ABSTRACTS

PLENARY SPEAKERS

Localization of energy in FPU chains

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The celebrated model of Fermi, Pasta and Ulam is revisited with the aim of investigating the relaxation of the system towards equipartition. We perform an extensive numerical exploration starting with all the energy E initially concentrated on the lowest frequency mode, as in the original FPU report. We produce evidence of the existence of two well separated different time scales. In a short time we observe the formation of a packet of low frequency modes which share most of the total energy,while the high frequency modes appear to be frozen in a exponentially decreasing distribution. After this fast relaxation a second phase begins during which the energy flows at a definitely slower rate towards the higher frequency modes. The numerical exploration shows that the fraction of the total number N of modes that take active part in the formation of the initial packet is a function of the specific energy E/N. Furthermore, the speed of the energy flow during the second phase tends quite rapidly to zero with E/N. Thus, this phenomenon appears to be relevant for the thermodynamic limit.

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Singularity Formation in 3-D Vortex Sheets

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One of the classical examples of hydrodynamic instability occurs when two fluids are separated by a free surface across which the tangential velocity has a jump discontinuity. This is called Kelvin-Helmholtz instability. Kelvin-Helmholtz instability is a fundamental instability of incompressible fluid flow at high Reynolds number. The idealization of a shear layered flow as a vortex sheet separating two regions of potential flow has often been used as a model to study mixing properties, boundary layers and coherent structures of fluids. It is well known that small initial perturbations on a vortex sheet may grow rapidly due to Kelvin-Helmholtz instability. The problem is ill-posed in the Hadamard sense. Most analytical studies of vortex sheet singularity to date rely heavily on complexifying the interface variables. It is not clear how to generalize this technique to 3-D vortex sheets in a natural way. In a joint work with G. Hu and P. Zhang, we study the singularity of 3-D vortex sheets using a new approach. First, we derive a leading order approximation to the boundary integral equation governing the 3-D vortex sheet. This leading order equation captures the most singular contribution of the integral equation. Moreover, after applying a transformation to the physical variables, we found that this leading order 3-D vortex sheet equation de-generates into a two-dimensional vortex sheet equation in the direction of the tangential velocity jump. This rather surprising result confirms that the tangential velocity jump is the physical driving force of the vortex sheet singularities. We show that the singularity type of the three-dimensional problem is similar to that of the two-dimensional problem. Moreover, we introduce a generalized Moore's approximation to 3-D vortex sheets. This model equation captures the same singularity structure of the full 3-D vortex sheet equation, and it can be computed efficiently using Fast Fourier Transform. This enables us to perform well-resolved calculations to study the generic type of 3-D vortex sheet singularities. We will provide detailed numerical results to support the analytic prediction, and to reveal the generic form of the vortex sheet singularity.

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A variational characterization of travelling waves in quasi-periodic media

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Travelling waves in heterogeneous media have gained much attention in the past decade in various fields of science such as ecology, physiology and combustion theory. Previously most of the mathematical studies were focused on spatially periodic cases, and little was known about the nature of travelling waves in spatially aperiodic media. This is in contrast with the case of temporally varying media, for which there is a comprehensive study by Shen (1999). Recently I have introduced the notion of travelling waves in spatially almost-periodic media, including quasi-periodic ones as special cases. The concept is a natural extension of the classical notion of travelling waves, and I have discussed existence, uniqueness and stability of those travelling waves. To be more precise, a travelling wave is defined to be a solution whose current profile depends continuously on its current landscape. Here, roughly speaking, the current profile means the shape of the solution (at each time moment) viewed from the position of the "front", and the current landscape refers to the spatial environment viewed from that position. In this lecture I will discuss two variational problems associate with travelling waves for nonlinear diffusion equations. The first is the mini-max characterization of propagation speed, which is introduced by Volpert et al for homogeneous problems and later extended to periodic problems by Heinze, Papanicolaou and Stevens in the case of bistable nonlinearity, and by Berestycki and Hamel in the case of KPP nonlinearity. This method enables one to obtain fine rigorous estimates of propagation speed. One can extend this method to quasi-periodic problems with bistable nonlinearity, but it raises a very intriguing question, which I will discuss in my lecture. The second is concerned with the minimal speed of travelling waves for KPP type equations. As conjectured by Kawasaki-Shigesada (1986),and later proved by Hudson-Zinner (1995 for 1-dim) and Berestycki-Hamel (2002 for higher dim), the minimal speed is characterized by a positive eigenfunction of a certain elliptic eigenvalue problem. In the case of quasi-periodic inhomogeneity, I can formulate a similar eigenvalue problem on a torus, but this eigenvalue problem may not have a positive eigenfunction because of the degeneracy of the differential operator. Despite this difficulty, this eigenvalue problems can still play a principal role for proving the existence of pseudo-travelling waves and to obtain estimates of their minimal speed.

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Diffusion and Cross-Diffusion in Pattern Formation: from Single Equations to Systems

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In this lecture I plan to explain how diffusions and cross-diffusions are used in modelling pattern formation, both from a practical point of view and from a mathematical point of view in studying the potential of these equations and systems. Examples will be used to illustrate various approaches.

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Macroscopic Equations from Microscopic Models: The Bacterial Example

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In recent years it has become clear that a reductionist approach to biological systems is generally inadequate; complex systems cannot be understood simply by dissecting them into their components. In a number of systems detailed information on transduction of extracellular signals into a behavioral change is now available, and a current problem is to incorporate this information into macroscopic equations that describe population-level behavior. We use bacterial chemotaxis as an example to describe several different levels at which this can be done, and show how a general theory can be developed. We also describe a new computational approach aimed at bridging the gap between microscopic models and macroscopic evolution.

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Method of Upper and Lower Solutions for Reaction-Diffusion

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The method of upper and lower solutions and its associated monotone iteration are powerful tools for the treatment of reaction diffusion systems both analytically and numerically. The analytical treatment includes the existence and uniqueness of the time dependent solution, existence, multiplicity, and bifurcation of steady-state solutions, stability or instability of a steady state solution, and blow-up and quenching of time-dependent solutions. Numerically, this method can be used to develop stable and reliable computational algorithms for numerical solutions of both time-dependent and steady state solutions, convergence of discrete solutions to their corresponding continuous solutions, and bounds and error estimates of the discrete solutions. The aim of this talk is to give an overview of the method, and to describe some of the basic ideas and main elements for various types of reaction diffusion systems, including systems with nonlinear or nonlocal boundary conditions, systems with finite or infinite time delays, and periodic solutions of time dependent problems. Applications and numerical results are given to some specific reaction diffusion equations to illustrate the practical aspect of the method.

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Nonlinear boundary value problems with multivalued terms

Nikolas Papageorgiou
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We examine vector nonlinear differential inclusions driven by the p-Laplacian or by more general differential operators which are not necessarily homogeneous.We impose nonlinear set-valued boundary conditions which incorporate the Dirichlet, Neumann and periodic conditions.We prove existence and relaxation results. We also consider evolution equations with multivalued terms defined in the framework of evolution triples.We obtain existence theorems for nonmonotone problems, while for the monotone problem we prove the topological triviality of the solution set.We also prove sensitivity results using the G-convergence of operators. Finally we examine optimal control problems (sensitivity analysis and minimax control).

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Spatial heteroclinics for a semilinear elliptic PDE

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The existence of spatially heteroclinic solutions for a semilinear elliptic PDE in an infinite cylinder in R^n will be discussed. Minimization arguments will be used to obtain the solutions.

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Dynamics - a panorama of recent results

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There has been substantial recent progress in the field of Dynamics, from which an understanding of the typical behaviour of very general systems is emerging, extending the scope of the classical hyperbolic theory. I'll discuss some of these developments. In particular, I'll report some very recent results in the theory of Lyapunov exponents of smooth systems and linear cocycles.

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Mini Symposia
Singular Perturbations and Propagation of Waves

Organizer: Didier Bresch, LMA-UBP, Clermont-Ferrand

Theory and application of initial-boundary-value problems for nonlinear wave equations.

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We will be concerned with the mathematical theory of initial-boundary-value problems for nonlinear, dispersive wave equations. The theory is somewhat more subtle than the corresponding theory for pure initial-value problems. After outlining the current state of the theory, attention will be turned to the application of this configuration in laboratory settings and in field studies connected with nearshore zone sediment transport and beach protection strategies.

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High frequency limit of the Helmholtz equation, with a source concentrating on a surface

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J.D. Benamou, **Th.Katsaounis**
B.Perthame and **O. Runborg**

This talk is a review about two papers involving the above authors. We consider the high-frequency Helmholtz equation, with source term. It describes, typically, an antenna emitting waves in the whole space. The frequency is measured by the parameter epsilon. Also, and this is the specificity of the analysis presented here, the source itself presents oscillation and concentration effects at the same scale epsilon. Using a Wigner transform approach, we analyze the asymptotic propagation of energy as epsilon goes to zero (high-frequency limit). This leads to analyzing the interactions between the oscillations carried by the source, and the oscillations induced by the Helmholtz operator, i.e. those induced by the mere propagation of waves. We prove that asymptotically the energy transport is well described by a kinetic equation with a specific source term. The latter has prescribed support both in space and frequency.

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Propagation of a laser beam in a gaz

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The propagation of a laser beam in a uranium gas is described by the Maxwell-Bloch equations. We perform a geometrical optics type limit in order to obtain the Schrödinger-Bloch system that is used for the propagation. A small parameter is introduced and an existence result on a time interval that does not depend on this parameter is proved. Some numerical simulations are given.

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The k^{-2} spectrum in rotating, forced turbulence

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Recent experiments in Swinney's lab in Texas show an interesting departure from conventional wisdom concerning spectra of rotating turbulence: there is evidence for a k^{-2} spectrum. I'll give a brief description of the experimental results and discuss some of the related mathematical results.

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Mathematical results on stability problems in fluid mechanics

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Global existence of Leray–type weak solutions for the compressible barotropic Navier–Stokes equations was proven by P.L. Lions at the beginning of the 90's. This result was the beginning of a series of studies regarding the stability of globally defined solutions, ranging from the coupling problem between weakly deformable solids evolving in a compressible viscous fluid, or the low Mach Number limit of compressible flows. Some of the filtering techniques introduced in the preceding problem have been successfully applied in the case of rapidly rotating Navier–Stokes system relevant in geophysical modelling. On the other hand, nonlinear results have been proven regarding the small time behavior of linearly unstable systems arising in fluid mechanics.

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Compressible fluids in moving domains

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The time evolution of the density $\rho = \rho(t,x)$ and the velocity $u = u(t,x)$ of a compressible viscous fluid is governed by the Navier-Stokes system

$$\rho_t + \mathrm{div}(\rho u) = 0,$$

$$(\rho u)_t + \mathrm{div}(\rho u \otimes u) + \nabla p = \mu \Delta u + (\lambda + \mu)\nabla(\mathrm{div} u) + \rho f$$

complemented by suitable boundary conditions as the case may be. We shall discuss some recent results concerning the dependence of solutions to this problem on the spatial domain. The topics we want to address include: 1) A general existence theorem for global weak solutions on any spatial domain with no restriction on the regularity of the boundary. 2) Problems in optimal design. The dependence of the solutions on the variations of the boundary of the corresponding spatial domains. The existence of the weak solutions to certain minimization problems. 3) A general existence theorem for the weak solutions in the situation when the fluid contains moving rigid objects. The theorem takes into account possible collisions and includes rather general class of rigid objects.Numerical simulations are given.

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Large time behavior for vortex evolution in the half-plane.

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In this talk we discuss the rigorous study of the time asymptotic behavior of solutions of the two dimensional Euler equations for incompressible and ideal fluids in the half-plane. Our main result is that possible asymptotic states for half-plane vortex dynamics must be a discrete superposition of states with well-defined asymptotic mean velocity. This is joint work with M. Lopes Filho and H. Nussenzveig Lopes.

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Coupling effect for Kadomtsev-Petviashvili equations

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Under the long wave approximation, the Euler equations yield, in the general $1D$ case, a system of two uncoupled KdV equations, corresponding to two counterpropagating waves. The error term is $O(\epsilon)$, where ϵ is the long wave small parameter. Under slight transverse perturbations, KdV equations must be replaced by KP equations. This can be done formally for the Euler equations, and can be proved for a model case. As for KdV, one obtains a system of uncoupled KP equations, but the error term is only $o(1)$ here. We propose to explain this difference by slight coupling effects between the two counterpropagating waves.

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On the short-wave limit of the water wave interaction equations

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This paper is devoted to the study of short-wave limit for the water wave interaction equations which arise in the study of surface waves with both gravity and capillary modes present and also in plasma physics. For the smooth solution, the limiting equation is given by the compressible Euler equation with a nonlocal pressure caused by the long wave. For weak solution, the nonlocal wave map equation is derived.

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Gravity travelling waves for two superposed fluid layers, one being of infinite depth: a new type of bifurcation

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In this talk, we study the travelling gravity waves in a system of two layers of perfect fluids, the bottom one being infinitely deep, the upper one having a finite thickness h. We assume that the flow is potential, and the dimensionless parameters are the ratio between densities $\rho = \rho_2/\rho_1$ and $\lambda = gh/c^2$. We study special values of the parameters such that $\lambda(1-\rho)$ is near 1^-, where a bifurcation of a new type occurs: Formulating the problem as a spatial reversible dynamical system, where $U = 0$ corresponds to a uniform state (velocity c in a moving reference frame), and considering the linearized operator around 0, we show that its spectrum contains the entire real axis (essential spectrum), with in addition a double eigenvalue in 0, a pair of simple imaginary eigenvalues $\pm i\lambda$ at a distance $O(1)$ from 0, and for $\lambda(1-\rho)$ above 1, another pair of simple imaginary eigenvalues tending towards 0 as $\lambda(1-\rho) \to 1^+$. When $\lambda(1-\rho) \leq 1$ this pair disappears into the essential spectrum. The rest of the spectrum lies at a distance at least $O(1)$ from the imaginary axis. Such a bifurcation in infinite dimensions cannot be reduced to finite dimensions using the center manifold theorem because of the lack of spectral gap of the linear operator induced by the essential spectrum lying on the whole real line. Nevertheless, we show in this talk that for $\lambda(1-\rho)$ close to 1^-, there is a family of reversible periodic solutions like in the Lyapunov-Devaney theorem (despite the resonance due to the point 0 in the spectrum). Moreover, showing that the full system can be seen as a perturbation of the Benjamin-Ono equation, coupled with a nonlinear oscillation, we also prove the existence of a family of homoclinic connections to these periodic orbits, provided that these ones are not too small.

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3D Navier-Stokes equations with initial data characterized by uniformly large vorticity

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We prove existence on infinite time intervals of regular solutions to the 3D Navier-Stokes Equations for fully three-dimensional initial data in R^3 characterized by uniformly large vorticity and infinite energy; smoothness assumptions for initial data are the same as in local existence theorems. The global existence is proven using techniques of fast singular oscillating limits and the Littlewood-Paley dyadic decomposition. Infinite time regularity is obtained by bootstrapping from global regularity of the limit equations and strong convergence theorems.

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Large viscous boundary layers for noncharacteristic nonlinear hyperbolic problems

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We study linear and nonlinear stability of large-amplitude multidimensional viscous boundary layers arising through the small viscosity perturbation of a hyperbolic initial-boundary value problem with noncharacteristic boundary. The main result is to show that, provided there holds the necessary condition that all "frozen," planar boundary layers associated with the inner layer of the profile satisfy an appropriate Evans function condition, then the linearized equations about the full profile are well-posed in L^2, with sufficiently strong estimates on the solution and its derivatives as to yield a full nonlinear stability result and thereby nonlinear continuation/validation of the formal boundary layer expansion.

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3D Euler Equations with initial data characterized by uniformly large vorticity

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We prove existence on arbitrary long time intervals of regular solutions to the 3D Euler Equations with initial data characterized by uniformly large vorticity; smoothness assumptions for initial data are the same as in local existence theorems. The above results are obtained in the cases of (i) bounded cylindrical domains with no flux boundary conditions, generic aspect ratios and weak alignment of the initial vorticity with the cylinder axis at t=0; (ii) 3D Euler Equations for fully three-dimensional initial data in R^3, with probability one in Vishik-Fursikov spaces of statistical solutions, characterized by uniformly large vorticity and infinite energy. The long-time existence theorems are proven using techniques of fast singular oscillating limits; the case of cylindrical geometry requires new techniques compared to periodic, infinite and/or slab domains. Algebraic geometry of resonant Poincaré curves is investigated in both cases.

The existence of a countable set of finite dimensional manifolds invariant under the nonlinear dynamics is demonstrated for the limit “2 1/2- dimensional” nonlinear Euler equations in generic cases.

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Refocusing for classical waves in complicated media

Leonid Ryzhik
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We will discuss the following remarkable property of solutions of wave equations in random media (or ergodic billiards) with localized initial data. Let us record the solution at some time T, restrict it to a finite (small) domain, transform it linearly in some fashion, possibly smooth it and use the resulting signal as a new initial data for the wave equation. It turns out that the new re-propagated solution will concentrate at the original source location at the same time T for a very large class of signal processing. This is somewhat surprising since recording domain is very far from being the full space. In particular this explains the time-reversal experiments.

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Long-time averaging and fast correctors

Steve Schochet
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An alternative derivation of the averaging method for systems with two time scales will be presented, and the relationship between the correctors and the applicability of the method to large time scales will be discussed.

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Control in presence of oscillations

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In this lecture we will consider differential systems which might include, but are not limited to the discretized form of equations of the atmosphere and the ocean, such as the Lorentz equations or more involved ones. The presence of a small parameter (the Rossby number for the equations of the atmosphere) produces oscillations in the system which are well understood following recent work in the area by many authors (S. Schochet and others). We consider the control problem for these equations with the long range aim of stabilizing unstable modes by a minimal injection of energy. In this preliminary report on an ongoing work, we introduce and study the control problem, and study its behavior when the small parameter converges to zero.

