A FLUID DYNAMIC MODEL FOR SUPPLY CHAINS

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Abstract. The paper deals with a fluid dynamic model for supply chains. A mixed continuum-discrete model is proposed and possible choices of solutions at nodes guaranteeing the conservation of fluxes are discussed. Fixing a rule a Riemann solver is defined and existence of solutions to Cauchy problems is proved.

1. Introduction. There are various approaches to treat supply chains dynamics, many of which, needless to say, focusing on discrete mathematics models. On the contrary, recent works proposed equations for macroscopic quantities measured by a continuum variable. In particular we focus on [2], where the authors, passing to the limit in discrete models, obtained the conservation law:

\[ \rho_t + (\min\{\mu(t, x), \rho\})_x = 0, \]  

(1)

where \( \rho \) is the density of objects processed by the supply chain (represented by a real line) and \( \mu \) is the processing rate.

It is well known that equation (1) may admit no solution for non smooth \( \mu \) (in [2] this equation is obtained differentiating another equation which may admit only discontinuous weak solutions). Our aim is thus to propose some special choice of the processing rate function \( \mu(t, x) \), included a time evolution, guaranteeing the existence of solutions. Our approach is based on a mixed continuum-discrete model, to describe which we need some notation and motivations.

A first idea is to consider piecewise constant data for \( \mu \) and an evolution equation of semi-linear type:

\[ \mu_t + \bar{v}\mu_x = 0, \]  

(2)

where \( \bar{v} \) is some constant velocity. Taking \( \bar{v} = 0 \), we may have no solution to a Riemann problem for the system (1)–(2) with data \( (\rho_l, \mu_l) \) and \( (\rho_r, \mu_r) \) if \( \min\{\mu_l, \rho_l\} > \mu_r \). Since we expect the chain to influence backward the processing rate we assume \( \bar{v} < 0 \) and for simplicity we set \( \bar{v} = -1 \).

We define a mixed continuum-discrete model in the following way. The supply chain is modelled by a real line seen as a sequence of sub-chains corresponding to intervals \( I_k \) such that \( I_k \cap I_{k+1} = P_k \): a vertex separating sub-chains. We want to model each sub-chain with a continuum system of type (1)–(2), while we interpret the evolution at nodes \( P_k \) thinking to it as Riemann problems for the density equation (1) with \( \mu \) data as parameters. Such Riemann problems may still