GAUSSIAN ESTIMATES FOR A HEAT EQUATION ON A NETWORK

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Abstract. We consider a diffusion problem on a network on whose nodes we impose Dirichlet and generalized, non-local Kirchhoff-type conditions. We prove well-posedness of the associated initial value problem, and we exploit the theory of sub-Markovian and ultracontractive semigroups in order to obtain upper Gaussian estimates for the integral kernel. We conclude that the same diffusion problem is governed by an analytic semigroup acting on all $L^p$-type spaces as well as on suitable spaces of continuous functions. Stability and spectral issues are also discussed. As an application we discuss a system of semilinear equations on a network related to potential transmission problems arising in neurobiology.

1. Introduction. Evolution equations taking place in networks or, more generally, in ramified structures have been first considered in pioneering articles by K. Ruedenberg and C. Scherr back in the 1950s, cf. [35], and, at a more mathematical level, in a series of papers by R. Mills and E. Montroll and by G. Lumer in the 1970s, cf. [29]–[28] and [24]–[25], respectively. Shortly afterwards, F. Ali Mehmeti, J. von Below, S. Nicaise, and J.P. Roth among others began a systematical study of properties of elliptic operators acting on spaces of functions over networks, cf. e.g. the monographs [33], [2], [23], and references therein. Ever since, such problems have aroused broad interest among mathematicians working on partial differential equations, control, and spectral theory – as well as among theoretical physicists interested in scattering theory of guided waves, photonic crystals, and quantum wires, resulting in a literature so vast that it can by no means be summarized here.

Throughout this paper we consider a finite, unitarily parametrized, connected network whose structure is given by a suitable graph. On it we study a general diffusion equation. Adopting a setting which is standard in literature, the node...