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Structural analysis and boundary layer separations of 2D incompressible flows

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We present in this talk a new dynamical systems theory for 2D incompressible flows. With this theory, we derive a rigorous characterization of boundary layer separations for 2D incompressible fluids, a long standing problem in fluid dynamics.

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Approximating equations for water waves

C. Eugene Wayne
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G. Schneider and **D. Wright**

I will review recent work on deriving and justifying approximating model equations for water waves.Among the equations we consider are the Korteweg-de Vries, Kawahara, and three-wave-interaction model. In each case we give rigorous estimates relating solutions of the model equation to the solutions of the full water wave equations. I will also describe some results on the form of the next order corrections to the KdV approximation.

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Stability of multidimensional viscous shocks

Mark Williams
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We discuss a new method involving degenerate Kreiss symmetrizers for studying the stability of multidimensional viscous shocks. We discuss applications to the questions of longtime stability and also to the small viscosity limit.

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Hamiltonian Systems

Organizer: Amadeu Delshams, Universitat Politcnica de Catalunya
Antonio Giorgilli, Universitá di Milano Bicocca

Symbolic dynamics, homoclinic and heteroclinic connections for the Hénon-Heiles system

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We consider the Hénon-Heiles Hamiltonian system $H = (p_x^2 + p_y^2)/2 + (x^2 + y^2)/2 + x^2y - y^3/3$ at the critical energy level. We present a computer assisted proof of the existence of a rich symbolic dynamic structure and of infinitely many periodic, homoclinic and heteroclinic orbits.

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Global regularization for the n-center problem on a manifold

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The n-center problem is a Lagrangian system on a 3-dimensional manifold with the potential energy V having n Newtonian singularities. We describe a global version of the KS regularization of singularities. The regularized configuration space turns out to be 4 or 5-dimensional manifold. As an application, using the results of Gromov and Paternain, we show that the n-center problem in R^3 has positive topological entropy for $n > 2$ and energy $E > \sup V$. For the n-center problem in S^3 the same holds for $n > 4$. These are purely topological phenomena independent of the concrete form of the Lagrangian.

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Geometric methods for instability in Hamiltonian systems

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Tere M. Seara and **Rafael de la Llave**

We describe some geometric features of a mechanism for detecting global instability in "a priori-unstable nearly integrable Hamiltonian systems". The mechanism presented is based on decomposing the motion in two types of different dynamics, one called "inner" that takes place inside a normally hyperbolic invariant manifold, where a lot of regular objects (i.e., invariant tori) live, and another one called "outer", that takes into account the asymptotic motions to the hyperbolic manifold. The combination of both types of dynamics gives rise to chaotic dynamics and instability. This mechanism has been applied to several problems. For instance, it was first applied to the existence of orbits of unbounded energy in generic geodesic flows with a (periodic or quasi-periodic) time-dependent potential. More recently, it has been applied to overcome the large gaps problem in Arnold diffusion.

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The problem of the rate of thermalization in the light of the Boltzmann–Jeans conjecture: the case of diatomic molecules

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In the modern theory of dynamical systems, after the classical work of Fermi, Pasta and Ulam, much emphasis was given to the point of view of Boltzmann and Jeans, concerning systems of physical interest which can be mathematically described as consisting of weakly coupled harmonic oscillators. Indeed, according to such authors, the observed lack of equipartition of energy among the various degrees of freedom could be explained by taking into account the time actually needed for the relaxation to equilibrium, while the existence of extremely long relaxation times has by now become familiar in the theory of dynamical systems after the work of Nehoroshev. On the other hand, quantitative estimates were lacking. In the present paper an attempt is made at considering realistic models for the simplest case of interest, namely that of diatomic molecules, in order to make a comparison with the experimental data.

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Triality and new phenomena in non-smooth, non-convex and non-conservative Hamilton systems

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This talk will present a brief survey and some new developments on the duality and triality theories in general nonsmooth/nonconvex, nonconservative Hamilton systems. By using the canonical dual transformation method developed by the speaker recently, many nonsmooth problems can be transformed into smooth dual problems, and certain nonconvex partial differential systems can be converted into the so-called Differential-Algebraic equations (DAEs). The triality theory reveals some interesting extremality properties and intrinsic symmetry in nonconvex dynamical systems. Based on this theory, an alternative algorithms is proposed for solving nonsmooth/nonconvex dynamical problems. Application is illustrated by a semi-linear, nonconvex parametric variational problem. Some interesting phenomena, i.e. meta-chaos and post-chaos are discovered in nonconvex/nonconservative Hamilton systems with double-well potential. Dual feed-back control against chaos is discussed.

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Topologically crossing heteroclinic connections and symbolic dynamics

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Piotr Zgliczynski

We present a topological refinement of Easton's method of windows. In our approach, two windows are correctly aligned with respect to a given diffeomorphism provided that the boundary of each window decomposes into naturally defined exit and entry components, and the image of some horizontal in the first window can be homotopically deformed onto a horizontal in the second window through a map with nonzero local Brouwer degree. This method does not require any transverse intersections of horizontals and verticals under iteration, so it can be used to study pathological cases in which heteroclinic connections with topological crossings are present. The main result in this line is that "one can see through a sequence of correctly aligned windows". We apply this technique to investigate a class of Hamiltonian systems possessing invariant tori whose stable and unstable manifolds cross topologically along solution curves, relative to some Poincar sections. We show that symbolic dynamics and 'small' Arnold diffusion can be detected in this case. Our method addresses situations in which one infers the existence of homoclinic and heteroclic orbits by solving for the critical points of a Melnikov potential, without necessarily checking their nondegeneracy.

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Hyperbolicity in time-dependent Hamiltonians with applications to fluid mechanics

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Two-dimensional incompressible fluid flows are described by 1DOF time-dependent Hamiltonian systems. In realistic applications, these systems are often far from integrable, admit general time dependence, and are only known for finite times in the form of experimental or numerical data. For all these reasons, finding hyperbolic structures that cause chaotic or turbulent mixing in such flows remains a great challenge. In this talk we survey recent analytic results on locating finite-time hyperbolic invariant manifolds in two-dimensional fluid flows. We also show how these results can be used to locate Lagrangian coherent structures in geophysical turbulence.

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On the existence of stable quasiperiodic motions near the triangular points of the real Earth-Moon system

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In this work we consider the motion of an infinitesimal particle near the (geometrically defined) equilateral points of the real Earth-Moon system. We use, as real system, the one provided by the JPL ephemeris: the ephemeris give the positions of the main bodies of the solar system (Earth, Moon, Sun and planets) so it is not difficult to write the vectorfield for the motion of a small particle under the attraction of those bodies. Numerical integrations of this vectorfield show that trajectories with initial conditions in a vicinity of the equilateral points escape after a short time. To do a preliminary study, we will introduce an analytic model that can be written as a quasi-periodic perturbation of the Restricted Three-Body Problem, and that tries to model the effect on the Sun and the eccentricity of the Moon. Then, we will compute some families of normally elliptic 3-D invariant tori near the triangular points, that give rise to regions of effective stability. By means of numerical integrations, we will show that they seem to persist in the real system, at least for time spans of 1000 years.

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A geometric proof of Mather's connecting and accelerating theorems

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We shall present a geometric proof of two important theorems by J.Mather about diffusing orbits of Hamiltonian systems. One (connecting) says that for an area-preserving twist map inside a region of instability there are trajectories connecting any two Aubry-Mather sets. Area-preserving twist maps naturally arise in Hamiltonian systems in two-degrees of freedom. Another (accelerating) says that for a generic Hamiltonian systems on a two-torus which is time periodic (2 1/2 degrees of freedom) there is a trajectory whose speed gradually accelerates to an arbitrary large speed. Other proofs of the last theorem were given by Bolotin-Treschev and Delschams-de la Llave-Seara.

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Renormalization of Hamiltonian flows, and critical invariant tori

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We describe a renormalization group (RG) transformation for partially analytic Hamiltonians in 2n variables, and its relation to a RG transformation for n-tuples of commuting maps. The conjecture is that a transformation of this type has a fixed point with nontrivial scaling properties. We show that, under some technical assumptions, such a fixed point (Hamiltonian) has an invariant torus that is critical, in the sense that its degree of differentiability is positive but less than one. We will also report on progress toward a proof of the above mentioned conjecture.

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Global behavior of mechanical systems with cubic homogeneous potentials

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M. Falconi and **C. Vidal**

We will review our previous results on the global behavior of two degrees of freedom classical mechanical systems with cubic homogeneous potentials for the simple case of negative energy. Then we will present new results for the case of positive energy. This case is harder than the case of negative energy, because in McGehee like blow up coordinates at infinity the flow is gradient-like only at the infinity manifold. Besides, solutions may reach the origin of configuration plane, which is sent to the unbounded part in blow up coordinates.

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Twistless bifurcations in Hamiltonian systems

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Near a nonresonant, elliptic fixed point, a symplectic map can be transformed into Birkhoff normal form. In these coordinates, the dynamics is represented entirely by the Lagrangian frequency map which gives the rotation number as a function of the action. The twist matrix, given by the Jacobian of the rotation number, describes the anharmonicity in the system. When the twist is singular the frequency map is in general not locally one-to-one. We will discuss the occurrence of fold and cusp singularities in the frequency map and show that these necessarily occur near third order resonances. We

illustrate the results by numerical computations of frequency maps for a quadratic, symplectic map.

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On central λ - stability zone for linear discrete-time Hamiltonian systems

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In this paper we start from the discrete version of linear Hamiltonian systems with periodic coefficients

$$y_{k+1} - y_k = \lambda(B_k y_k + D_k z_{k+1})$$
$$z_{k+1} - z_k = -\lambda(A_k y_k + B_k^* z_{k+1})$$

where A_k and D_k are Hermitian matrices, A_k, B_k, D_k define N - periodic sequences, and λ is a complex parameter. For this system a Krein - type theory of the λ - zones of strong (robust) stability may be constructed. Within this theory the side λ - zones' width may be estimated using the multipliers' "traffic rules" of Krein while the central stability zone (centered around $\lambda = 0$) is estimated using the eigenvalues of a certain self-adjoint boundary value problem. In the discrete-time there occur some specific differences with respect to the continuous time case due to the fact that the transition matrix (hence the monodromy matrix also) is not entire with respect to λ but rational. During the paper we consider some specific cases (the matrix analogue of the discretized Hill equation, the J - unitary and symplectic systems, real scalar systems) for which the results on the eigenvalues are complete and obtain some simplified estimates of the central stability zones. AMS (MOS) Subject Classification: 39A10, 39A11, 39A12 **Keywords**: Discrete-time, Hamiltonian systems, stability.

⟶ ∞ ⋄ ∞ ⟵

Transition state via normal forms

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A. Burbanks, **C. Jaffé**, **J. F. Palacián**
T. Uzer and **S. S. Wiggins**

In this talk we show the normal form approach as a way of getting an analytical handle on geometric objects, such as normally hyperbolic invariant manifolds (NHIM), and the breakthrough this is in chemistry problems. Computer visualization is used in order to "see" these objects. In order to illustrate the technique we focus on the Rydberg atom problem. Concretely, we show how to determine analytically the transition state (TS) in this type of chemical reactions. For that we calculate the normal form and transform the original three-degree-of-freedom (3DOF) Hamiltonian to one of 0DOF. In fact, we are able to construct three asymptotic integrals of the original Hamiltonian by inverting the normal form transformation. Moreover, we calculate in the original system the three-dimensional normally hyperbolic invariant manifold (NHIM), its stable and unstable manifolds, as well as the transition state. We compute trajectories that start on the NHIM in the five-dimensional energy surface. Besides, we determine trajectories in either the forward or backward stable and unstable manifolds associated to the NHIM. These trajectories are simply chosen and computed from the normal form vector field. The normal form transformation then allows us to visualize them in the original coordinates. Thus, we have complete control and knowledge of the exact dynamical trajectories near the TS in a 3DOF system. This is the first time this has been demonstrated for a 3DOF chemical or atomic system.

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Phase-space transition states

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We use dynamical systems theory to construct a general phase space version of transition state theory. Special multidimensional separatrices are found which act as impenetrable barriers in phase space between reacting and nonreacting trajectories. The elusive momentum-dependent transition state between reactants and products is thereby characterized. A practical algorithm is presented and applied to a strongly coupled Hamiltonian.

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Mathematical Issues of Geophysical Fluid Dynamics

Organizer: Michael Ghil, University of California, Los Angeles
Shouhong Wang, Indiana University

Global well–posedness for a planetary geostrophic model with hyper–viscosity

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We consider a three dimensional planetary geostrophic hyper–viscous model of the gyre-scale midlatitude ocean. We show the global existence and uniqueness of the weak and strong solutions to this model. Moreover, we establish the existence of a finite dimensional global attractor to this dissipative evolution system.

⟶ ∞ ⋄ ∞ ⟵

The effect of a bathymetry vanishing on the shore on different geophysical models

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In this talk, I present some recent results on different geophysical models in a domain with a depth vanishing on the shore. This has some applications in oceanography or in limnology. For example, the hydrostatic Navier-Stokes equations, the quasi-geostrophic equations, the planetary geostrophic equations will be discussed.

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Model of the shallow-water equations with variable resolution

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John Drake

A formulation of a semi-implicit, semi-Lagrangian spectral method is given together with a conformal mapping of the underlying discrete grid. The mapping based on the Schmidt transformation, focuses grid resolution on a particular region. A new advective form of the vorticity-divergence equations allows for the conformal map to be incorporated while maintaining an efficient spectral transform algorithm. A shallow water model on the sphere is used to test the variable resolution spectral model. We are able to focus on a specified location resolving local details of the flow.

⟶ ∞ ⋄ ∞ ⟵

Strong solution of primitive equations under small depth assumption

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We study the global (in time) existence and the uniqueness of the solutions of the Primitive Equations for the large scale ocean in thin domains M_ε. The thickness parameter ε of the domain is the aspect ratio between its vertical and horizonal scales. Using and generalizing the methods developed Temam and Ziane, we obtain the global existence of solutions for initial data and volume and boundary 'force', which belong to large sets in their respective phase spaces, provided ε is sufficiently small. Our proof of the existence results for the PEs is based on precise estimates of the dependence of a number of classical constants on the thickness ε of the domain. We establish various anisotropic Sobolev-type inequalities, such as the Poincaré, Ladyzhenskaya, and Agmon inequalities, to the domains with complicated geometry corresponding to the shape of the ocean, and to the case of non-homogeneous boundary conditions. Due to the lack of smoothness of the domains under consideration, we are led to directly prove the Cattabriga-Solonnikov regularity result for the Stokes-type problem arising in the PEs, and some related type of regularity results for the temperature and salinity. The aforementioned inequalities and regularity results are important tools in the study of the PEs.

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Some remarks on the theory of multiple equilibria in the atmosphere

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Shouhong Wang

Multiple flow equilibria in the atmosphere is a basic characteristic of the atmospheric circulation. However, the necessary condition of multiple equilibria in the atmosphere is an important and still unsolved problem. Theoretically, this problem is equivalent to finding the condition of a multi-solution existing in stationary equations of the atmosphere. Based on the full primitive equations of stationary atmospheric motion the problem is discussed in this paper. Through studying three simplified equations of the atmospheric stationary equations, the linearized equations, adiabatic dissipative equations and forced non-dissipative equations, we prove that for the stationary equations of the atmosphere with its stationary solution is either unique or non-existential at the absence of any one of nonlinearity, dissipation and external forcing. Therefore, the joint action of nonlinearity, dissipation and external forcing is the source of multiple equilibria in the atmosphere.

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The 3-D basic state of the atmospheric motion

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Shikuo Liu

In this paper, a simple partial differential equation with respect to the basic state of 3-D convergence upward wind is deduced. The convergence upward wind results form balance among pressure gradient force, Coriolis force and viscous force. The status of 3-D convergence upward wind represents convergence upward motion for low pressure with cyclonic vorticity and divergence downward motion for high pressure with anticyclonic vorticity in atmosphere lower level. The 3-D velocity of convergence upward wind can be decomposed by stream function and convection velocity potential. As Reynolds number $Re \to \infty$, convergence upward wind reduces to geostrophic wind.

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Global bifurcation in the double-gyre ocean circulation

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Benjamin Luce LANL

The dynamics of an idealized wind-driven double-gyre circulation in an ocean basin are studied from a dynamical systems point of view in an effort to better understand its variability. While previous analyzes of this circulation have mostly dealt with local bifurcations of steady states and limit cycles, this study demonstrates the importance of considering global bifurcations as well. In one case, a coherent picture of the global dynamics spanning a range of parameters from where there are only stable steady-state solutions to where there is chaotic eddy-shedding presented. A simple but novel use of power spectra along with dynamical projections of the dynamics suggests that just beyond the regime in which there are only stable steady-states, the system exhibits a complicated global bifurcation known as "Shilnikov Phenomenon".

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The anisotropic Lagrangian averaged Navier-Stokes equations

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Lagrangian averaging for the Navier-Stokes (LANS-alpha) equations on bounded domains, such as channels and pipes, yields a closed system of partial differential equations for the mean velocity field of a turbulent fluid and its covariance tensor. The analysis of these equations requires dealing with elliptic operators that degenerate at the boundary. We shall discuss well-posedness results for this system, and then consider the problem of finding the mean velocity profile in a channel using weighted Sobolev spaces. We shall show that only one possible weight function describing the degeneracy of the covariance tensor yields physical velocity profiles.

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Homoclinic bifurcations in the wind-driven double-gyre circulation

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Only recently, the impact of homoclinic explosions in various models of the double-gyre circulation has been pointed out as an efficient source of low-frequency variability in mid-latitude oceanic flows. We focus here on a simple barotropic quasi-geostrophic model of the double-gyre circulation. We discuss the dynamical origin and genericity of the first global (homoclinic) bifurcation, the possible existence of different chaotic regimes, and the influence of the symmetry on the dynamics.

