 and c_6, respectively. Then, the constructed IGO-ISC sequence set $\mathbf{\Gamma}$ in this section can be denoted by IGO-ISC$_N^{N\cdot P}(G \cdot N \cdot K, \{G; N\}, N)$.

4. Performance of the constructed IGO-ISC sequence set

In this section, the correlation properties of IGO-ISC sequence set, such as ACF, intra-subgroup CCF, inter-subgroup CCF and inter-group CCF, are firstly presented. Then, a construction example of IGO-ISC sequence set is provided to show how the proposed construction algorithm works. Finally, comparison between the IGO-ISC sequence construction and others several constructions is given.

4.1. IGO-ISC correlation properties.

The aperiodic and periodic correlation properties of the constructed IGO-ISC sequence set $\mathbf{\Gamma}$ can be expressed as follows.

Theorem 4.1. *Let $\mathbf{\Gamma}_{g_1,r_1,k_1}$ and $\mathbf{\Gamma}_{g_2,r_2,k_2}$ denote any two IGO-ISC sequences in $\mathbf{\Gamma}$. Then, we have*

$$\sum_{n=0}^{N-1} \psi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_1,k_1,n}}(\tau) = \sum_{n=0}^{N-1} \phi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_1,k_1,n}}(\tau) = 0, \quad \text{if } (\tau)_N \neq 0, \tag{13}$$

$$\sum_{n=0}^{N-1} \psi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_1,k_2,n}}(\tau) = \sum_{n=0}^{N-1} \phi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_1,k_2,n}}(\tau) = 0, \quad \text{if } k_1 \neq k_2 \text{ and } (\tau)_N \neq 0 \text{ except } \tau = 0, \tag{14}$$

$$\sum_{n=0}^{N-1} \psi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_2,k_2,n}}(\tau) = \sum_{n=0}^{N-1} \phi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_1,r_2,k_2,n}}(\tau) = 0, \quad \text{if } r_1 \neq r_2 \text{ and } \forall \tau, \tag{15}$$

$$\sum_{n=0}^{N-1} \psi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_2,r_2,k_2,n}}(\tau) = \sum_{n=0}^{N-1} \phi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n},\, \mathbf{\Gamma}_{g_2,r_2,k_2,n}}(\tau) = 0, \quad \text{if } g_1 \neq g_2 \text{ and } (\tau)_N = 0, \tag{16}$$

where $0 \leq g_1, g_2 \leq G-1$, $0 \leq r_1, r_2 \leq N-1$ and $0 \leq k_1, k_2 \leq K-1$.

In Theorem 4.1, Eqs. (13)-(16) describe aperiodic and periodic ACF, intra-subgroup CCF, inter-subgroup CCF and inter-group CCF, respectively. The proof of Theorem 4.1 can be given as follows.

Proof. We first discuss aperiodic correlation properties of the constructed IGO-ISC sequence set $\mathbf{\Gamma}$.

In terms of the proposed construction algorithm, the aperiodic correlation function between any two IGO-ISC sequences $\mathbf{\Gamma}_{g_1,r_1,k_1}$ and $\mathbf{\Gamma}_{g_2,r_2,k_2}$ in $\mathbf{\Gamma}$ can be calculated as follows,

$$\sum_{n=0}^{N-1} \psi_{\mathbf{\Gamma}_{g_1,r_1,k_1,n}, \mathbf{\Gamma}_{g_2,r_2,k_2,n}}(\tau)$$

$$
= \begin{cases} \left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-(\tau)_N-1} E^{(n)}_{g_1,r_1,s}\cdot\left(E^{(n)}_{g_2,r_2,s+(\tau)_N}\right)^*\right)\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\lfloor\tau/N\rfloor\right) \\ \quad +\left(\sum_{n=0}^{N-1}\sum_{s=0}^{(\tau)_N-1} E^{(n)}_{g_1,r_1,N-(\tau)_N+s}\cdot\left(E^{(n)}_{g_2,r_2,s}\right)^*\right)\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\lfloor\tau/N\rfloor+1\right), & (\tau)_N\neq 0; \\ \left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-1} E^{(n)}_{g_1,r_1,s}\cdot\left(E^{(n)}_{g_2,r_2,s}\right)^*\right)\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\tau/N\right), & (\tau)_N=0, \end{cases}
$$

