PERIODIC SOLUTIONS FOR A SEMI-RATIO-DEPENDENT PREDATOR-PREY DYNAMICAL SYSTEM WITH A CLASS OF FUNCTIONAL RESPONSES ON TIME SCALES

MOSTAFA FAZLY
Department of Mathematical Sciences
Sharif University of Technology, P.O.Box 11365-9415, Tehran, Iran

MAHMOUD HESAARAKI
Department of Mathematical Sciences
Sharif University of Technology, P.O.Box 11365-9415, Tehran, Iran

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Abstract. In this paper we explore the existence of periodic solutions of a nonautonomous semi-ratio-dependent predator-prey dynamical system with functional responses on time scales. To illustrate the utility of this work, we should mention that, in our results this system with a large class of monotone functional responses, always has at least one periodic solution. For instance, this system with some celebrated functional responses such as Holling type-II (or Michaelis-Menten), Holling type-III, Ivlev, $mx/(A + x)(B + x)$ and some other monotone functions, has always at least one $\omega$-periodic solution. Besides, for some well-known functional responses which are not monotone such as Monod-Haldane or Holling type-IV, the existence of periodic solutions is proved. Our results extend and improve previous results presented in [4], [10], [22] and [38].

1. Introduction. In this paper we investigate the existence of periodic solutions of a nonautonomous semi-ratio-dependent predator-prey dynamical system with some well-known functional responses, on time scales. The study of dynamic equations on time scales is an area of mathematics that tries to unify the study of differential and difference equations, and provides new powerful tools for exploring connections between the traditionally separated fields. It goes back to its founder Stefan Hilger [18] in 1988 but, in the past few years, it has found a considerable amount of interest and attracted many researchers' attention.

The general idea is to prove a result for a dynamic equation where the domain of the unknown function is a so-called time scale, which may be an arbitrary closed subset of the real numbers. In this way, not only are the results related to the set of real numbers or to the set of integers but also those pertaining to more general time scales are obtained. In a short sentence, we may state that unification and extension are the two main features of the time scales calculus.

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