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Numerical simulation of the primitive equations formulated in mean vorticity

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The primitive equations (PEs) of large-scale oceanic flow formulated in mean vorticity is proposed and the numerical methods based on the corresponding formulation are presented in the talk. Since the vertical average of the horizontal velocity field is divergence-free, we can introduce mean vorticity and mean stream function which are connected by a 2-D Poisson equation. As a result, the PEs can be reformulated such that the prognostic equation for the horizontal velocity is replaced by evolutionary equations for the mean vorticity field and the vertical derivative of the horizontal velocity. The total velocity field (both horizontal and vertical) is statically determined by either recovery formulae or differential equations at each fixed horizontal point. Both the second order and the fourth order finite difference approximation can be used for the prognostic equations and the determination of numerical values for the total velocity field is implemented by FFT-based solvers. Accuracy check and numerical results for a physical example of thermocline circulation are also addressed.

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Hopf bifurcation in quasi-geostrophic channel flow

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Zhi-Min Chen and **Michael Ghil**

We conduct a rigorous stability and bifurcation analysis for a highly idealized model of planetary-scale, atmospheric and oceanic flows. The model is governed by the two-dimensional, quasi-geostrophic equation for the conservation of vorticity in an East-West oriented, periodic channel. The main result is the existence of Hopf bifurcation of the flow as the Reynolds number crosses a critical value.

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Selective decay principle

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Andrew Majda

We generalize the physicist's selective decay principle from one layer model to two layer model and continuously stratified model. These generalized selective decay principle will enable us to explain computational experiments regarding certain decaying geophysical flows.

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Recent Trends in Nonlinear Analysis

Organizer: Hiroshi Matano, University of Tokyo
Paul Rabinowitz, University of Wisconsin

Spatial heterogeneity and the coexistence of competing species

Steve Cantrell
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In this talk we use reaction-diffusion models to examine the impact of spatial heterogeneity on the coexistence of two competing species simultaneously inhabiting a bounded isolated habitat patch. Specifically, we are interested in the issue of when and how incorporating spatial variation in one or more coefficients in the model equations can lead to a prediction of coexistence in a model which fails to predict coexistence in the absence of the spatial variation. We employ elements of persistence theory, singular perturbation theory and eigenvalue analysis in our study.

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Recent results on asymptotically linear problems in R^N

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We present some recent results (joint work with H. Tehrani) on the question of existence of solution for a class of semilinear equations of the form

$$Lu = g(x, u) \ , \ x \in R^N \ ,$$

where L is a linear selfadjoint operator on a Hilbert space of L^2 functions on R^N (having both *point* and *essential* spectrum) and the nonlinearity $g(x, u)$ is *asymptotically linear* in a suitable sense as $|u| \to \infty$.

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Generalized min-max theory and sequential canonical dual transformation methods in nonconvex analysis

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Duality and associated min-max theory are beautiful and fundamental concepts that underlie almost all natural phenomena. In finite deformation theory, modern physics, chaotic dynamical systems, non-convex variational analysis, global optimization and computational science, duality principles and methods are playing more and more important roles. Motivated by many very interesting problems in nonconvex mechanics, the speaker will present a newly developed, potentially useful canonic dual transformation method and associated generalized min-max theories. He will show that by this method, many fully nonlinear, non-smooth partial differential equations in mathematical physics can be converted into certain smooth algebraic systems. Therefore, some closed form solutions can be obtained. In addition to the traditional saddle-Lagrange duality in static systems, a nice bi-duality theory is proposed for solving dynamical problems. The extremality conditions in general nonconvex problems are governed by a very interesting tri-duality theory, which reveals the intrinsic symmetry in fully nonlinear systems. As a typical example, the semi-linear nonconvex variational problem with applications in super-conductivity and dynamical systems will be illustrated. Some very interesting new phenomena in chaotic systems are discovered. Potential applications in computational science will be discussed.

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Stability and asymptotic behavior of solutions of a semilinear heat equation on R^N

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Eiji Yanagida

We consider the Cauchy problem for a semilinear heat equation on R^N with a power nonlinearity. It is known that if the exponent of the nonlinearity is sufficiently large, then there is a family of steady states that are stable in a suitable sense. We describe the domain of attraction of the steady states, and also of general time-dependent solutions, by giving sharp conditions on the behavior of initial data at spatial infinity. We further discuss the asymptotic behavior of solutions bounded above and below by a singular steady state. In particular, we show the existence of solutions that approach a continuum of steady states (rather than a single steady state) as time approaches infinity.

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Multiplicity of solutions for quasilinear elliptic problems involving critical Sobolev exponents

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Magda S. Xavier

The main results of this paper establish, via the variational method, the multiplicity of solutions for quasilinear elliptic problems involving critical Sobolev exponents under the presence of symmetry. The concentration-compactness principle allows to prove that the Palais-Smale condition is satisfied below a certain level.

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Singularly perturbed elliptic equations on unbounded domains

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When one searches for stan wave solutions of the nonlinear Schrodinger equation, the equation reduces to a semilinear elliptic equation. The equation has a small parameter attached to the highest derivative term. For small parameter values, ground states of the solution, resembling "spikes," may exist. The existence and location of spikes are determined by properties of the coefficient functions in the equation. I will give a history of findings for these equations, including results by myself and many others.

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Periodic solutions to some N body problems

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A remarkable new solution of the 3-body problem was recently discovered by Chenciner and Mongomery, using some symmetry argument. We shall show how many non collision periodic solutions can be constructed by exploiting symmetries of the systems.

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Semilinear parabolic and elliptic equations on a thin domain with a node

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Solutions of semilinear parabolic and elliptic equations on a thin domain can be approximated by those on an interval. When a thin domain has a bump or a narrow neck, the limiting equation is defined on two intervals with a node and a matching condition is imposed at the node. It is shown in this talk that, depending on the size of the bump and the narrow neck, various types of matching conditions appear.

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Solution for a resonant elliptic system with coupling in $\mathbb{R}^N$

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Existence and multiplicity of solutions are established, via the Variational Method, for a class of resonant semilinear elliptic system in under a local nonquadraticity condition at infinity. The main goal is to consider systems with coupling where one of the potentials does not satisfy any coercivity condition. The existence of solution is proved under a critical growth condition on the nonlinearity.

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Existence and multiplicity results for a class of elliptic problems in $\mathbb{R}^N$

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We are interested in positive solutions for the problem

$$\begin{cases} -\mathrm{div}\big(a(x)\nabla u\big) + \lambda^2\, u = b(x), \quad |u|^{p-2}u \quad \mathrm{in}\mathbb{R}^N \\ \lim\limits_{|x|\to\infty} u(x) = 0, \qquad (1) \end{cases}$$

where $N \geq 2$, $2 < p < 2^*$, $\lambda > 0$ and a, b are continuous and bounded coefficients. We assume that $0 \leq a(x) \leq \lim_{|x|\to\infty} a(x)$ and $b(x) \geq \lim_{|x|\to\infty} b(x) > 0$. In particular, a may vanish somewhere in $\mathbb{R}^N$; in such a case, we impose additional conditions on the location of zeros of a and the local summability of some negative power of a. We study the effect of the properties of the coefficients a and b on the existence and multiplicity of solutions to (1).

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Mathematical Aspects of Wave Propagation

Organizer: Boris Belinskiy, University of Tennessee at Chattanooga

Exact control of a string under an axial stretching tension

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Boris Belinskiy

We study the exact controllability problem for a string under an axial stretching tension. We are looking for an exterior force $g(x)f(t)$ that drives the state solution to the rest. The tension is a sum of two terms. The first term is a positive constant and the second one is small and slowly varies in time. The function $f(t)$ is considered as a control. The problem of exact controllability is reduced to a moment problem for $f(t)$. If the tension is a constant, the moment problem for a system of non-harmonic exponentials appears (see the classical papers by D.L. Russell). In our case, the tension varies in time, and the moment problem is more complex. Its solution requires the proof of the basis property for a system of special solutions of a second order differential equation with variable coefficients.

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Stability of mechanical system with multiplicative white noise

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Boris Belinskiy

The asymptotic behavior as $t \to \infty$ of some linear mechanical systems described by stochastic PDE with multiplicative white noise is considered. Ito and Stratonovich interpretations are used. They lead to different conditions of stability with respect to the expected energy. If the initial data are random (independent on white noise) but the parameters are not, the systems are stable. Otherwise the expected energy may be infinite, approach zero, remain bounded, or increase with no bound. The necessary and sufficient conditions for stability in terms of the structure of roots of an auxiliary equation are formulated.

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Analyticity of semigroup for Mead-Markus sandwich plate

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The classical sandwich plate model of Mead and Markus has been very useful in explaining vibrational properties of composite structures. In particular this model explains why damping due to shear in the middle layer (i.e., constrained layer damping) leads to much higher dissipation of energy at low frequencies than other forms of damping. In this paper we formulate a general version (for n layers) of the sandwich plate model and show that shear damping in alternate layers leads to analyticity of the associated semigroup.

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The resolvents of operators in several classes of integrodifferential equations

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The resolvent of an operator plays a crucial role in determining properties of the semigroup generated by the operator. The interest of this paper is to develop an effective way of determining properties of semigroups (thus leading to the properties of solutions) with respect to parameters through obtaining the properties of resolvents in abstract integrodifferential equations. A specific discussion focuses on a problem arising in viscoelasticity .

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Some recent findings concerning unsteady dipolar fluid flows

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P. Puri

Stokes' first and second problems, as well as that of plane Couette flow, are considered for incompressible dipolar fluids, dipolar fluids being the simplest example of a class of non-Newtonian fluids known as multipolar fluids. Laplace transform methods are used to determine exact solutions, for arbitrary values of the dipolar constants $d(\geq 0)$ and $l(> 0)$, to these unsteady flow problems. In considering special/limiting cases of the dipolar constants, exact solutions are also determined for Rivlin-Ericksen, couple stresses, and viscous Newtonian fluids. In particular, steady-state development time, displacement thickness/mass flux, and boundary layer thickness are determined for the various fluid types and results for all fluids are then compared. The influence of d, l, and the other physical parameters on the velocity field is illustrated, as well as the effects of start/stop plate motion in the Couette flow case. Most significantly, we show that the velocity field suffers a jump discontinuity at start-up when $d > 0$, a backflow condition is possible when $l > d$, a phase velocity term is negative when $l > d$, and for special values of the physical parameters the flow instantly attains steady-state. Lastly, bounds are placed on the value of the dipolar traction at the plate(s).

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An averaging method for the Helmholtz equation

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Boris Pavlov

The well-known J. Schauder result on the existence of $\mathrm{Lip}_\alpha(\Omega)$ solutions of the Dirichlet problem for bounded domains with smooth boundaries is obviously true for the Helmholtz equation $-\triangle u = \lambda u$, with spectral parameter different from eigenvalues of the corresponding homogeneous problem. We suggest a new method of construction of the solution, based on an averaging procedure and mean-value theorem. We show that, for $0 < \alpha < 1$, and $\lambda \leq \lambda_0$, where λ_0 is a positive parameter depending on the "effective width" of the domain, the sequence of iterated averages converges in $\mathrm{Lip}_\alpha(\Omega)$ as a geometric progression.

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Optimal design of turbines

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Boris Belinskiy

We minimize, with respect to shape, the lowest frequency of the torsional oscillations of a turbine, subject to fixed moment of inertia. The problem is modelled by a Sturm-Liouville problem with eigenparameter dependence in the boundary conditions. Rearrangements with respect to a weight lead to the existence of an optimal design. Optimality conditions are also established.

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Some aspects of output determination for distributed parameter

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We consider a linear input-output system in the abstract form $dx/dt = Ax + bu$, x,b in H, $y =_H$, c in H, u scalar, where H is a Hilbert space. The goal is to be able to determine the output function $y(t)$ on an interval $[t_0, \infty)$, $t_0 > 0$, through appropriate choice of $u(t)$ on $[0, \infty)$. The values of $y(t)$ on $[0, t_0)$ are ignored. Clearly we need $U(s) = Y(s)/T(s)$, where $T(s) =< (sI - A)^{-1}b, c >$ is the transfer function and $Y(s)$ and $U(s)$ are the Laplace transforms of $y(t)$ and $u(t)$, respectively. Since $T(s)$ will, in most cases of interest, have an infinite collection of right half plane zeros; the requirement u in $L^2(0, \infty)$ implies that $Y(s)$ must have zeros at those same points. This is arranged, if it can be arranged, through the choice of $y(t)$ on $[0, t_0)$; the choice of t_0 depends on the system in question. Additionally $T(s)$ may tend to zero at various rates depending on how s approaches infinity; this leads to further requirements on $y(t)$, not necessarily restricted to $[0, t_0)$; e.g. differentiability and/or related requirements on the whole output $y(t)$ may thereby be indicated. A number of specific cases related to applications will be considered.

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Realization theory of Herglotz-Nevanlinna matrix-valued functions

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A survey of recent results in realization theory of Herglotz-Nevanlinna matrix- valued functions, interpolation problems and explicit system solutions is presented. An extension of Brodskiĭ-Livšic systems (operator colligations), involving an unbounded main operator and an additional orthogonal projection, is studied. It leads to new types of representation and realization results for certain classes of Herglotz-Nevanlinna functions and for the associated transfer functions. The realization criterion of general Herglotz-Nevanlinna matrix-valued function in terms of linear fractional transformation of transfer function of the systems involving triplets of Hilbert spaces (rigged operator colligations) is established. We consider also a new type of solutions of Nevanlinna–Pick interpolation problem, so–called explicit system solutions generated by Brodskii–Livsic time-invariant scattering systems(colligations), and find conditions on interpolation data of their existence, uniqueness and restoration.

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Nonlinear Boundary Value Problems

Organizer: Anna Capietto, University of Torino, Italy

Critical and subcritical nonlinear Schrödinger equation with magnetic field

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We consider the nonlinear stationary Schrödinger equation in a magnetic field $\hat{H}\psi = f(\psi)$, where $\hat{H} = \frac{1}{2m}\left(-i\hbar\nabla - \frac{e}{c}A\right)^2 + eV$. V is the scalar potential and A is the vector potential. We study the existence and nonexistence of solutions for the equation under different assumptions on the potentials and on the nonlinear part f. We consider both the critical case $f(x) = |x|^{2^*-2}x$ and the subcritical case $|f(x)| < |x|^p$, $p < 2^* - 1$. The results are obtained by variational methods; in particular the solutions are obtained either as constrained minima or as mountain pass points.

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Multiple positive solutions for quasilinear elliptic boundary value problems

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R. Shivaji

We will discuss multiplicity of positive solutions for a class of quasilinear elliptic boundary value problem. We prove our result by using the method of sub and super solutions.

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On the existence of solutions and their long time behavior of a nonlocal thermistor inequality

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We study an obstacle problem which models the behavior of certain micromachined microsensor devices:

$$r\frac{du}{dt} - \Delta u + \eta \int_\Omega G(x,y)u(y,t)dy + \gamma u^4 \geq \nabla[\sigma(u)\phi\nabla\phi],$$
$$-\nabla[\sigma(u)\nabla\phi] = 0.$$

Here the unknown functions u and ϕ denotes the distributions of the temperature and the electrical potential in the electrical devices respectively. Let Ω be a domain in R^3. Its boundary $\partial\Omega$ is divided into three parts, i.e., $\partial\Omega = \Gamma_0 \cup \Gamma_1 \cup \Gamma_N$. On the boundary, the temperature u satisfies a homogenous Dirichlet boundary condition. While the potential ϕ satisfies $\phi|_{\Gamma_0} = \phi_0(x,t)$, $\frac{\partial\phi}{\partial n}|_{\Gamma_N} = 0$ and $\phi|_{\Gamma_1} = \xi(t)$. Here $\xi(t)$ is an unknown constant for each t. But the total current $I(t)$ through Γ_1 is given for each time t. Thus another nonlocal boundary condition is given by

$$I(t) = \int_{\Gamma_1} \sigma(u)\frac{d\phi}{dn}ds.$$

We first consider the initial value case, i.e., $u(x,0) = u_0(x)$ with $u_0(x)$ a known function. The existence of a unique solution is established by a penalized method. Next, if $\phi_0(x,t)$ and $I(t)$ are time periodic functions with period T, by setting up a Poicaré map and applying Schauder's fixed point theorem

we show that there exists a time periodic solution such that $u(x, t+T) = u(x,t)$. Finally the long time behavior of the solutions is studied. Actually it is characterized by a uniform attractor. The upper bound of the dimensions of the attractors are also investigated.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some multi-point boundary value problems containing the operator $-(\phi(u'))'$

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In this talk we will consider some result for problems of the form

$$(\phi(u'))' = f(t, u, u'), \quad t \in (a,b),$$

under the multi-point boundary conditions

$$u(a) = 0, \quad u'(\eta) = u'(b),$$

and

$$u'(a) = 0, \quad u(\eta) = u(b),$$

where $\eta \in (a,b)$ is given. We will consider the case when problem (P) is at resonance. Three-point boundary value problems at resonance have been studied in several papers, we present some new result as well as generalizations to the general operator of other results valid for particular forms of the operator $-(\phi(u'))'$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Boundary value problems on sequence spaces

Jesus Rodriguez
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In this paper we study the solvability and qualitative properties of solutions of nonlinear operator equations of the form

$$Lx = F(u,x)$$

subject to

$$H(u,x) = 0$$

where x belongs to a sequence space,u is a parameter, L is a linear map from the sequence space into itself, F and H are smooth nonlinear maps, the range of F is contained in the sequence space and the range of H is contained in some n-dimensional Euclidean space. Connections with differential equations will be established.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlinear boundary value problems of the Calculus of Variations

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We consider various nonlinear boundary value problems arising in the extremal theory for the functional

$$I(x) = \int_a^b L(t, x, x')\, dt,$$

where L is sufficiently smooth. The free end point problem, the basic problem of the calculus of variations, the Bolza problem and others are considered. We are interested mainly in the existence of extremals (solutions of the Euler equation $L_x = \frac{d}{dt} L_{x'}$ together with related boundary conditions) and properties of extremals. First, we discuss the method of upper and lower solutions (functions) and its variational meaning. Second, we derive several the Bernstein - Nagumo type conditions, which are formulated directly in terms of the Lagrangian L. We show the role of those conditions in the theory of coercive, non-coercive (slow-growth) variational problems and discuss also the so called problem of regularity of solutions in the calculus of variations. Third, we consider properties of extremals, which are expressed in terms of the linearized equation (the Jacobi equation) and discuss several problems arising in the theory of the second variation for the functional $I(x)$.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Existence and uniqueness for a class of quasilinear elliptic boundary value problems

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We prove existence and uniqueness of positive solutions for the boundary value problem $(r^{N-1}\phi(u'))' = -\lambda r^{N-1} f(u)$, $u'(0) = u(1) = 0$, where $\phi(x) = |x|^{p-2}x$, $\frac{f(x)}{x^{p-1}}$ may not be decreasing on $(0,\infty)$, and λ is a large parameter.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some multiplicity results for polyharmonic elliptic problems with broken of symmetry

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By means of a perturbation argument devised by P. Bolle we investigate the existence of infinitely many solutions for the problem $(-\Delta)^K u = |u|^{\sigma-2}u + \varphi$ in Ω with nonhomogeneous Dirichlet boundary conditions $\left(\frac{\partial}{\partial\nu}\right)^j u\Big|_{\partial\Omega} = \phi_j \quad j = 0, \ldots, K-1$, provided that suitable growth restrictions on σ are assumed.