$$
= \begin{cases} \left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-(\tau)_N-1} V_{g_1,n}\cdot U_{s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s+(\tau)_N,(r_2+n)_N}\right)\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}(\lfloor\tau/N\rfloor) \\ \quad +\left(\sum_{n=0}^{N-1}\sum_{s=0}^{(\tau)_N-1} V_{g_1,n}\cdot U_{N-(\tau)_N+s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s,(r_2+n)_N}\right) \\ \quad \cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\lfloor\tau/N\rfloor+1\right), & (\tau)_N\neq 0; \\ \left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-1} V_{g_1,n}\cdot U_{s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s,(r_2+n)_N}\right)\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\tau/N\right), & (\tau)_N=0, \end{cases}
$$

(17)

$$
= \begin{cases} \mathcal{C}_1\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\lfloor\tau/N\rfloor\right)+\mathcal{C}_2\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\lfloor\tau/N\rfloor+1\right), & (\tau)_N\neq 0; \\ \mathcal{C}_3\cdot\psi_{\mathbf{S}_{k_1},\mathbf{S}_{k_2}}\left(\tau/N\right), & (\tau)_N=0, \end{cases}
$$

where

$$
\mathcal{C}_1=\left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-(\tau)_N-1} V_{g_1,n}\cdot U_{s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s+(\tau)_N,(r_2+n)_N}\right), \tag{18}
$$

$$
\mathcal{C}_2=\left(\sum_{n=0}^{N-1}\sum_{s=0}^{(\tau)_N-1} V_{g_1,n}\cdot U_{N-(\tau)_N+s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s,(r_2+n)_N}\right), \tag{19}
$$

$$
\mathcal{C}_3=\left(\sum_{n=0}^{N-1}\sum_{s=0}^{N-1} V_{g_1,n}\cdot U_{s,(r_1+n)_N}\cdot V^*_{g_2,n}\cdot U^*_{s,(r_2+n)_N}\right). \tag{20}
$$

For the fist equality in Eq. (17), we employ the properties of interleaving operation in Eq. (3). When $(\tau)_N \neq 0$, the calculation of aperiodic correlation function can be divided into two parts. In terms of Eq. (12), two cases of adjacent shifts should be considered. Since the number of columns of sub-matrix $\mathbf{E}^{(n)}$ is equal to N, we can obtain the fist equality in Eq. (17) for $(\tau)_N \neq 0$ by combining Eq. (12) with Eq. (3). For the case of $(\tau)_N = 0$, there only exists one shift τ/N and the calculation expression can be easily obtained.

It is obvious that the aperiodic correlation function between $\mathbf{\Gamma}_{g_1,r_1,k_1}$ and $\mathbf{\Gamma}_{g_2,r_2,k_2}$ is determined by correlation properties of three matrices of $\mathbf{S}$, $\mathbf{U}$ and $\mathbf{V}$. We will discuss Eq. (17) in the following four cases.

1). ACF, namely the case of $g_1 = g_2$, $r_1 = r_2$ and $k_1 = k_2$.

Due to the orthogonal property of matrix $\mathbf{U}$, we have $\mathcal{C}_1 = \mathcal{C}_2 = 0$ when $(\tau)_N \neq 0$. Then, Eq. (17) is equal to zero when $(\tau)_N \neq 0$.

2). Intra-subgroup CCF, namely the case of $g_1 = g_2$, $r_1 = r_2$ and $k_1 \neq k_2$.

When $(\tau)_N \neq 0$, it is the same with the case of ACF, namely $\mathcal{C}_1 = \mathcal{C}_2 = 0$.

