COMPETITION AND DISPERAL DELAYS IN PATCHY ENVIRONMENTS

NANCY AZER
Department of Mathematics and Statistics, University of Victoria
PO BOX 3045 STN CSC, Victoria, B.C. V8W 3P4

P. VAN DEN DRIESSCHE
Department of Mathematics and Statistics, University of Victoria
PO BOX 3045 STN CSC, Victoria, B.C. V8W 3P4

(Communicated by Stephen Gourley)

Abstract. Dispersal delays are introduced into a competition model for two species that disperse among $n$ identical patches. The model is formulated as a system of integro-differential equations with an arbitrary distribution of dispersal times between patches. By identifying steady states and analyzing local stability, conditions for competitive exclusion, coexistence or extinction are determined in terms of the system parameters. These are confirmed by numerical simulations with a delta function distribution, showing that all solutions approach a steady state and that high dispersal is generally a disadvantage to a species. However, if the two species have identical local dynamics, then small dispersal rates (with certain parameter restrictions) can be an advantage to the dispersing species. If the number of species is increased to three, then oscillatory coexistence with dispersal delay is possible.

1. Introduction. Lotka (1932) and Volterra (1926) developed a classical model of competition; see, for example, [4, Section 3.5]. Realistically, most species exist in patchy environments; thus, dispersal needs to be incorporated into the classical Lotka-Volterra competition model. For two species competing on two patches, ordinary differential equation (ODE) models have been studied by several authors; see, for example, Levin [3], Takeuchi [7, Chapter 5, Section 5.6], Gourley and Kuang [1].

In Takeuchi [7] a four-dimensional ODE system model is formulated for two species that compete on two non-identical patches. Let $N_{ij}$ (for $i, j = 1, 2$) denote the density of species $i$ on patch $j$, $r_{ij}$ and $K_{ij}$ denote the growth rate and carrying capacity for species $i$ on patch $j$, respectively, and $\alpha_{12j}$ and $\alpha_{21j}$ denote the interspecific competition parameters on patch $j$. Furthermore, let $D_i$ be the dispersal rate for species $i$ (the model given in [7, page 121] assumes that dispersal is both species and patch dependent). All parameters are assumed to be positive. The model equations [7, page 121] are

$$\frac{dN_{ij}}{dt} = \frac{r_{ij}}{K_{ij}} N_{ij}(K_{ij} - N_{ij} - \alpha_{ikj}N_{kj}) + D_i[N_{it} - N_{ij}], \quad (1)$$