Moreover, when σ reaches the critical growth and $\phi_j = 0$, we show the existence of multiple solutions when the domain and the nonhomogenous term are invariant with respect to some group of symmetries.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Periodic solutions to some N body problems

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A remarkable new solution of the 3-body problem was recently discovered by Chenciner and Mongomery, using some symmtery argument. We shall show how many non collision periodic solutions can be constructed exploiting symmetries of the systems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the existence of periodic solutions to second order ordinary differential equations

James Ward
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We consider second order differential equations and systems of the form

$$u'' + g(u) = p(t),$$

where g is a continuous function and $p(t+T) = p(t)$. In the scalar case we assume $g(s)s \geq 0$ for $|s|$ sufficiently large. We provide sufficient conditions for the existence of T-periodic solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive solutions for second order nonlinear equations with singularities

Fabio Zanolin
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We consider the two-point (Dirichlet) boundary value problem for the second order scalar equation

$$u'' + f(t, u) = 0, \quad u(0) = u(1) = 0$$

where the presence of singularities both in t and in u is allowed.

In particular, we present some recent results (jointly with Gaudenzi and Habets) on the existence of positive solutions in the case when $f(t, u) = a(t)g(u)$ and $a : [0, 1] \to I\!R$ is not necessarily of constant sign and may have singularities at $t = 0$ and $t = 1$.

Proofs are based on a development of the method of lower and upper solutions to the singular case. The situation in which $\alpha \not\leq \beta$ (with $\alpha(t)$ and $\beta(t)$ a lower and an upper solution, respectively), is considered as well.

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Stabilization and Optimal Control of Dynamical Systems

Organizer: Yacine Chitour, University of Paris Sud
Ugo Boscain, University of Bourgogne

Optimal control and generalized rigid body dynamics

Anthony Bloch
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In this talk I will discuss the geometry of certain optimal control problems and relate them to geodesic flows on symmetric spaces, smooth and discrete generalized rigid body problems and inviscid fluid flow. In particular I will show how to obtain a discrete symmetric form of the rigid body equations and relate this form to the discrete rigid body equations of Moser and Veselov. In the fluid case I will relate the problem to the so-called impulse equations. This is joint work with P. Crouch and also with D. Holm, J. Marsden and T. Ratiu.

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Lower and upper bounds for the number of switchings for time optimal trajectories of the Dubin's problem on SO(3)

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Yacine Chitour

In this paper, we investigate the structure of time-optimal trajectories for the Dubin's control system on $SO(3)$: $\dot{x} = x(f_1 + uf_2)$, $|u| \leq 1$, where $f_1, f_2 \in SO(3)$ define two linearly independent left-invariant vector fields on $SO(3)$. In particular we are interested to find a lower and an upper bound on the number of switchings for bang-bang trajectories as function of the parameters of the problem. The lower bound is obtained by studying the projection of this control system on the two dimensional sphere by an appropriate Hopf fibration. For the projected problem we compute the optimal synthesis. This is a joint work with Yacine Chitour.

⟶ ∞ ⋄ ∞ ⟵

On the stabilizability of controllable switching systems

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P. Perera

We consider a linear switching system of the form $\dot{x} = Ax + Bu$, where $A \in \{A_i, i = 1, \cdots, k\}$, and B is fixed. It has been shown recently that the controllability of such systems is equivalent to the Lie ideal consisting of constant vector fields contained in the Lie algebra generated by $\{A_i x,\ i = 1, \cdots, k\}$ and B, span the state space. An important unresolved question is whether controllability implies stabilizability in an appropriate sense, and if so, what is the analog of the pole placement theorem for fixed linear control systems. In this note we take an indirect approach to studying this problem. Our first aim is to ask whether it is possible to find positive feedback functions $\alpha_i(x)$, $i = 1, \cdots, k$ such that $\dot{x} = \sum_{i=1}^{k} \alpha_i(x) A_i(x) + Bu$ is controllable. In this note it will be shown that such feedback functions always exist, and in addition, they can be found in such a way that the closed loop system becomes a weighted homogeneous control system. It will be shown that the trajectories of the closed loop system can be approximated via switched trajectories of the open loop system. This will be used to derive a stabilization theorem.

⟶ ∞ ⋄ ∞ ⟵

Verification theorems for Hamilton-Jacobi-Bellman equations

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We study an optimal control problem in Bolza form and we consider the value function associated to this problem. We prove two verification theorems which ensure that, if a function W satisfies some suitable weak continuity assumptions and a Hamilton-Jacobi-Bellman inequality outside a rectifiable set of codimension one, then it coincides with the value function.

⟶ ∞ ⋄ ∞ ⟵

On noncoercive quadratic forms

Manuel Guerra
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Nonautonomous linear-quadratic (LQ) problems are an important class of optimal control problems that arise naturally in the study of second-order optimality conditions for nonlinear problems. The properties of LQ problems that satisfy a strengthened Legendre condition are, in most aspects, well understood. In particular, the cost functional is coercive for time intervals up to the first conjugate point. LQ problems that don't satisfy this strengthened Legendre condition are far less understood. In this cases it can be difficult to ascertain whether the second variation of the cost is nonnegative. The loss of coercivity also implies that, even if the second variation is nonnegative, the existence of a minimum is not guaranteed. We present some conditions that are necessary for the second variation of a problem of this type to be nonnegative. These conditions are extensions of the well known Goh and generalized Legendre-Clebsch conditions. For LQ problems that satisfy these conditions, we show how to provide the space of controls with a new topology, weaker then the usual topology of

L_2, such that the problem can be extended by continuity onto the topological completion of L_2. If the second variation is positive definite, then the cost functional is coercive with respect to this new topology and hence standard theorems can be used to prove existence and uniqueness of generalized optimal solutions.

⟶ ∞ ◇ ∞ ⟵

Measures of transverse paths in sub-Riemannian geometry

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Elisha Falbel

We define a class of lengths of paths in a sub-Riemannian manifold. It includes the length of horizontal paths but it also measures the length of transverse paths. It is obtained by integrating an infinitesimal measure which generalizes the norm on the tangent space. This requires to define and study the metric tangent space (in Gromov sense). As an example we compute geometrically those measures in the case of contact sub-Riemannian manifolds.

⟶ ∞ ◇ ∞ ⟵

On flatness in the smallest nontrivial dimensions

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D. Avanessoff

This talk will be a review of results on flatness of control systems with two inputs and three states. According to a previously known necessary condition for flatness, systems of these dimensions which are flat may be transformed into a *control affine* system with four states and two inputs. A characterization of (x, u)-flatness for these systems have been given by the second author (1997). The results are presented here in more geometrical manner, and with much simpler proofs.

⟶ ∞ ◇ ∞ ⟵

Control of quantum mechanical processes: models, techniques and practice

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Several applications motivate the control, observation and optimization of quantum mechanical systems. Included among them are ultrafast photochemistry and spectroscopy in the visible, infrared, ultrviolet and radiofrequency regimes; quantum information processing and of course, quantum computation. The talk will first address the issue of models for the control of quantum systems in such applications. It will be argued that for many problems (though by no means all) finite-dimensional models have good predictive power. The talk will, besides the usual unitary models, include other finite-dimensional models which arise when certain physical objectives can be reformulated as the control over certain observables, as opposed to the wave function itself. A brief synopsis of the kinds of approximations (at least within spectroscopy) that go into unitary models will be given, with emphasis on the issue that control of such systems has to naturally respect the regimes for these approximations. Next, the talk will dwell on two techniques for the control of such systems. The first is centered around the notion of factorization of unitary matrices and leads to new techniques even for classical systems. The second, dubbed learning control is an immensely popular method which has found immediate applications but is yet to be thoroughly, mathematically investigated. Finally, time permitting some discussion of pulse shaping laboratory techniques will be provided.

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Optimal control of a spatial shuttle

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The aim of this talk is to make some geometric remarks and some numerical simulations in order to construct the optimal atmospheric arc of a spatial shuttle (problem of reentry on Earth or Mars Sample Return project). The system describing the trajectories is in dimension 6, the control is the bank angle and the cost is the total thermal flux. Moreover there are state constraints (thermal flux, normal acceleration and dynamic pressure). Our study is mainly geometric and is founded on the evaluation of the accessibility set taking into account the state constraints. We make an analysis of the extremals of the Minimum Principle in the non-constrained case, and give a version of the Minimum Principle adapted to deal with the state constraints. Numerical simulations are done using shooting methods.

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Recent Developments in Mechanical Systems and Geometric Control Theory

Organizer: Manuel de Leon and Alberto Ibort,Spain

Advances on numerical integrators for nonholonomic systems

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S. Martnez Consejo

In the last few years, variational integrators derived from Veselov's discretization technique has grown out to be a very large and active area of research. These are numerical integrators that preserve the symplectic structure and the momentum conserved quantities for a given mechanical system. In this talk, we will focus our attention on one of the many interesting avenues of exploration that variational integrators open: namely, that which deals with mechanical systems subject to nonholonomic constraints. We will present a discretization of the Lagrange-d'Alembert principle, which will allow us to derive the discrete nonholonomic equations of motion. We will also present a study of the geometric invariance properties of this discrete flow, which provide an explanation for the good performance of the proposed method. Several examples will illustrate the results.

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Cartans approach applied to nonholonomic geometry

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I shall discuss the application of Cartans method equivalence to nonholonomic geometry. The method of equivalence is an algorithmic procedure for uncovering differential invariants associated to geometric structures. As an example, the method applied to Riemannian manifolds yields the Levi-Civita connection and Riemannian curvature apparatus. The problem can be described as follows. Let (M, D, g_1) and (N, C, g_2) be Riemannian manifolds with completely non-integrable distributions D and C. We take as metrics on D and C the restrictions of g_1 and g_2 to the distributions. The basic problem is then to find invariants characterizing the existence of local diffeomorphisms from M to N that preserve the nonholonomic geometry. As particular examples, I will discuss the examples of nonholonomic geometry on contact and Engel manifolds. Robert Bryant and his students have successfully applied the method to the study of sub-Riemannian geometries on several types of distributions. In the nonholonomic case, one begins the algorithm with a smaller structure group and this leads to additional differential invariants.

⟶ ∞ ⋄ ∞ ⟵

On the topology and geometry of singular optimal control problems

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We will discuss some of the geometrical and topological properties of singular optimal control problems. We will analyze a geometrical recursive algorithm, inspired both in the theory of implicit differential equations and Dirac's constraints algorithm, whose result will be a reduced Pontriaguine maximum principle for singular systems. A global singular perturbation theory will be discussed that will allow us to solve explicitly such reduced systems. Finally, some examples and applications will be presented.

⟶ ∞ ⋄ ∞ ⟵

An axially symmetric rolling sphere hitting a rough wall

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Non-Holonomic systems with symmetry are described by a reduced Lagrange-d'Alembert variational principle. The case of rolling constraints has a long history and it has been the purpose of many works in recent times, in part because of its applications to robotics. In this paper we study the case of a symmetric sphere, that is, a sphere where two of its three moments of inertia are equal, rolling on a plane, using an abelian group of symmetry. The presence of some impulsive constraints upon hitting a rough wall is also studied. This is joint work with H. Cendra and W. Reartes.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Some geometric aspects of control theory

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We introduce a quasi-order relation associated with an everywhere defined family of vector fields appearing in a geometric formulation of a control problem. A notion of variations to a concatenation of integral curves of vector fields is considered and is used to develop necessary conditions for an optimal control problem with fixed time.

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Control theory for affine connection control systems

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It is well-known that affine differential geometry is useful in classical mechanics as a means for organizing the description of certain systems, e.g., the geodesics of the Levi-Civita connection are the solutions of the Euler-Lagrange equations for kinetic energy Lagrangians. In this talk an overview will be given of how affine differential geometry is useful in the control theory for mechanical systems. As control systems, the problems considered are challenging as they are not amenable to well-established methods in control. The emphasis will be on optimal control, controllability, and motion planning for these systems.

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Reduction of symmetries

Jedrzej Sniatycki
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I shall describe the role played by manifolds of a given symmetry type and manifolds of a given orbit type in singular reduction of symmetries of differential equations. Reduction techniques for ODEs will be illustrated by the symplectic reduction of Hamiltonian systems, and the non-holonomic reduction of distributional Hamiltonian systems. PDEs will be represented by the Dedonder equation (multisymplectic reduction).

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Analysis and Computation of Nonlinear Elliptic PDEs

Organizer : Zhonghai Ding, University of Nevada
Hossein Tehrani, University of Nevada

A Bolza–type problem in a Riemannian manifold

Anna Maria Candela
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Consider the nonlinear equation

$$D_s \dot{x} + \lambda \nabla_x V(x,s) = 0$$

in a complete Riemannian manifold M. If V has a quadratic growth with respect to x, then a "best constant" $\bar{\lambda} > 0$ exists such that if $\lambda < \bar{\lambda}$ any couple of points in M can be joined by a curve which is solution of the given equation.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On a class of critical perturbations of a Neumann elliptic problem with limiting Sobolev exponent

David Costa
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P. Girao

We consider the question of existence of least-energy solutions for the Neumann problem

$$\begin{cases} -\Delta u + au = u^{2^*-1} - \alpha u^{q-1} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded smooth domain in $\Re^N$, $N \geq 5$, $a > 0$, $\alpha \geq 0$ and $2^* = \dfrac{2N}{N-2}$ is the limiting Sobolev exponent for the embedding $H^1(\Omega) \subset L^r(\Omega)$. We show that the exponent $q = \dfrac{2(N-1)}{N-2}$ plays a critical role on existence of such ground-state solutions for these problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple nonlinear periodic oscillations in a suspension bridge system

Zhonghai Ding
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In this talk, we present a recent result on multiple nonlinear periodic oscillations in a suspension bridge system governed by the coupled nonlinear wave and beam equations describing oscillations in the supporting cable and roadbed under periodic external forces. By applying a variational reduction method, it is proved that the suspension bridge system has at least three periodic oscillations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Schrödinger type equations with asymptotically linear nonlinearities

Francois Heerden
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Z.Q Wang.

We consider existence of solutions for the nonlinear Schrödinger type equation

(1) $\quad -\Delta u + (\lambda g(x) + 1)u = f(u), \quad x \in \mathbf{R}^N$

which satisfy $u(x) \to 0$ as $|x| \to \infty$. The potential $\lambda g(x) + 1$ is strictly positive, $\lambda \geq 0$ is a real parameter and there exists a nonempty potential well $\Omega := \text{int } g^{-1}(0)$. No limit for $g(x)$ is assumed as $|x| \to \infty$. The nonlinearity f is assumed to be asymptotically linear, i.e. there exists an $\alpha \in (0, \infty)$ such that $\lim_{|s|\to\infty} f(s)s^{-1} = \alpha$. The constant α will be further restricted in terms of the spectrum of the linear operator $-\Delta + 1$ under Dirichlet boundary conditions in Ω. I will show that for λ sufficiently large, a positive solution to (1) is obtained with minimal assumptions on f. I will also illustrate how, under the assumption that f is odd, the number of eigenvalues of the operator $-\Delta + 1$ beneath α effects the number of solutions for (1). The limiting behavior of solutions as $\lambda \to \infty$ will also be considered.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Solutions with internal jump for an autonomous elliptic system of bistable type

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We consider the following system of semilinear elliptic equations

$$(2) \qquad \begin{cases} -\varepsilon^2 \Delta u &= f(u) - v \quad \text{in} \quad \Omega; \\ \gamma v - \Delta v &= \delta u \qquad\quad \text{in} \quad \Omega; \\ u &= v = 0 \qquad \text{on} \quad \partial\Omega. \end{cases}$$

We assume Ω to be a smooth bounded domain in $\Re^N$, with $N \geq 1$, while γ is larger than the first eigenvalue of $-\Delta$ on Ω subjected to homogeneous Dirichlet boundary conditions. We take $\varepsilon > 0$ and $\delta \geq 0$ as parameters. The nonlinearity we assume for simplicity to be $f(u) = u(u-1)(a-u)$ with $0 < a < 1/2$, although other more general nonlinearities can also be treated. We observe that the system is coupled in a noncooperative way, and hence is not order preserving. This leads to a richer solution structure. In particular for small $\delta \geq 0$ the solutions to (1) are similar to the solutions to the scalar equation

$$(3) \qquad \begin{cases} -\varepsilon^2 \Delta u &= f(u) \quad \text{in} \quad \Omega; \\ u &= 0 \qquad \text{on} \quad \partial\Omega. \end{cases}$$

It is known that under certain assumptions on Ω and for ε small, equation (2) has only two nontrivial solutions, a stable one of boundary layer type and one of mountain pass type. We are interested in how these solutions change as δ increases. Using variational, continuation and order methods we show that the boundary layer solution continues to exist as δ increases up to some value, and as δ increases a second stable appears. This solution exhibits an internal jump. In this respect the autonomous system behaves very differently from the autonomous scalar equation. These results also lead to other multiplicity results.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Multiple solutions for perturbed elliptic equations in unbounded domains