In addition, Eq. (17) is also equal to zero when $\tau = 0$ since the matrix $\mathbf{S}$ is an orthogonal one.

3). Inter-subgroup CCF, namely the case of $g_1 = g_2$ and $r_1 \neq r_2$.

When $(\tau)_N = 0$, we have $\mathcal{C}_3 = 0$ since $\mathbf{U}$ is a symmetrical orthogonal matrix.

When $(\tau)_N \neq 0$, we have $\mathcal{C}_1 = \mathcal{C}_2 = 0$ due to Eq. (8).

4). Inter-group CCF, namely the case of $g_1 \neq g_2$.

We only discuss the case of $(\tau)_N = 0$.

When $r_1 = r_2$, we have $\mathcal{C}_3 = 0$ due to the orthogonal property of matrix $\mathbf{V}$.

When $r_1 \neq r_2$, we also have $\mathcal{C}_3 = 0$ due to the orthogonal and symmetrical properties of matrix $\mathbf{U}$.

According to the above discussion of four cases, aperiodic correlation properties in Theorem 4.1 have been proved. As for periodic correlation properties, similar analysis can be obtained and the corresponding proof is omitted. Thus, Theorem 4.1 has been proved. □

Corollary 1. *The IGO-ISC sequence set $\mathbf{\Gamma}$ is a ZC sequence set $(G \cdot N \cdot K, 1) - CS_N^{N \cdot P}$ and is optimal when $G = N$ and $K = P$.*

According to Theorem 4.1 and Eq. (6), we can easily obtain Corollary 1 and then its proof is omitted.

Corollary 2. *Each sequence group $\mathbf{\Gamma}_g$ in $\mathbf{\Gamma}$ can be seen as three different kinds of sequence sets as follows. When $G = N$ and $K = P$, the sequence group $\mathbf{\Gamma}_g$ is optimal.*

1). The ZC sequence set $(N \cdot K, N) - CS_N^{N \cdot P}$;

2). The generalized IGC sequence set $(N \cdot K, N, N) - IGC_N^{N \cdot P}$;

3). The T-ZCZ sequence set T-$ZCZ_N^{N \cdot P}(N \cdot K, N)$;

where $(x_1, x_2, x_3) - IGC_{x_5}^{x_4}$ denotes a generalized IGC sequence set with set size x_1, the number of groups x_2, minimum ZCZ width x_3, element sequence length x_4 and flock size x_5. The notation T-$ZCZ_{y_2}^{y_1}(y_3, y_4)$ denotes a T-ZCZ sequence set with element sequence length y_1, flock size y_2, set size y_3 and T-ZCZ width y_4.

According to Eqs. (13)-(15) and (6), we can easily obtain Corollary 2 and then its proof is omitted.

4.2. A construction example. In order to show the construction and performance of the proposed IGO-ISC sequence set, we give a simple example.

Example 1. Let three orthogonal matrices of $\mathbf{S}$, $\mathbf{U}$ and $\mathbf{V}$, be respectively expressed as $\mathbf{S} = \begin{bmatrix} 0\,0 \\ 0\,2 \end{bmatrix}$, $\mathbf{U} = \begin{bmatrix} 0\,0\,0\,0 \\ 0\,3\,2\,1 \\ 0\,2\,0\,2 \\ 0\,1\,2\,3 \end{bmatrix}$ and $\mathbf{V} = \begin{bmatrix} 0\,0\,0\,0 \\ 0\,2\,0\,2 \\ 0\,0\,2\,2 \\ 0\,2\,2\,0 \end{bmatrix}$, where 0, 1, 2 and 3 represent $+1$, $+j$, -1 and $-j$, respectively. As quadriphase matrices, the elements of $\mathbf{S}$, $\mathbf{U}$ and $\mathbf{V}$ satisfy $\{e^{j2\pi k/4} | k = 0, 1, 2, 3\} = \{+1, +j, -1, -j\}$, where $j = \sqrt{-1}$. $\mathbf{U}$ is a discrete Fourier transform (DFT) matrix which satisfies Eq. (8).