Addolorata Salvatore
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Let us consider the elliptic problem

$$\begin{cases} -\Delta u \ + \sigma(x)u = \ |u|^{p-2}u \ + \ \ f(x) & \text{in } \Omega \\ u = \varphi & \text{on } \partial\Omega \\ |u| \to 0 \text{ as } |x| \to +\infty \end{cases}$$

where Ω is an unbounded domain in $\mathbf{R}^N$ with smooth bounded boundary $\partial\Omega$, $N \geq 2$, $2 < p < \frac{2N}{N-2}$, $f \in L^{p'}(\Omega)$, $\frac{1}{p} + \frac{1}{p'} = 1$, $\varphi \in H^{\frac{1}{2}}(\partial\Omega)$.
If we assume that σ is a positive function on Ω which "grows enough at infinity", we can overcome the lake of compactness of the problem by introducing a suitable weighted Sobolev space X which is compactly embedded in $L^2(\Omega)$. Then, in spite of the loss of the symmetry, variational methods and perturbative arguments allows to find infinitely many solutions in X of the given equation when the exponent p is not too large.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Diffusive logistic equation with constant yield harvesting

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We study a fishery management problem using a reaction-diffusion equation model:

$$\begin{cases} u_t = d\Delta u + a(x)u - b(x)u^2 - ch(x), & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \\ u(0,x) = u_0(x) \geq 0, & x \in \Omega, \end{cases}$$

where u is the density of fish, $\Omega \in \mathbf{R}^2$ (a bounded region) is the habitat, and $c, d > 0$. Since the fishing quota is fixed but may vary geographically, the harvesting rate of the fish is assumed to be density independent and spatially nonhomogeneous. We discuss the existence and uniqueness of stable steady state solution and bifurcation diagram with bifurcation parameter c. Related dynamics will also be discussed.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On a class of Schrödinger equations with indefinite nonlinearities

Hossein Tehrani
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We will consider Schrödinger equations like the model

$$-\Delta u + V(x)u = a(x)|u|^{p-2}u \qquad\qquad x \in R^N$$

where the function $a(x)$ changes sign in $\Re^N$ (hence the indefinite nature of the nonlinearity) with $\lim_{|x|\to\infty} a(x) = a_\infty < 0$. We will investigate existence and multiplicity results for the solutions under the assumption that $\sigma(-\Delta + V(x)) \cap (-\infty, 0]$ is a finite set.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A minimax method for computing multiple critical points in Banach space

Xudong Yao
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A minimax method in Hilbert space setting has been developed by Li-Zhou, and successfully applied to find multiple numerical solutions to many nonlinear problems. Motivated by a semilinear p-Laplacian equation, whose solutions coincide with critical points of a variational functional in Lp space, this method is generalized to fit Banach space setting. Implementation of the method in Lp space setting will be discussed.

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Invariant properties in computing multiple critical points

Jianxin Zhou
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Invariant sets of negative gradient flow have been introduced to prove the existence of multiple solutions to a class of semilinear elliptic PDE by Z. Liu and J. Sun, and are generalized and intensively studied by S. Li and Z. Wang. In this talk, invariant sets of negative gradient-type numerical algorithms will be studied. It will be shown that how an invariant set may help or may collapse in computing multiple critical points.

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Hyperbolic Systems of Conservation Laws and Related Problems

Organizer: Haitao Fan, Georgetown University
Tong Yang, City University of Hong Kong

Ring formation in flows with liquid/vapor phase transitions

Haitao Fan
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In this talk, we shall explain the peculiar multi-dimensional phenomenon in flows with phase transitions: ring formation. The flow is described by a reactive flow type systems. The key to the explanation is the existence and properties of various travelling waves of the model, which we shall discuss.

⟶ ∞ ⋄ ∞ ⟵

L^1 stability estimate of semilinear hyperbolic systems with quadratic nonlinear source terms

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Athanasios E. Tzavaras

In this talk, we consider semilinear hyperbolic systems with quadratic nonlinear source terms: $\partial_t f_i + \partial_x(v_i(x,t)f_i) = \sum_{j,k} S_i^{jk} f_j f_k$. This system is assumed to be strictly hyperbolic in the sense that all characteristic velocities are different and this system contains the general one-dimensional discrete Boltzmann models as an example. Under various assumptions on the coefficients S_i^{jk} and initial data, we construct nonlinear functional which is equivalent to the L^1 distance and non-increasing in time t. Using this nonlinear functional, we prove the L^1 stability of mild solutions: $||f(\cdot,t) - \bar{f}(\cdot,t)||_{L^1(\mathbb{R})} \leq G||f_0(\cdot) - \bar{f}_0(\cdot)||_{L^1(\mathbb{R})}$, where f and $\bar{f}$ are mild solutions corresponding to two initial data f_0 and $\bar{f}_0$ and C is a constant independent of time t.

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Stability of the Dafermos regularization of conservation laws

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Stephen Schecter

In contrast to a viscous regularization of conservation laws, a Dafermos regularization admits many self-similar solutions. We refer to these smooth solutions as Riemann-Dafermos solutions. After a change of coordinates, Riemann-Dafermos solutions become stationary, and their stability can be studied by linearization. We study the stability of Riemann-Dafermos solutions near Riemann solutions consisting of n Lax shock waves. We show, by studying the essential spectrum of the linearized system in a weighted function space, that stability is determined by eigenvalues only. We then use asymptotic methods to study the eigenvalues and eigenfunctions. We find there are fast eigenvalues of order 1/epsilon and slow eigenvalues of order one. The fast eigenvalues correspond to fast convergence of initial data to travelling wave solutions in singular layers, while the slow eigenvalues correspond to convection in regular layers connected by travelling waves in singular layers. For an example from gas dynamics, we show that all the slow eigenvalues are stable.

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Global structure and asymptotic behavior to the solutions of flood wave equations

Tao Luo
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Tong Yang

In this talk, the global structure and large time asymptotic behavior of solutions to the Riemann problem of the flood wave equations will be discussed by solving the free boundary problems. The zero relaxation asymptotic behavior of the Cauchy problem with a class of initial data will be presented by using a modified Glimm's scheme.

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Atomic-scale localization of high-energy solitary waves on lattices

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Gero Friesecke

One-dimensional monatomic lattices with Hamiltonian $H = \sum_{n \in Z}(\frac{1}{2}p_n^2 + V(q_{n+1} - q_n))$ are known to carry localized travelling wave solutions, for generic nonlinear potentials V. In this paper we derive the asymptotic profile of these waves in the high-energy limit $H \to \infty$, for Lennard-Jones type interactions. The limit profile is proved to be a universal, highly discrete, piecewise linear wave concentrated on a single atomic spacing. The limiting equation for the profile is a spatially discrete analogue of a system of hyperbolic conservation laws. This shows that dispersionless energy transport in these systems is not confined to the long-wave regime on which the theoretical literature has hitherto focused, but also occurs at atomic-scale localization.

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Dispersive effects in a modified Kuramoto-Sivashinsky equation

Judith Miller
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A. Iosevich

We study the limiting behavior of the Kuramoto-Sivashinsky/Korteweg-de Vries (KS/KdV) equation

$$u_t = -\beta_1 u_{xx} - \beta_2 u_{xxxx} - \delta u_{xxx} - uu_x.$$

We show that in the appropriate sense, the solutions of KS/KdV tend to the solutions of the standard Korteweg-de Vries equation

$$v_t = -\delta v_{xxx} - vv_x,$$

as $\delta \to \infty$. The proof relies, to a large extent, on precise estimates for oscillatory integrals that yield pointwise bounds on Green's functions.

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Remarks on the Chapman Enskog expansion

Marshall Slemrod
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This talk gives a survey on the use of the Chapman Enskog expansion to derive reduced orders models of the Boltzmann equation. Numerical results using this will also be given.

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Asymptotic analysis and regularity of solutions to models of compressible flows

Konstantina Trivisa
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Systems of conservation laws result from the balance laws of continuum physics and govern a broad spectrum of physical phenomena in compressible fluid dynamics, nonlinear materials science, etc. Such equations admit solutions that may exhibit various kinds of shocks and other nonlinear waves, which play an significant role in multiple areas of physics; astrophysics, dynamics of (solid-solid) material interfaces, multiphase (liquid-vapor) flows, combustion theory, etc. In this talk we present results on the well-posedness and qualitative behavior of solutions to various systems of conservation laws, including models for compressible flows with large, discontinuous initial data.

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Large-time behavior of real compressible reacting flows in combustion

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The equations for viscous compressible, heat-conductive, real reactive flows in combustion are considered, where the equations of state are nonlinear in temperature. The initial-boundary value problem with Dirichlet-Neumann mixed boundaries in a finite one-dimensional domain is studied. The existence, uniqueness, and regularity of global solutions are established with general large initial data in H^1. It has been proven that, although the solutions have large oscillations, there is no shock wave, turbulence, vacuum, mass or heat concentration developed in a finite time.

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Boundary layer solutions to Boltzmann equation

Tong Yang
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Seiji Ukai Yokohama and **Shih-Hsien Yu**

We study the half-plane problem of the nonlinear Boltzmann equation, assigning the Dirichlet data for outgoing particles at the boundary and a Maxwellian as the far field. We will show that the solvability condition of the problem changes with the Mach number $\mathbb{M}^\infty$ of the far field Maxwellian. If $\mathbb{M}^\infty < -1$, there exists a unique smooth solution connecting the Dirichlet data and the far field Maxwellian for any Dirichlet data sufficiently close to the far field Maxwellian. Otherwise, such a solution exists only for the Dirichlet data satisfying certain admissible conditions. The set of admissible Dirichlet data forms a smooth manifold of codimension 1 for the case $-1 < \mathbb{M}^\infty < 0$, 4 for $0 < \mathbb{M}^\infty < 1$ and 5 for $\mathbb{M}^\infty > 1$, respectively. We also show that the same is true for the linearized problem at the far field Maxwellian, and the manifold is, then, a hyperplane. The proof is essentially based on the macro-micro or hydrodynamic-kinetic decomposition of solutions combined with an artificial damping term and a spatially exponential decay weight.

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Nonlinear Evolution Equations and Related Topics

Organizer: Alain Haraux, Universit Pierre et Marie Curie
Mitsuharu Otani, Waseda University

Long-time convergence of solutions to a phase-field system

Sergiu Aizicovici
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E. Feireisl

We discuss the long-time stabilization of bounded solutions to a phase-field model with memory. The approach relies on the use of analyticity, in the spirit of L. Simon [Ann. of Math. 118 (1983), 525-571.]

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Evolution equations and subdifferentials in Banach spaces

Goro Akagi
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M. Ôtani

In this talk, we introduce a treatment of evolution equations governed by subdifferentials in reflexive Banach spaces. Since the pioneering work of H. Brézis, many results are obtained within the Hilbert space setting. However it seems that the study in reflexive Banach space setting is not fully pursued yet. On the other hand, J. L. Lions established some methods to solve evolution equations in reflexive Banach spaces by using Faedo-Galerkin's method. In our framework, we can obtain more detailed information on the regularity of solutions. Our approach relies on an approximation procedure in the Hilbert space in which the reflexive Banach space is embedded.

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Qualitative properties of a wave equation

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We present a qualitative analysis of the weak solutions of a nonlinear dissipative wave equation. We give a characterization of blow-up that improves previous versions. Also we characterize all the solutions which are global and unbounded, and those which are global and bounded. We use the concepts of stable (potential well) and unstable sets. We give interesting characterizations of these sets.

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Blow-up rates of solutions of a semilinear parabolic equation with nonlocal nonlinearity

Isamu Fukuda
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We treat total vs. single point blow-up of solutions of a semilinear parabolic equation with nonlocal nonlinearity. Moreover, we obtain blow-up rates of solutions depending on space dimensions.

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Existence and nonexistence of nontrivial solutions of some nonlinear fourth order elliptic equations

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In this talk, we are concerned with the existence and nonexistence of nontrivial solutions for nonlinear elliptic equations involving a biharmonic operator. Concerning the second order equations, a complementary result was obtained for the problem of interior, exterior and whole space. The main purpose of this talk is to discuss whether the complementary result mentioned above is still valid for the nonlinear fourth order equations. We introduce "Kelvin type transformation" for a biharmonic operator to convert an exterior problem to an interior problem. The existence results in case of supercritical exterior problem are shown by introducing a weighted version of Sobolev-Poincaré type inequality, and the nonexistence results are shown by giving a Pohozaev-type identity for fourth order equations.

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Analysis of nonlinear polarization and magnetization models

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This talk is concerned with the anharmonic oscillator model and the Landau-Lifschitz equation coupled with Maxwell's equations describing the electromagnetic field in generally nonlinear polarizable or magnetized media. The main subject are existence and the asymptotic behavior of the solutions to these models. References F. Jochmann, Long time asymptotics of solutions to the anharmonic oscillator model from nonlinear optics, SIAM J. Math. Anal., 32, 4, (2000), 887-915 . F. Jochmann, Asymptotic behaviour of solutions to a class of semilinear hyperbolic systems in arbitrary domains J. Diff. Equations, 160, (2000), 439-466. G. Carbou and P. Fabrie and F. Jochmann, A remark on the weak ω-limit set in a micromagnetism model, to appear in Appl. Math. Lett., Vol. 15, no. 1, (2001). F. Jochmann, Convergence to stationary states in the Maxwell Bloch system from nonlinear optics, to appear in Quart. Appl. Math. F. Jochmann, Existence of solutions and a quasistationary limit for a hyperbolic system describing ferromagnetism, in preparation.

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Principle of symmetric criticality and evolution equations

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Mitsuharu Ôtani

Let X be a Banach space where a symmetry group G linearly acts and let J be a G-invariant functional defined on X. In 1979, R. Palais gave some sufficient conditions to guarantee the so-called "Principle of Symmetric Criticality": every critical point of J restricted on the subspace of symmetric points becomes also a critical point of J on the whole space X. In this talk this principle is extended to the case where J is non-smooth and the problem does not have full variational structure. This "extended" principle is applied to a parabolic problem associated with p-Laplacian in unbounded domains.

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Asymptotic behavior of solutions to semilinear Euler-Poisson-Darboux type equations

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We investigate the asymptotic property of the solution to the following boundary value problem (BV) for a semilinear Euler-Poisson-Darboux type of equation as $t \to \infty$.

$$P[u(t,x)] = g(t,x,\partial_t u, D_x u, u) \text{ in } \quad (t,x) \in [0,\infty)\times\Omega \tag{0.1}$$

(BV)

$$u(t,x) = 0 \text{ on } [0,\infty)\times\partial\Omega \tag{0.2}$$

where Ω is assumed to be a bounded domain in $\boldsymbol{R}^n$ with a smooth boundary $\partial\Omega$,

$$P[\cdot] = \partial_t^2 \cdot - \sum_{i,j=1}^{n} \partial_i(a_{ij}(x)\partial_j \cdot) + \frac{2\beta}{t+T}\partial_t \cdot , \tag{0.3}$$

T and $\beta > 0$, $\partial_t = \frac{\partial}{\partial t}$, $\partial_i = \frac{\partial}{\partial x_i}$, $i = 1, \cdots, n$, $D_x u = (\partial_1 u, \cdots, \partial_n u)$. Based on this result, we discuss the optimality of decay estimates and lower bounds of solutions to the mixed problem corresponding to (BV).

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Strong solutions of magneto-micropolar fluid equation

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Mitsuharu Ôtani and **Hiroshi Inoue**

We show the existence and the uniqueness of strong solutions for the initial-boundary value problems of magneto-micropolar fluid equations. Our result is similar to Fujita-Kato's result for the classical Navier-Stokes equations. The method of proof relies on the abstract results in the nonmonotone perturbation theory developed by M. Otani.

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Global attractor for one-dimensional Fremond model of shape memory alloys

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Pierluigi Colli

In this talk, we consider a one-dimensional Fremond model of shape memory alloys. Here let us imagine a bar (with the length 1) of a shape memory alloy whose left hand side is fixed, and assume that the external stress on the right hand side vanishes as time goes to infinity. Under the above assumptions, we shall discuss the asymptotic stability for the dynamical system from the viewpoint of the global attractor. More precisely, we first show the existence of the global attractor for the limiting autonomous dynamical system (the case of zero external stress), and secondly, we shall characterize the asymptotic stability for nonautonomous case by the limiting global attractor.

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Global solutions for quasilinear parabolic systems with cross-diffusion

Yoshio Yamada
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My talk is concerned with the following Lotka-Volterra competition system with cross-diffusion effects: $u_t = d_1\Delta[(1+\alpha v)u] + u(a_1 - b_1 u - c_1 v)$ in $\Omega \times (0,\infty)$, $v_t = d_2\Delta v + v(a_2 - b_2 u - c_1 v)$ in $\Omega \times (0,\infty)$, where $a_i, b_i, c_i, d_i (i = 1,2)$ and α are positive constants and Ω is a bounded domain in R^N with smooth boundary. This system is supplemented with Neumann or Dirichlet boundary conditions and nonnegative initial functions. Note that the global existence of solutions has been established only in case N is less than or equal to 3. In this talk we can prove that the system admits a unique global solution without any restrictions on N and norms of initial functions.

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Global attractors for non-autonomous multivalued dynamical systems generated by subdifferentials

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In a real separable Hilbert space we treat nonautonomous evolution equations including time-dependent subdifferentials and their non-monotone multivalued perturbations. In this talk we consider the non-autonomous multivalued dynamical systems associated with time-dependent subdifferentials, in which the solution is not unique for a given initial state. In particular we discuss the asymptotic behavior of our multivalued semiflows from the view-point of attractors. In fact, assuming that the time-dependent subdifferential converges asymptotically to a time-independent one (in a sense) as time goes to infinity, we shall construct global attractors for non-autonomous multivalued dynamical systems and its limiting autonomous system.

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Invariant Manifolds and Their Applications

Organizer: Kresimir Josic, Boston University

Noise induced chaos

Lora Billings
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Ira Schwartz and **Erik Bollt**

We identify a global mechanism to induce chaos by stochastic perturbations. Two systems in which we find this are the class B laser and the SEIR population dynamics model. The bifurcation to chaos requires two co-existing saddle periodic orbits in a multistable system, which we call a bi-instability. The noise induces a heteroclinic connection between the invariant manifolds of the saddle periodic orbits, therefore inducing a chaotic attractor. To refine the possibility of control, we have also analyzed the stochastic transport between basins. This is joint work with Ira Schwartz and Erik Bollt.

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Approximating the dynamics of thin elastic media

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C. Eugene Wayne

In this talk, we describe a method for deriving and justifying a hierarchy of "reduced equations" for the dynamical motion of thin elastic media, i.e., starting with a PDE defined on a three-dimensional domain, we will show that its solutions can be approximated by the solutions of equations defined on a two-dimensional domain, and, furthermore, there is a sequence of approximating equations, each of which affords a successively better approximation. The approach is based on ideas from Hamiltonian mechanics, and is related to Nekhoroshev theory.

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Bubbling bifurcations

Brian Hunt
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For a one-parameter family of dynamical systems with a persistent invariant submanifold, I will characterize how a chaotic attractor in the invariant manifold loses asymptotic stability transverse to the manifold as the parameter is varied. After this bifurcation, the attractor generically remains weakly stable, having a basin of attraction that is "riddled" – it has positive Lebesgue measure but is not open. Small perturbations of the system can then lead to intermittent behavior called "bubbling" – trajectories spend most of their time near the (formerly) invariant manifold but occasionally burst far away. I will describe different types of bifurcations that can lead to bubbling and the resulting size and frequency of bursts near the bifurcation. The results are relevant to the synchronization of coupled chaotic systems, where bursting represents temporary loss of synchronization.

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Limits to the detection of nonlinear synchrony

Kresimir Josic
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It has long been recognized that the phenomenon of synchronization of chaotic systems can be naturally described in terms of smooth invariant manifolds. Recent evidence suggests that systems exhibiting complex behavior may be synchronized in a weaker sense. A number of such examples and the analytical methods needed to study them will be discussed. I will address the effect of nonsmoothnes of the synchronization manifold on the detectability of the synchronized state. Moreover, in the case the driving system is not invertible the synchronization set is no longer even a manifold but a far more complicated set. I will discuss how the usual graph transform methods can be extended to this case to gain information about the structure of this set. In conclusion I will discuss how these examples provide clues about the dynamical nature of weak synchrony and discuss practical methods for the detection of such coherent states.

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Oscillation properties of the complex Ginzburg-Landau equation

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We present estimates on complexity of solutions of the 1D Complex Ginzburg-Landau equation. We will discuss optimality of bounds and discuss extensions to the 2D case. The methods are based on analyticity and unique continuation properties of solutions to the equation.

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On the viscous shock profiles and viscous wave fan profiles of Riemann solutions

Weishi Liu
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Marchesin, Plohr and Schecter

In this talk, I will first describe briefly the Exchange Lemmas for singularly perturbed systems with a class of turning points. As an application, we consider a system of conservation laws in one space dimension and study the structural stability of Riemann solutions. We show that, in particular, there are Riemann solutions which are generically structurally unstable in terms of viscous shock profile criterion but are generically structurally stable in terms of viscous wave fan profile criterion.

This research is closely related and motivated by some of the works of Marchesin, Plohr, and Schecter.

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Invariant manifolds and the Navier-Stokes equation

C. Eugene Wayne
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T. Gallay

We will explain some recent work on the construction of finite dimensional invariant manifolds in the phase space of the Navier-Stokes equation on $\mathbf{R}^n$. These manifolds control the long-term behavior of small solutions, give geometric insight into the host of existing results on the asymptotics of such solutions, and allow one to extend those results in a number of ways. Our results also allow us to prove the stability of certain vortex solutions of the Navier-Stokes equation, even at very large Reynolds's number.

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Approximate normally hyperbolic invariant manifolds

Chongchun Zeng
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In this talk, we consider a semiflow in a Banach space where a C^1 submanifold is approximately invariant and normally hyperbolic. Assuming the semiflow is inflowing (overflowing) along the boundary, we prove there exists a unique positively invariant stable (unstable) manifold which has an invariant stable (unstable) foliation.

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Smooth Dynamical Systems

Organizer: Vadim Y. Kaloshin, MIT

Critical saddle-node bifurcations and Morse-Smale maps

Brian Hunt
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We study the dynamics of a family of diffeomorphisms just beyond a saddle-node bifurcation that destroys a stable periodic orbit, assuming that at the bifurcation parameter the orbit has a homoclinic tangency. We show that if the tangency is near critical (cubic), the family generically includes diffeomorphisms that are locally Morse-Smale for a set of parameters with positive Lebesgue density at the bifurcation parameter, while if the tangency is sufficiently far from critical, there are no Morse-Smale diffeomorphisms in the family. These results rely heavily on projecting the dynamics to circle endomorphisms. We conclude with some numerical results that indicate how common the Morse-Smale property is for near-critical circle endomorphisms.

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On the derivative formula of SRB measures

Miaohua Jiang
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We give the derivative formula of the Sinai-Ruelle-Bowen (SRB) measure with respect to the hyperbolic dynamical system when the foliation of the unstable manifold is smooth. Ruelle's simple formula is valid when the potential function of the SRB measure is a constant. As a consequence of this formula, we obtain the derivative formula of the entropy of the SRB measure for hyperbolic attractors.

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Dynamics of an oil spill

Vadim Kaloshin
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D. Dolgopyat and **L.Koralov**

We consider the evolution of a passive scalar, e.g. an oil spill, on surface of ocean (the plane), where the motion is modelled by a periodic incompressible stochastic flow. We show that for almost every realization of the random flow at time t most of the particles are at a distance of order $\sqrt{t}$ away from the origin and there is a measure zero and full Hausdorff dimension set of points, which escape to infinity at the linear rate. We study the set of points visited by the original set by time t, and show that such a set, when scaled down by the factor of t, has a limiting non random shape.

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Random perturbations of 2-dimensional Hamiltonian flows

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We find the asymptotics of the effective diffusivity (which is the main term of the homogenized flow) of 2-dimensional incompressible flows perturbed by small diffusion. In the case when the stream function of the flow has some symmetry properties, the asymptotics of the effective diffusivity was obtained by Fannjiang and Papanicolaou using variational methods. We consider the general case using the probabilistic approach.

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Smooth dynamical systems

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In the early seventies Michael Shub posed the following conjecture about the topological entropy: If the universal covering space of a manifold M is homeomorphic to the Euclidean space then the topological entropy $h(f)$ of any continuous self-map $f : M \to M$ is estimated from below by the logarithm of the spectral radius $\rho(f)$ of the map $H^*(f)$ induced by f on real cohomology. This conjecture was confirmed for a torus map by Misiurewicz and Przytycki in 1977. In this work we prove the conjecture in a weaker form, in the case when M is a compact nilmanifold. An assumption we need requires that the Lefschetz number $L(f) \neq 0$, i.e. f is not homotopic to a fixed point free map. The presented method uses a notion of linearization of a self-map of nilmanifold to compare the spectral radius $\rho(f)$ with the asymptotic Nielsen number $N^\infty(f)$, and apply the Ivanov theorem. In a second theorem we drop out the assumption $L(f) \neq 0$, but we can prove then only that $h(f) > 0$ if $\rho(f) > 0$.

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Dynamics of Dominated Splitting

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We will show that surface diffeomorphisms with dominated splitting admit a spectral decomposition and we present some consequences of it. Also, we will discuss the robustness of sets having dominated splitting and the integrability of these subbundless in dimensions greater than 2.

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Chaos in Classic and Quantum Systems

Organizer: Helmut Kroger, Laval University

Open quantum dots as a probe of quantum chaos: going beyond convenient mathematical approaches

J. Bird
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R. Akis, **D. K. Ferry**
A. P. de Moura and **Y.-C.Lai**

Semiconductor quantum dots are sub-micron sized structures that consist of a mesoscopic scattering region, fabricated on length scales much smaller than the electron mean-free path. The relevant current-flow process through such structures is therefore one in which electrons are injected into the cavity, and undergo multiple scattering from the walls of the dot before finally escaping to the external reservoirs. For this reason, these ballistic structures may be viewed as the solid-state analog of classical scattering billiards, and their electrical properties have attracted much interest as an experimental probe of quantum chaos (for a recent review, see Ref. [1]).In the development of a theoretical description of these structures, there has unfortunately been a tendency to introduce a number of limiting assumptions, which provide for mathematical simplicity at the expense of physical reality. In this presentation, we will therefore discuss the results of experimental and theoretical studies of open quantum dots, which go beyond these artificial approaches to develop a better understanding of electron dynamics in these structures. From quantum-mechanical simulations of our experimental results, we find strong evidence for the role of scarred wavefunction states in transport through the dots [2]. This behavior is in turn shown to be related to an effect analogous to resonance trapping, in which transport through the open system can be understood in terms of selected eigenstates of the closed structure. We will also discuss the results of a more resent semiclassical analysis, which has pointed to a consistent picture in which the scars are found to be associated with classically-inaccessible orbits, which end up playing an important role in transport due to the process of dynamical (or phase- space) tunnelling [3].

[1] J. P. Bird, "Recent experimental studies of electron transport in open quantum dots", J. Phys.: Condens. Matter 11, R413 (1999).

[2] J. P. Bird, R. Akis, D. K. Ferry, D. Vasileska, J. Cooper, Y. Aoyagi, and T. Sugano, "Lead orientation dependent wavefunction scarring in open quantum dots", Phys. Rev. Lett. 82, 4691 (1999).

[3] A. P. S. de Moura, Y.-C. Lai, R. Akis, J. P. Bird, and D. K. Ferry, "Tunneling and nonhyperbolicity in quantum dots", submitted for publication.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Chaotic ionization of hydrogen in parallel fields

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John Delos

We examine the classical ionization of photoexcited states of hydrogen in parallel electric and magnetic fields. This system reduces to an area-preserving chaotic map of the plane and is a useful and experimentally accessible model of chaotic decay and scattering. Decay is studied by examining segments of a line of initial conditions that escape at various iterates of the map. These segments exhibit what we call "epistrophic self-similarity": the segments organize themselves into self-similar geometric sequences, but the beginnings of these sequences are only partially predictable. Points that remain from a Cantor set which we call an "epistrophic fractal". These studies should be important for the analysis of classical decay rates, which numerical studies have shown typically exhibit algebraic rather than exponential decay. They should also be important for the analysis of semiclassical decay and scattering, including the construction of the semi-classical propagator and S-matrix.

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Semiclassical and anticlassical limits of the Schrödinger equation

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The natural place to study how the chaotic nature of a classical system manifests itself in the corresponding quantum system is in or near the semiclassical limit of the quantum system. For billiard systems this is, quite generally, the high-energy limit, but for more realistic one- or many-body systems described by the appropriate Schrödinger equation,the semiclassical limit may not be so easy to identify. In this talk I identify the semiclassical limit of the Schrödinger equation - and the anticlassical or extreme quantum limit - for several examples including one-electron atoms in external fields and two- or more-electron atoms. As a curiosity I mention a class of one-dimensional bound systems where the quantum number grows to infinity without approaching the semiclassical limit.

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Long period orbits from symbolic dynamics

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Petr Braun and **Stefan Heusler**

Working with a sequence of local sections of the symbol sequence one can construct periodic orbits of extremely long periods. The accuracy attained grows exponentially with the length of the local sections. By properly including correlations between symbols one obtains orbits behaving ergodically.

$\longrightarrow \infty \diamond \infty \longleftarrow$

One-dimensional quantum chaos: solved

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Yuri Dabaghian

We have identified a class of quantum graphs (corresponding to particles in one-dimensional potential wells) with unique and precisely defined spectral properties that we call "regular quantum graphs". Although these physical systems are chaotic in the classical limit (with positive topological entropy), the regular quantum graphs are explicitly solvable. Exact, convergent periodic orbit expansions can not only be developed for the quantum density of states (a la Gutzwiller) but explicit periodic orbit formulas can be developed for individual quantum energy levels.

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Quantum chaos at finite temperature in the Paul trap

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In quantum chaos, we have no "local" information of the degree of chaoticity, being available in classical chaos via Lyapunov exponents and Poincaré sections from phase space. Also little is known about the role of temperature in quantum chaos. For example, an analysis of level densities is insensitive to temperature. Here we present a new method aiming to overcome those problems.Recently, my co-workers and I have proposed the concept of a quantum action. This action has a mathematical structure like the classical action, but takes into account quantum effects (quantum fluctuations) via renormalized action parameters. This bridges the gap between classical physics and quantum physics. Lyapunov exponents, Poincaré sections etc. can be computed to study quantum chaos via the quantum action. The quantum action has been applied and proven useful to precisely define and quantitatively compute quantum instantons.Here we use the quantum action to study quantum chaos at finite temperature. We present a numerical study of 2-D Hamiltonian systems which are classically chaotic. First we consider harmonic oscillators with weak anharmonic coupling. We construct the quantum action non-perturbatively and find temperature dependent quantum corrections in the action parameters. We compare Poincaré sections of the quantum action at finite temperature with those of the classical action. We observe chaotic behavior for both. Secondly, we consider the Hamiltonian of the Paul trap, which is quite important for atomic clocks, in Bose-Einstein condensation, etc. We present Poincaré sections as function of temperature and compare its chaotic behavior with its classical counterpart.

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Classical and quantum chaos in fundamental field theories

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The role of chaotic field dynamics for the confinement of quarks is a longstanding question. Concerning classical chaos, we analyze the leading Lyapunov exponents of Yang-Mills field configurations on the lattice. Concerning the quantum case, we investigate the eigenvalue spectrum of the staggered Dirac operator in QCD at nonzero temperature. The quasi-zero modes and their role for chiral symmetry breaking and the deconfinement transition are examined. The bulk of the spectrum and its relation to quantum chaos is considered. Our results demonstrate that chaos is present when particles are confined, but it persists also into the quark-gluon-plasma phase. Further, we decompose U(1) gauge fields into a monopole and photon part across the phase transition from the confinement to the Coulomb phase. We analyze the leading Lyapunov exponents of such gauge field configurations on the lattice which are initialized by quantum Monte Carlo simulations. It turns out that there is a strong relation between the sizes of the monopole density and the Lyapunov exponent.

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Photon induced chaotic scattering

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Strong electromagnetic fields, interacting with atoms, can induce chaotic structures in open space. These chaotic structures may support new quasi-bound electronic states and delay ionization of electrons. Scattering processes in the presence of electromagnetic fields can be analyzed using Floquet theory. Signatures of chaos-induced quasi-bound states appear in scattering phase shifts and delay times.

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Quantum chaos in fractal repellers and disordered systems: experiments using microwaves

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I discuss some recent themes from our microwave experiments designed to explore issues in Quantum Chaos. In n-disk geometries, we have seen the quantum fingerprints of classical Ruelle-Pollicott resonances. The wave experiments thus are laboratory realizations of a fractal repeller, a prototypical Axiom A dynamical system. In disordered geometries, which are realizations of electrons in disordered solids, we have studied correlations of eigenfunctions which manifest localization. The results are shown to be well described by recent theories based upon non-linear sigma models of supersymmetry. Earlier microwave experiments have led to observation of scars in quantum eigenfunctions of Sinai and other chaotic billiards, precision tests of random matrix theory in microwave spectra and eigenfunctions, and studies of experimental mathematics of isospectral domains. (1) Quantum fingerprints of classical Ruelle-Pollicot resonances, K.Pance, W.T.Lu and S.Sridhar, Phys. Rev. Lett., 85, 2737 (2000) (2) Correlations due to localization in quantum eigenfunctions of disordered microwave cavities", Prabhakar Pradhan and S. Sridhar, Phys.Rev. Lett., 85, 2360 (2000) (3) Quantum Correlations and Classical Resonances in an open chaotic system", W.T.Lu, K.Pance, P.Pradhan and S.Sridhar, Physica Scripta :Special Issue : Nobel Symposium on "Quantum Chaos Y2K", T90, 238 (2001).

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A simple scheme for extracting internal motions from spectroscopic Hamiltonians

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The problem addressed is to determine the internal vibrational motions that when quantized yield the vibrational bound and resonant states of a molecule. In particular we consider systems with two or more vibrational resonances due to frequencies in rational ratio, where motions at variance with simple normal or local modes exist. We restrict ourselves to problems where a spectroscopic normal form Hamiltonians (Heff) can be obtained either from fits to experimental lines, or with the use of a PES, to calculated eigenvalues. The application of canonical perturbation theory to a system with a known PES can also supply such an Heff. Presently we also require that the number of degrees of freedom minus the number of constants of motion, one being the polyad quantum number (P) that will exist when resonances are present, is no greater than three. We treat the non-trivial cases where more than one resonance exists and hence chaos can occur and where the reduced phase space, obtained by canonical transform using constants of motion, corresponds to two or three degrees of freedom. We further limit ourselves to the challenging cases where the dynamics is not susceptible to any simple adiabatic or other separation scheme and where wave functions and trajectories in the full dimension are too complex to be interpreted by any graphical representations. In such cases it will be demonstrated that most if not all of the dynamics can be uncovered and that dynamic quantum numbers representing quasiconserved quantities can be assigned given only the already existing eigenfunction-basis transformation matrix used in fitting the Heff to the experimentally or theoretically generated spectrum. The method in conception depends on the ability previously gained(1), in studying problems where nonlinear dynamics was used to find the motions underlying the simple patterns seen in plots of the density and the phases of semiclassical eigenfunctions created from the information in the transformation matrix calculated in the above "fit". These eigenfunctions are parametric in the constants of the motion (the polyad number P in particular) and lie for DCO on a 2D toroidal configuration space described by two angle variables. Since the features of these 2D wave functions are generally simple to recognize once one is comfortable working in this unusual space; just viewing the patterns allows the sorting of the interleaved states of different dynamics into suits (like a deck of cards) each based on different dynamics. Then nodal counts and/or phase advances (since we are in a space for angles) allows sequential quantum number assignment. The contours of the 2D angle space wave function density and phases can be used for each suite to infer the type of classical internal motion that the atoms are undergoing in normal mode, local mode or displacement coordinate space. These motions are those that when quantized gives rise to the levels in the suit. A discussion, as to why wave functions represented in compact angle (of action-angle) spaces as opposed to the usual open coordinate spaces are so much simpler to interpret is given. The assignments and dynamics of DCO are presented(4). Because of a strong 1:1:2 resonance assignment and interpretation alluded previous workers who computed eigenfunctions from both a high quality potential surface(2) and from a spectroscopic Hamiltonian(3). Again we stress no serious computation was needed to extract dynamics and to assign once the Heff was available. Graphical representations of the phase and densities of eigenfunctions in reduced configuration (angle) space, the principles of nonlinear dynamic and semiclassical ideas on how wave functions accumulate about phase space organizing structure are the keys to the analysis. 1. (a) M.P. Jacobson, C. Jung, H.S. Taylor and R.W. Field, J. Chem. Phys., 111, 600 (1999). (b) C. Jung, H.S. Taylor and M. Jacobson, J. Phys. Chem. A., 105, 681 (2001). 2. H.-M. Keller, H. Floethmann, A. J. Dobbyn, R. Schinke, H.-J. Werner, C. Bauer and P. Rosmus, J.Chem. Phys. 105, 4983 (1996). 3. A. Troellsch and F. Temps, Zeitschreft for Physikalische Chemie, 215, 207 (2000). 4. E.Atligan, C. Jung and H.S. Taylor, J. Phys. Chem. (2002) in press.