According to Eq. (9), we can obtain a coefficient matrix $\mathbf{E}$ in Eq. (21). In terms of the proposed construction scheme, an IGO-ISC sequence set $\mathbf{\Gamma}$ can be constructed in Eq. (22). This is a set IGO-ISC$_4^8(32, \{4; 4\}, 4)$. Its absolute value distributions of aperiodic ACF and CCF can be shown in Fig.1, where four figures (a)-(d) show examples of ACF, intra-subgroup CCF , inter-subgroup CCF and inter-group CCF, respectively. The significance of Eqs. (13)-(16) in Theorem 4.1 can be clearly illustrated in Fig. 1. In addition, we can also see the T-ZCZ properties with T-ZCZ width being equal to 4 from Fig. 1 (a)-(b), the ISC properties form Fig. 1 (c) and the IGO properties form Fig. 1 (d).

$$\mathbf{E} = \begin{bmatrix} \mathbf{E}^{(0)} & \mathbf{E}^{(1)} & \mathbf{E}^{(2)} & \mathbf{E}^{(3)} \end{bmatrix}$$

$$= \begin{bmatrix} 0000 & 0321 & 0202 & 0123 \\ 0321 & 0202 & 0123 & 0000 \\ 0202 & 0123 & 0000 & 0321 \\ 0123 & 0000 & 0321 & 0202 \\ 0000 & 2103 & 0202 & 2301 \\ 0321 & 2020 & 0123 & 2222 \\ 0202 & 2301 & 0000 & 2103 \\ 0123 & 2222 & 0321 & 2020 \\ 0000 & 0321 & 2020 & 2301 \\ 0321 & 0202 & 2301 & 2222 \\ 0202 & 0123 & 2222 & 2103 \\ 0123 & 0000 & 2103 & 2020 \\ 0000 & 2103 & 2020 & 0123 \\ 0321 & 2020 & 2301 & 0000 \\ 0202 & 2301 & 2222 & 0321 \\ 0123 & 2222 & 2103 & 0202 \end{bmatrix}. \tag{21}$$

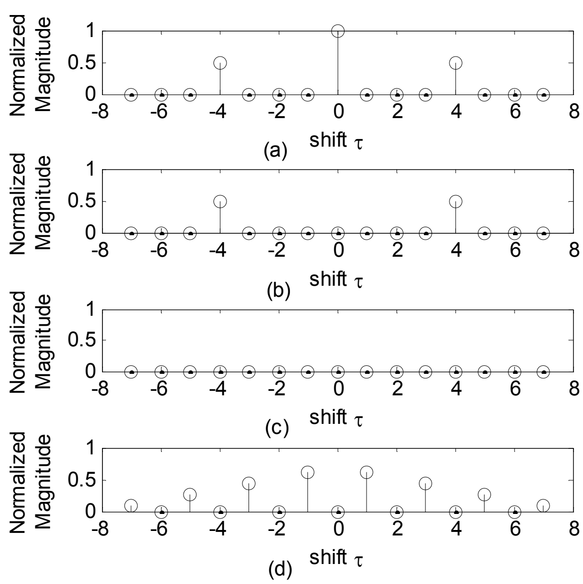

FIGURE 1. The absolute value distributions of aperiodic ACF and CCF of $\mathbf{\Gamma}$. (a) ACF of $\mathbf{\Gamma}_{0,0,0}$; (b) The intra-subgroup CCF between $\mathbf{\Gamma}_{0,0,0}$ and $\mathbf{\Gamma}_{0,0,1}$; (c) The inter-subgroup CCF between $\mathbf{\Gamma}_{0,0,0}$ and $\mathbf{\Gamma}_{0,1,0}$; (d) The inter-group CCF between $\mathbf{\Gamma}_{0,0,0}$ and $\mathbf{\Gamma}_{2,0,0}$.