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Mathematical Fluid Dynamics

Organizer:Igor Kukavica, University of Southern California
James Robinson, University of Warwick

Boundary layer for the Navier-Stokes-alpha model of fluid turbulence

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We study a boundary layer problem for the Navier-Stokes-alpha model obtaining a generalization of the Prandtl equations which we conjecture to represent the averaged flow in a turbulent boundary layer. We study the equations for the semi-infinite plate, both theoretically and numerically. Solutions agree with some experimental data in a part of the turbulent boundary layer.

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Inertial manifolds and Gevrey regularity for the Moore–Greitzer model of turbo–machine engine

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Edriss S. Titi

We study the regularity and long-time behavior of the solutions to the Moore–Greitzer model

of turbo-machine engine. In particular, we prove that this dissipative system of evolution equations possesses a global invariant inertial manifold, and therefore its underlying long-time dynamics reduces to that of an ordinary differential system. Furthermore, we show that the solutions of this model belong to a Gevrey class of regularity (real analytic in the spatial variables). As a result, one can show the exponentially fast convergence of the Galerkin approximation method to the exact solution, an evidence of the reliability of the Galerkin method as a computational scheme in this case. The rigorous results presented here justify the readily available low dimensional numerical experiments and control designs for stabilizing certain states and travelling wave solutions for this model.

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Remarks on rotating fluids

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We will discuss bounds for transport and spectra in rotating Navier-Stokes equations forced at the boundary.

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On interpolation between algebraic and geometric conditions for smoothness of the vorticity in the 3D NSE

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We formulate some sufficient conditions for smoothness of the vorticity consisting of space-time integrability mixed with the regularity of the vorticity directions.

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Recurrent estimates for the Navier-Stokes equations

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C. Foias and **O. P. Manley**

In the Kraichnan theory of two-dimensional turbulence it is assumed that the enstrophy, rather than the energy that is the most relevant quantity. We briefly recall recent rigorous results which show that indeed the enstrophy cascade is more pronounced than that of energy, in the 2-D case. These results pertain to averages of the enstrophy transfer, or flux, through a specified wavenumber, and thus do not provide any information on how these quantities can fluctuate in time. Toward answering this question, we also present two estimates relating the enstrophy beyond a given wavenumber (the so-called high modes) and the enstrophy flux through that wavenumber. They are recurrent in that they hold at least once within certain bounded time intervals. The first effectively provides a bound on how long the enstrophy flux can remain negative, the second a bound on the enstrophy of the high modes, valid for wavenumbers on the order of the square root of the Grashof number. Some numerical results are presented to illustrate these estimates. This work is joint with C. Foias and O.P. Manley, in that much of it was completed before O.P.M. passed away.

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The number of determining modes in 2D turbulence: A computational study

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Edriss Titi

The method of continuous data assimilation from weather forecasting is used to study the number of determining modes for the two-dimensional incompressible Navier–Stokes equations. Our focus is on how the body forcing affects the rate of continuous data assimilation and the number of determining modes. These quantities are shown to depend strongly on the length scales present in the forcing.

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Finite-dimensional dynamics on global attractors

James Robinson
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C. Foias

The talk will discuss approaches to reproducing the dynamics on a finite-dimensional attractor using a finite-dimensional dynamical system. In particular, the aim is to obtain something akin to an "inertial form" for the 2d Navier-Stokes equations even though the existence of an inertial manifold is still an open question.

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Nonlinear wave equations and the Melnikov problem

Alain Schenkel
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Bricmont and **A. Kupiainen**

I will describe a new proof of the Melnikov problem in infinite dimensional systems, namely, persistence of quasi-periodic, low dimensional elliptic tori. Our result covers situations in which the so-called normal frequencies are multiple. In particular, it provides a new proof of the existence of small amplitude, quasi-periodic solutions of nonlinear wave equations with periodic boundary conditions.

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Sharp interface limits and global existence for the phase field Navier-Stokes equations

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We will introduce the phase-field Navier-Stokes equations, and prove that they possess Leray global weak solutions. We will then show that for smooth initial data, solutions of the phase-field model converge weakly to solutions of the sharp-interface Navier-Stokes equations. In the convergence proof, an auxiliary PDE is introduced which couples the Navier-Stokes equations with the classical geometric problem of motion by mean curvature.

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Attracting fixed points for Kuramoto-Sivashinsky equation - a computer assisted proof'

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We present a computer assisted proof of an existence of several attracting fixed points for the Kuramoto-Sivashinsky equation

$$u_t = (u^2)_x - u_{xx} - \nu u_{xxxx},$$
$$u(x,t) = u(x+2\pi,t), \quad u(x,t) = -u(-x,t),$$

where $\nu > 0$. The approach based on the concept of self-consistent a priori bounds introduced in [ZM]. The method is general and can be applied to other dissipative PDEs, for example Navier-Stokes or Ginzburg-Landau equations, not only to obtain fixed points, but also more complex dynamical objects like periodic orbits and hopefully topological horseshoes. The partial results concerning a rigorous steady-state bifurcation diagram for Kuramoto-Sivashinsky equation will be also mentioned.

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Topological Methods for Boundary Value Problems

Organizer: J. R. L. Webb, University of Glasgow, UK
K. Q. Lan, Ryerson University, CA

Singular nonlinear boundary value problems with multiple positive solutions

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Philip T. Carroll

We extend recent results of Henderson and Thompson, Baxley and Haywood, and Graef, Qian, and Yang, which provided conditions on the nonlinear function $f(y)$ in order that the boundary value problem $(-1)^n y^{(2n)} = f(y)$, $y^{(2k)}(0) = 0$, $y^{(2k)}(1) = 0$, for $k = 0, \cdots, n-1$ have multiple symmetric positive solutions. Since such solutions must satisfy $y^{(2k+1)}(1/2) = 0$, for $k = 0, \cdots, n-1$, we consider the more general problem $L(y) = f(y)$, where L is the nth iterate of the Sturm-Liouville operator dis $-\frac{1}{w}(py')'$, with boundary conditions $y^{(2k)}(0) = 0$, $y^{(2k+1)}(b) = 0$, $b > 0$, for $k = 0, \cdots, n-1$. The conditions we obtain allow singular behavior in the operator L at $x = 0$. In the case $w \equiv p \equiv 1$, $b = 1/2$, our conclusions reduce to the earlier results mentioned above. Previous work using the Leggett-Williams fixed point theorem or a fixed point theorem of Krasnosel'skiĭ has used properties of relevant Green's functions. Here we use a refined version of the same theorem of Krasnosel'skiĭ, but need no use of Green's functions to obtain the necessary estimates.

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Existence theorems for weakly inward semilinear operators

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We obtain existence theorems for semilinear equations of the form Lx = Nx, where the operators L and N satisfy a weakly inward condition and are such that L - N is A proper. In particular, results involving positive and multiple solutions are proved.

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Kelvin-Helmholtz instability waves and upstream propagating acoustic waves in supersonic multiple jets

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Jet aircraft were introduced right after that the Second World War. Shortly after that, jet noise prediction and reduction became an important research topic. Because of the need for large thrust, many high performance military aircraft are propelled by two or more jet engines housed close to each other. Three Physics Laws, Conservation of Mass, Momentum and Energy, in differentiation form are employed to formulate the Kelvin-Helmholtz Instability problem of supersonic triple jets. The general solution of the system about pressure of the jets and the dispersion relation for instability waves about jet noise are derived.

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Spectrum of positively homogeneous operators and applications

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The spectral theory for nonlinear operators has been extensively studied by many authors. After the theory of Furi, Martelli and Vignoli, a new definition was introduced by the author. Later, the work was generated to semilinear operators. In this paper, some results on the relationship between the eigenvalues and the spectrum of a positively homogeneous operators were obtained. Applying the results, we prove a theorem that gives a condition for a compact, positive operator to have a positive eigenvalue and eigenvector. The theorem can be used in the study of a second order differential equations with a three point boundary value conditions that has been studied recently by some authors.

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Positive solutions of differential equations with nonlinear boundary conditions

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Using the theory of fixed point index, we establish new results for some differential equations subject to nonlinear boundary conditions. We obtain existence of at least one or of multiple positive solutions.

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On the solvability of implicit complementarity problem and implicit variational inequalities —— A unified approach and implicit projected dynamical system

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In this first part of this paper we will present a unified approach of the study of Implicit Complementary Problems and Implicit Variational Inequalities. This study is based on the concept of "Exceptional Family of Elements" for a function. This concept is obtained in this case using a kind of implicit Leray-Schauder alternative. In the second part of this paper we will present a study of

solutions of Implicit Complementary Problems and Implicit Variational Inequalities, from the dynamical point of view. This study is obtained using an implicit global projected dynamical system. This paper will be finished by comments and open problems.

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Multiple positive solutions of conjugate boundary value problems with singularities

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We consider the existence of one or several nonzero positive solutions for a higher order nonlinear ordinary differential equation with conjugate boundary conditions. The conjugate boundary value problems can be changed into a Hammerstein integral equation with a suitable kernel. We shall show that the kernel has upper and lower bounds. This enables us not only to exhibit a new property of positive solutions for the conjugate boundary value problems but also to derive new results on the conjugate boundary value problems from the well-known results on the existence of one or several positive solutions of Hammerstein integral equations with singularities obtained by the author recently. Our results generalize some known results where stronger conditions were imposed and the theory of fixed point index for compact maps defined on cones was used directly.

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Riccati equations

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Lynn Erbe

We will use the Riccati equation to prove oscillation theorems for self-adjoint vector differential equations on time scales.

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Existence and uniqueness for a class of quasilinear elliptic boundary value problems

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We prove existence and uniqueness of positive solutions for the boundary value problem

$$(r^{N-1}\phi(u'))' = -\lambda r^{N-1} f(u),\ u'(0) = u(1) = 0,$$

where $\phi(x) = |x|^{p-2}x$, $\frac{f(x)}{x^{p-1}}$ may not be decreasing on $(0,\infty)$, and λ is a large parameter.

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Remarks on positive solutions of some 3-point boundary value problem

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Some recent work on existence of one or of multiple solutions of a nonlinear second order differential equation with nonlocal boundary conditions will be discussed by the method of fixed point index. An optimal value will be given for a constant that appears in the definition of the cone being used and in some of the other hypotheses.

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On the existence of explosive solutions for semilinear elliptic problems

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Optimal Control and Control Systems

Organizer: Urszula Ledzewicz,Southern Illinois University
Heinz Schaettler,Washington University

On optimality of bang-bang extremal controls for bilinear systems

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Application of Pontryagin Maximum Principle - a necessary optimality condition for optimal control problems with constrained controls - provides information on extremal controls, among which optimal controls have to be searched. In some cases, like the one of chained control systems with two inputs, one can provide complete description of the structure of Pontryagin extremals for time-optimal problems (see A.Sarychev, H.Nijmeijer, J. Dynam. Control Systems, v.2, 1996, pp. 503-527). It is known however that the bang-bang Pontryagin extremals may cease to be optimal. In our talk we are going to present some results on optimality of bang-bang extremals for bilinear control systems with constrained controls and in particular on time optimality for chained systems.

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Local controllability of linear systems on Lie groups

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In a recent paper by Ayala and Tirao, the notion of linear control systems on Lie groups was introduced. On a real connected finite-dimensional Lie group G with Lie algebra g, a linear control system Σ on G is given by

$$\dot{x} = \vec{X}(x) + \sum_{j=1}^{k} u_j \vec{Y_j}(x), \qquad (1)$$

where $\vec{X}$ is an infinitesimal automorphism, u_j are piecewise constant functions, and the control vectors $\vec{Y_j} \in g$ are left-invariant vector fields. Under conditions similar to the Kalman condition that is needed for controllability of linear control systems on R^n, Ayala and Tirao showed local controllability of the system Σ at the group identity e. Another proof of this result is obtained using Lie Theory of Semigroups. More importantly, using a Lie-wedge approach, the result is extended to local controllability on H, the subgroup of G defined by $H = \langle \exp(\mathcal{H}) \rangle$. Here $\mathcal{H} \subseteq g$ denotes the Lie subalgebra generated by the control vector fields, that is, $\mathcal{H} = \langle Y_1, \ldots, Y_k \rangle$. The fundamental idea is to work in a related group using the Lie theory of semigroups. The fact that X is an infinitesimal automorphism is used to construct a related group $\hat{G}$. In fact, G is diffeomorphic to a homogeneous manifold of the form $\hat{G}/K$. The system Σ on G is then lifted into a system $\hat{\Sigma}$ on $\hat{G}$. The latter has the advantage of being a right-invariant system. Many results can be obtained for $\hat{\Sigma}$. The conclusions on Σ are drawn from these results.

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Relaxation of elliptic control systems

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We consider relaxation methods for nonlinear optimal control problems.We present four such methods, showing that they are admissible and compare them.We also prove an general existence theorem which motivates one of the relaxation methods.Also we present a relaxation method based on lower semicontinuity regularization techniques.

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Optimal controller for third degree polynomial systems

Michael Basin
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Maria Aracelia

This paper presents the optimal controller for a stochastic system state given by a third degree polynomial equation with a linear control input in the presence of linear observations. The optimal controller equations are based on the separation principle applied to the optimal filter for a third degree polynomial state and linear observations and the optimal regulator for a third degree polynomial system with a linear control input and a quadratic criterion. The obtained results are applied to the problem of controlling an automotive system with unobservable states and compared with the best linear controller available for a linearized model. Simulation graphs are given.

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Minimization of the base transit time in semiconductor devices using optimal control

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Paolo Rinaldi

In this paper we consider the problem of determining the optimal profile of doping concentration that minimizes the base transit time in homojunction bipolar transistors. This is a well-studied problem in the electronics literature , but attempts at giving analytical solutions so far only have produced incorrect results which were easily improved upon by numerical optimization. The reason for this lies in the fact that the problem becomes an optimal control problem with state-space constraints to which standard results in the calculus of variations which have been used in earlier attempts to find analytical solutions cannot directly be applied. In this paper we give an explicit analytic solution to the problem using the Pontryagin Maximum Principle with state-space constraints and prove its optimality using synthesis type arguments.

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Optimal control for a general class of model in chemotherapy of cancer and HIV

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Heinz Schaettler

In the past years there has been a strong interest in the analysis of mathematical models arising in the problems of cancer chemotherapy and treatment of HIV infections. Even for a specific disease, like for example AIDS, in the literature an abundance of models is considered which arises by considering different specific aspects and neglecting others. Similarly, many different models exist in the chemotherapy of cancer depending on the detail of modelling like whether drug resistance is taken into account or not. Still, all these models are normally based on one underlying biological structure and many of them, although different in their specifics, exhibit a significant common structure as mathematical models. In particular this holds if optimal control problems are considered. It therefore is natural to combine these different approaches and analyze a general model which would encompasse this common structure. In this paper we present such a general optimal control problem and point out the main properties of optimal controls which all these models have in common. We simultaneously consider optimal control problems with objectives which are linear respectively quadratic in the control, so-called L_1- and L_2-type objectives.

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Numerical Solutions of Evolution Equations

Organizer: Yanping Lin, University of Alberta
Tong Sun, Bowling Green State University

Adaptive moving mesh methods for problems with blow-up solutions

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We consider the numerical solution of partial differential equations with blow-up solutions for which scaling invariance plays a natural role in describing the underlying solution structures. It is a challenging numerical problem to capture the qualitative behavior in the blow-up region, and the use of nonuniform meshes is essential. We consider moving mesh methods for which the mesh is determined using so-called moving mesh partial differential equations (MMPDEs). Based on error analysis and motivated by the desire for the MMPDE to preserve the scaling invariance of the underlying problem, we study the effect of different choices of MMPDEs and monitor functions. Numerical results are also presented to highlight features of moving mesh methods.

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Numerical Schemes for a highly coupled elliptic-parabolic problem

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In this talk we study a finite element method for the following strongly coupled nonlinear parabolic variational inequality:

$$\frac{\partial u}{\partial t} - \nabla[k(u)\nabla u] + \eta \int_{\Omega} G(x,y)u(y,t)dy + \gamma u^4 \geq \nabla[\sigma(u)\phi\nabla\phi].$$
$$-\nabla[\sigma(u)\nabla\phi] = 0.$$

This is a mathematical system which models the behavior of certain micromachined thermistor sensors where heat losses to the surrounding gas and radiation effects play a significant role. We discuss the case where mixed boundary conditions are satisfied for both u and ϕ. The difficulty arising from this case is the low regularity of the exact solutions and hence the usual analysis tools can not be applied here directly. We intend to apply a nonstandard method and obtain suitable error estimate for the scheme.