The constructed IGO-ISC sequence set $\mathbf{\Gamma}$ in Example 1 includes 32 sequences in total and each sequence group includes 8 sequences. Since all of 32 sequences are mutually orthogonal and the minimum ZCZ width of each sequence group is equal to 4, it is obvious that the set $\mathbf{\Gamma}$ and its each sequence group are optimal in terms of Eq. (6).

$$
\mathbf{\Gamma} = \begin{bmatrix} \mathbf{\Gamma}_{0,0,0} \\ \mathbf{\Gamma}_{0,0,1} \\ \mathbf{\Gamma}_{0,1,0} \\ \mathbf{\Gamma}_{0,1,1} \\ \mathbf{\Gamma}_{0,2,0} \\ \mathbf{\Gamma}_{0,2,1} \\ \mathbf{\Gamma}_{0,3,0} \\ \mathbf{\Gamma}_{0,3,1} \\ \mathbf{\Gamma}_{1,0,0} \\ \mathbf{\Gamma}_{1,0,1} \\ \mathbf{\Gamma}_{1,1,0} \\ \mathbf{\Gamma}_{1,1,1} \\ \mathbf{\Gamma}_{1,2,0} \\ \mathbf{\Gamma}_{1,2,1} \\ \mathbf{\Gamma}_{1,3,0} \\ \mathbf{\Gamma}_{1,3,1} \\ \mathbf{\Gamma}_{2,0,0} \\ \mathbf{\Gamma}_{2,0,1} \\ \mathbf{\Gamma}_{2,1,0} \\ \mathbf{\Gamma}_{2,1,1} \\ \mathbf{\Gamma}_{2,2,0} \\ \mathbf{\Gamma}_{2,2,1} \\ \mathbf{\Gamma}_{2,3,0} \\ \mathbf{\Gamma}_{2,3,1} \\ \mathbf{\Gamma}_{3,0,0} \\ \mathbf{\Gamma}_{3,0,1} \\ \mathbf{\Gamma}_{3,1,0} \\ \mathbf{\Gamma}_{3,1,1} \\ \mathbf{\Gamma}_{3,2,0} \\ \mathbf{\Gamma}_{3,2,1} \\ \mathbf{\Gamma}_{3,3,0} \\ \mathbf{\Gamma}_{3,3,1} \end{bmatrix} = \begin{bmatrix}
00000000 & 03210321 & 02020202 & 01230123 \\
00002222 & 03212103 & 02022020 & 01232301 \\
03210321 & 02020202 & 01230123 & 00000000 \\
03212103 & 02022020 & 01232301 & 00002222 \\
02020202 & 01230123 & 00000000 & 03210321 \\
02022020 & 01232301 & 00002222 & 03212103 \\
01230123 & 00000000 & 03210321 & 02020202 \\
01232301 & 00002222 & 03212103 & 02022020 \\
00000000 & 21032103 & 02020202 & 23012301 \\
00002222 & 21030321 & 02022020 & 23010123 \\
03210321 & 20202020 & 01230123 & 22222222 \\
03212103 & 20200202 & 01232301 & 22220000 \\
02020202 & 23012301 & 00000000 & 21032103 \\
02022020 & 23010123 & 00002222 & 21030321 \\
01230123 & 22222222 & 03210321 & 20202020 \\
01232301 & 22220000 & 03212103 & 20200202 \\
00000000 & 03210321 & 20202020 & 23012301 \\
00002222 & 03212103 & 20200202 & 23010123 \\
03210321 & 02020202 & 23012301 & 22222222 \\
03212103 & 02022020 & 23010123 & 22220000 \\
02020202 & 01230123 & 22222222 & 21032103 \\
02022020 & 01232301 & 22220000 & 21030321 \\
01230123 & 00000000 & 21032103 & 20202020 \\
01232301 & 00002222 & 21030321 & 20200202 \\
00000000 & 21032103 & 20202020 & 01230123 \\
00002222 & 21030321 & 20200202 & 01232301 \\
03210321 & 20202020 & 23012301 & 00000000 \\
03212103 & 20200202 & 23010123 & 00002222 \\
02020202 & 23012301 & 22222222 & 03210321 \\
02022020 & 23010123 & 22220000 & 03212103 \\
01230123 & 22222222 & 21032103 & 02020202 \\
01232301 & 22220000 & 21030321 & 02022020
\end{bmatrix}. \tag{22}
$$

4.3. CONTRAST OF CONSTRUCTIONS. The set-dividing method has been used in the constructions of different kinds of sequence sets. Such sequence set can be divided into multiple sequence groups, which generally will ensure better performance. Table 1 lists several constructions of ZCZ sequence sets with multiple sequence groups, where $ZCZ-(c_1, c_2, c_3)$ denotes a ZCZ sequence set with the sequence length c_1, the set size c_2 and the ZCZ width c_3.

From Table 1, the number of sequence groups of GPC sequence set in [1] is fixed to 2 while other constructions can contain more sequence groups. As a kind of unitary sequences working on a one-sequence-per-user basis, that is, the number of elements sequences is equal to 1, the sequence sets constructed in [9] [33] [18] can not possess both of ideal auto-correlation and cross-correlation properties even if their out-of-phase ACFs and CCFs are equal to zero within ZCZ. In contrast, the GPC sequences in [1], the IGC sequences in [16] and the IGO-ISC sequences in this paper, as a kind of ZC sequences working on a one-clock-per-user basis, can

TABLE 1. Contrast of constructions of ZCZ sequence sets with multiple sequence groups

Constructions	Number of sequence groups	Number of sequence subgroups	Number of ZCCZ widths	Number of element sequences	Performance parameter
$ZCZ-(L_t, N_t, Z_t)$ in [9]	N_0	1	2	1	$\eta=\frac{Z_0+1-\Lambda}{Z_0+1}$
$ZCZ-(T\cdot N, N, T-1)$ in [18]	T	1	2	1	$\eta=1$
$ZCZ-(T\cdot N, \lfloor\frac{N}{T}\rfloor, T-1)$ in [33]	T	1	2	1	$\eta=\frac{1}{N}\cdot\lfloor\frac{N}{T}\rfloor$
$(2K, 2, 4N)-GPC_2^{4NK}$ in [1]	2	1	2	2	$\eta=1$
$(M\cdot P, M, L_0)-IGC_M^{P\cdot L_0}$ in [16]	M	1	2	M	$\eta=1$
$IGO-ISC_N^{N\cdot P}(G\cdot N\cdot K, \{G;N\}, N)$ in this paper	G	N	3	N	$\eta=1$

obtain not only ZCZ properties for the whole sequence set but also ideal correlation properties among some special sequences.

Although the number of sequence groups of the first five constructions in Table 1 are larger or equal to 2, the number of sequence subgroups are equal to 1. Compared with the first five constructions, the IGO-ISC sequence set proposed in this paper further divides each sequence group into multiple sequence subgroups. As a result, the proposed IGO-ISC sequence set will possess three different ZCCZ widths, namely inter-group ZCCZ width, inter-subgroup ZCCZ width and intra-subgroup ZCCZ width, which is different from previous results of two ZCCZ widths of inter-group ZCCZ width and intra-group ZCCZ width.

5. Conclusions

This paper proposes a kind of IGO-ISC sequences based on three orthogonal matrices and the interleaving technique. The constructed IGO-ISC sequence set can be divided into multiple sequence groups with each sequence group including multiple sequence subgroups. Each sequence subgroup possesses T-ZCZ properties. The sequences from different sequence subgroups in one sequence group have ideal aperiodic and periodic cross-correlation properties while the sequences from different sequence groups are mutually orthogonal. As a kind of ZC sequences, both of the presented IGO-ISC sequence set and its each sequence group can achieve the theoretical bound on the set size for the ZCZ width and sequence length.

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