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Numerical simulation of capillary formation during the onset of tumor angiogenesis

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A mathematical model of the process of angiogenesis, the sprouting of new capillary vessels from pre-existing capillaries, as it relates to tumor vascularization will be briefly described. To fully understand the implications of the model, which is a coupled system of nonlinear ordinary and partial differential equations describing biochemical and cellular processes in the extracellular matrix between a tumor and an existing capillary, numerical simulations are needed. Due to the nature of the modelling equations both smooth and front-like regimes must be numerically resolved. A discussion of these and other numerical issues will be given and results of numerical simulations will be presented. This is a report of work done in collaboration with H.A. Levine and M. Nilsen-Hamilton, two of the proposers of the model.

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Long-time error estimation for semi-linear parabolic PDEs

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The new long-time error estimation approach is applied here for the error analysis and estimation of linear and semi-linear parabolic partial differential equations. The new concepts and techniques include the stability-smoothing indicator, the smoothing assumption, the moving attractor, the exact error propagation and the two-level error propagation analysis. Moreover, an inverse elliptic projection is employed here as a key technique in dealing with the spatial discretization error.

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On numerical solutions of a dynamic contact problem

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We consider the dynamics of a cantilever beam which is imperfectly connected to a rigid wall. The contact condition consists in the fact that the loose part of the beam cannot penetrate the wall. We discuss a discretization procedure which reduces the originally three dimensional problem to a two-point boundary value problem with contact boundary conditions and establish the well-posedness of the discretized problem.

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On American put options on zero-coupon bonds

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In this paper I shall study the existence and uniqueness of weak solutions to parabolic free boundary problems for American put options on zero-coupon bonds. Numerical methods and results are also presented.

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Traveling Waves and Shock Waves

Organizer: Xiao-Biao Lin , North Carolina State University
Stephen Schecter , North Carolina State University

Waves for bistable equations with non-local Mexican Hat interaction

Peter Bates
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Xinfu Chen and **Adam Chamj**

We establish the existence of stationary or travelling waves for a nonlocal dissipative equation with bistable kinetics. The nonlocal self-interaction is indefinite and so no maximum principle holds. The equation arises in biological and material sciences.

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Shock waves in a hemodynamics problem

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I will discuss formation of shock waves in a hyperbolic model of blood flow through compliant axisymmetric large vessels. Although the model and its closely related versions have been extensively used by many authors, a mathematical analysis of shock formation with pulsatile flow boundary data has not been performed. The aim is to show that, in healthy humans, the model does not produce shock waves. In this talk I will present estimates on the initial and boundary data that imply strict hyperbolicity in the region of smooth flow, and prove a general theorem which gives conditions under which an initial-boundary value problem for a quasilinear hyperbolic system admits a smooth solution. Using this result I will show that pulsatile flow boundary data always gives rise to shock formation (high gradients in the velocity and inner vessel radius) but the time and the location of the first shock formation is well outside the physiologically interesting region (2.8 meters downstream from the inlet boundary). In the end I will present a study of the influence of vessel tapering on shock formation and show movies of the dynamics of the vessel wall and shock formation in the pulsatile blood flow regime.

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Nongeneric heteroclinic loop bifurcation

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Homoclinic and heteroclinic bifurcations from a heteroclinic loop are considered. The system in consideration has three parameters two of which are not suitable for generic unfoldings. Analytical criteria in terms of derivatives to Melnikov's functions are given for nongeneric parameters. Four qualitatively distinct bifurcation diagrams are obtained. The result is used to give an explanation to a numerical finding on the generation of travelling pulsing waves in a two-phase flow problem.

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Exponential dichotomies and semidiscrete profiles of shock waves

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Looking for travelling waves in conservation laws which are discretized in space leads to the study of forward–backward functional differential equations. Even in the linear constant-coefficient case the Cauchy problem for these equations is not well posed. The main result of the talk states that for a linear non-autonomous forward-backward functional differential equation the state space can be split into a direct sum of two subspaces: One of them contains all initial conditions that allow for a solution in forward time, the other consists of those initial conditions for which a solution exists for all negative times. Moreover, solutions in both subspaces decay exponentially fast. It is shown how this result can be used to prove the existence of a center manifold containing the weak shock profiles near a constant state. Such a center manifold has only recently been constructed by Benzoni-Gavage and Huot using a different method. It is also indicated how the exponential dichotomies may be used to identify geometric objects which determine the linear stability of semidiscrete shock profiles. The talk is based on joint work with B.Sandstede and

A.Scheel.

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Local tracking and stability for degenerate viscous shock waves

Peter Howard
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It is well known that the stability of certain distinguished waves arising in evolutionary PDE can be determined by the spectrum of the linear operator found by linearizing the PDE about the wave. Indeed, work over the last fifteen years has shown that *spectral stability* implies nonlinear stability in a broad range of cases, including asymptotically constant travelling waves in both reaction–diffusion equations and viscous conservation laws. A critical step toward analyzing the spectrum of such operators was taken in the late eighties by Alexander, Gardner, and Jones, whose *Evans function* (generalizing earlier work of John W. Evans) serves as a characteristic function for the above- mentioned operators. Thus far, results obtained through working with the Evans function have made critical use of the function's analyticity at the origin (or its analyticity over an appropriate Riemann surface). In the case of degenerate (or sonic) viscous shock waves, however, the Evans function is certainly not analytic in a neighborhood of the origin, and does not appear to admit analytic extension to a Riemann manifold. I will show how this obstacle can be surmounted by dividing the Evans function (plus related objects) into two pieces: one analytic in a neighborhood of the origin, and one sufficiently small.

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Edge bifurcations for near integrable systems

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Bjorn Sandstede

When studying the linear stability of waves for near integrable systems, a fundamental problem is the location of the point spectrum of the linearized operator. Internal modes may be created upon the perturbation, i.e., eigenvalues may bifurcate out of the continuous spectrum, even if the corresponding eigenfunction is initially not localized. This phenomenon is also known as an edge bifurcation. It has recently been shown that the Evans function is a powerful tool when one wishes to detect an edge bifurcation and track the resulting eigenvalues. It has been an open question as to the role played by the solutions to the Lax pair, associated with the integrable problem, in the construction of the Evans function and the detection of edge bifurcations. Using the Zakharov–Shabat eigenvalue problem as an illustration, we show the connection between the inverse scattering formalism and the linear stability analysis of waves. In particular, we show a direct connection between the scattering coefficients and the Evans function.

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Travelling waves in thermal multiphase flow in porous media

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Combustion in-situ and steam injection are methods used to improve oil recovery, especially when it is very viscous. The main reason for their effectiveness is that they reduce oil viscosity by heating. We discuss the main nonstandard wave appearing in such flows, a shock where the chemical reaction or the condensation occurs. The internal structure of this shock is crucial to the nature of the overall flow, so it has to be studied carefully as a travelling wave. We show that such a wave is described as an orbit connecting two equilibria of an associated system of ODE's. Typically, at least one of the equilibria is not hyperbolic.

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Shock structure in two- and three-phase flow with permeability hysteresis

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Dan Marchesin

Two-phase flow in a porous medium can be modelled, using Darcy's law, in terms of the relative permeability functions of the two fluids (say, oil and water). The relative permeabilities generally depend not only on the fluid saturations but also on the direction in which the saturations are changing. During water injection, for example, the relative oil permeability falls gradually until a threshold is reached, at which stage the begins to decrease sharply. This stage is termed imbibition. If oil is subsequently injected, then the relative oil permeability does not recover along the imbibition path, but rather increases only gradually until another threshold is reached, whereupon it rises sharply. This second stage is called drainage, and the type of flow that occurs between the imbibition and drainage stages is called scanning flow. Changes in permeability during scanning flow are approximately reversible, whereas changes during drainage an d imbibition are irreversible. In our lecture, we describe a model of permeability hysteresis based on relaxation. The distinctive features of our model are that it allows the scanning flow to extend beyond the drainage and imbibition curves and it treats these two curves as attractors of states outside the scanning region. Through a rigorous stud y of travelling waves, we determine the shock waves that have diffusive profiles, and by means of a formal Chapman-Enskog expansion, we make a connection between our model and a standard one in the limit of vanishing relaxation time. Numerical experiments confirm our analysis.

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Traveling waves in networks of synaptically coupled spiking neurons

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Bard Ermentrout
Remus Osan

Experiments have suggested the existence of propagating waves of activity in neuronal media. Such waves result from the intrinsic behavior of the neuronal circuitry, as well as synaptic interactions between neurons, which are nonlocal and time-dependent. We analyze travelling waves in one-dimensional models for such media, which take the form of integrodifferential equations. Specifically, we use shooting methods to prove the existence of smoothly propagating waves in the theta model for neurons near a saddle-node bifurcation, and we consider stability of these waves. Further, we study single-spike, multi-spike, and periodic waves in an integrate-and-fire model.

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Period-doubling bifurcations of spiral waves

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Period-doubling bifurcations of spiral waves have been observed in light-sensitive BZ-type experiments. These observations are puzzling since period-doubling bifurcations of spirals appear to be impossible from a mathematical viewpoint. We report on preliminary and partial explanations of these phenomena that involve the computation of spectra, and of the associated eigenfunctions, of spirals on bounded and unbounded domains.

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Active and passive defects

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Motivated by the observation of one-dimensional spirals in the CIMA reaction and travelling defects in convection experiments, we study interaction between a large, standing pulse and small-amplitude plain waves. We approximately describe the dynamics of the plain waves by Burgers' equation and investigate the role of the pulse as a point defect.

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Thin film flow near a dynamic contact line

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A fourth order nonlinear PDE describes the flow of a thin liquid film up an inclined surface, under the opposing forces of gravity and shear stress. The Navier slip condition allows a small amount of slip between the liquid and the solid surface near the contact line; it is designed to remove the stress singularity that occurs under the usual no-slip condition. I present analytic and numerical results for the ODE satisfied by travelling wave solutions representing the contact line dynamics. Some of these solutions can be identified with compressive shocks, and others with undercompressive shocks, depending on whether upstream characteristics approach the contact line, or are slower than the contact line, respectively.

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Travelling waves in random media

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The current talk deals with travelling waves in random diffusive media, including time and/or space recurrent, almost periodic, quasi-periodic, periodic ones as special cases. It first introduces a notion of travelling waves in general random media. A theory of existence of travelling waves is then established. Applications of the general theory to bistable and KPP type media are discussed. The results obtained generalize many existing ones on travelling waves.

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Spectral stability of small amplitude shock waves

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TBA

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Deformations of homoclinic and periodic solutions under singular perturbations

Alexander Tovbis
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We will consider a family of singular perturbations for certain model second order ODE (quadratic nonlinearity). Deformatons of both periodic and homoclinic solutions under these perturbations will be discussed.

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Stability of viscous shock and relaxation profiles

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Corrado Mascia

Under the weak assumption of spectral stability, or stable point spectrum of the linearized operator about the wave, we establish sharp pointwise Green's function bounds and consequent linear and nonlinear stability for shock profiles of relaxation and real viscosity systems satisfying the dissipativity condition of Zeng/Kawashima. These include in particular compressible Navier–Stokes and MHD equations, and essentially all standard relaxation models: in particular, the discrete kinetic models of Broadwell, Jin-Xin, Natalini, Bouchut, Platkowski–Illner, and the moment closure models of Grad, Levermore, Müller–Ruggeri. A consequence is stability of small-amplitude profiles of Broadwell and Jin-Xin models and of general real viscosity systems, for each of which spectral stability has been verified in other works. These are the first complete stability results for profiles of a real viscosity system, and the first for relaxation models with nonscalar equilibrium equations. Our results apply also in principle large-amplitude shocks, an important direction for future investigation.

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Recent Progress in the Theory of Exponential Attractors

Organizer: A. Miranville, University of Poitiers

Chaotic attractors in discrete population models

Jerry Chen
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Denis Blackmore

We investigate and compare the dynamics of the discrete Exponentially Self-Regulating (ESR), Lotka-Volterra (LV) and Pioneer-Climax (PC) models for competing populations along with some variants of these models. Special attention is paid to the existence of chaotic dynamics for these models in certain ranges of the critical parameters. It is shown, in particular, that several of these models have strange attractors of the twisted horseshoe with bending tail variety recently proven to exist for special ESR models.

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Exponential attractors for a class of cross-diffusion reaction systems on any dimensional domains

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It will be proven that a class of triangular cross diffusion systems possess exponential attractors in $W^{1,p}(\Omega)$. Here, Ω is smooth bounded domain in R^n with n can be arbitrary and $p > n$. Thus, the functional spaces considered here are Banach spaces.

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Gevrey regularity for the attractor of a damped wave equation

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Sergei Zelik IITP

The goal of this paper is to obtain a Gevrey regularity for the attractor of the following singularly perturbed damped wave equation in a cube domain $\Omega = [0, 2\pi]^3$

$$(1) \qquad \begin{array}{l} \varepsilon\partial_t^2 u^\varepsilon + \gamma\partial_t u^\varepsilon - \triangle u^\varepsilon + f(u^\varepsilon) = g, \\ u^\varepsilon_{|t=0} = u_0,\ \partial_t u^\varepsilon_{|t=0} = u_1, \end{array}$$

with periodic boundary conditions. We assume that $\varepsilon > 0$ and $\gamma > 0$. The nonlinear function f is required to be real analytic,
(2)

$$f(u) = \sum_{j=0}^{\infty} a_j u^j, \text{ where } \sum_{j=0}^{\infty} |a_j| s^j < +\infty \ \forall s \in \mathbb{R}.$$

We assume furthermore that the nonlinearity f satisfies

$$(3) \qquad \begin{array}{l} f'(u) \geq -K, \\ f(u)\cdot u \geq 0 \text{ if } |u| \geq L \\ |f''(u)| \leq C(1+|u|), \end{array}$$

where C, K, and L are fixed positive constants. The assumptions (2) and (3) are fulfilled for cubic nonlinearity $f(u) = u^3 - \alpha u,\ \alpha \in \mathbb{R}$.

Remark We can replace the assumption (3) by an other one, if we are able to obtain uniform (with respect to ε) absorbing sets in $L^\infty(\Omega)$. For example $f(u) = \sin u$. We assume that

$$(4) \qquad g \text{ is periodic and analytic.}$$

we already obtained, for this problem, the existence of exponential attractors with a rate of attraction, a diameter and a fractal dimension uniform with ε, in the variable u as well as u_t. For such a result, the appropriated spaces for solutions of (1) are $\mathcal{E}^k(\varepsilon) = H^{k+1}_{per}(\Omega) \times H^k_{per}(\Omega)$ equipped with the following norms

$$||(u^\varepsilon(t), u^\varepsilon_t(t))||^2_{\mathcal{E}^k(\varepsilon)} =$$

$$\varepsilon||u^\varepsilon_t(t)||^2_{H^k_{per}} + ||u^\varepsilon_t(t)||^2_{H^{k-1}_{per}} + ||u^\varepsilon(t)||^2_{H^{k+1}_{per}},$$

The aim here is to establish

Theorem Let $k > \frac{5}{2}$, let (u_0, u_1) be in $\mathcal{B} \subset \mathcal{E}^k$, under assumptions (2), (3), (4), there exists (v, v_t) uniformly bounded with respect to ε in $C^\infty(\mathbb{R}^+, \mathcal{F}^k_\sigma(\varepsilon))$ such that

$$||(u^\varepsilon(t), u^\varepsilon_t(t)) - (v(t), v_t(t))||_{\mathcal{E}^0(\varepsilon)} \leq d(t),\ \forall t \geq 0,$$

with $\lim_{t\to\infty} d(t) = 0$,

where the function $d(t)$ is independent of ε but depends on $\mathcal{B}$, g, f. The Gevrey space $\mathcal{F}^k_\sigma(\varepsilon)$ is equipped with the norm

$$||(u^\varepsilon(t), u^\varepsilon_t(t))||^2_{\mathcal{F}^k_\sigma(\varepsilon)} =$$

$$\varepsilon||u^\varepsilon_t(t)||^2_{G_\sigma^{\frac{k}{2}}} + ||u^\varepsilon_t(t)||^2_{G_\sigma^{\frac{k-1}{2}}} + ||u^\varepsilon(t)||^2_{G_\sigma^{\frac{k+1}{2}}}.$$

Corollary Under the same assumptions, the points of the attractor of (1) are uniformly bounded for the $\mathcal{F}^k_\sigma(\varepsilon)$-norm.

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Necessary and sufficient conditions for the existence of global attractors of semigroups and applications

Qingfeng Ma
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Checkups Zhong and **Shouhong Wang**

First we established some necessary and sufficient conditions for the existence of the global attractor of an infinite dimensional dynamical system, using the measure of noncompactness. Then we gave a new method/recipe for proving the existence of the global attractor. The main advantage of this new method/recipe is that one needs only to verify a necessary compactness condition with the same type of energy estimates as those for establishing the absorbing set. In other words, one doesn't need to obtain estimates in function spaces of higher regularity. In particular, this property is useful when higher regularity is not available, as demonstrated in the example on the Navier-Stokes equations on nonsmooth domains.

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A construction of exponential attractors in Banach spaces

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Our aim in this talk is to give a construction of exponential attractors in Banach spaces and present some applications and consequences.

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Exponential attractors in Banach spaces

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We extend the theory of exponential attractors from the Hilbert space setting to the Banach spaces context. No squeezing conditions are needed, in contrast to previously known constructions. The only requirements are for the semiflow to be differentiable in some absorbing ball; and for the linearized semiflow at every point inside the absorbing ball to split into the sum of a compact operator plus a contraction. This turns out to be equivalent to the weakest conditions known for a semiflow to possess a global classical attractor of finite fractal dimension and seems to indicate that exponential attractors are as general as global attractors of finite dimension. Applications are made to 3D Navier-Stokes equations with fast rotation. This is a joint work with Le Dung (University of Texas in San Antonio). Ref. "Exponential Attractors in Banach Spaces",L.Dung and B. Nicolaenko, J. of Dynamics and Diff. Equations, Vol. 13, 4,791-806(2001).

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Uniform attractors for dynamical systems with memory

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We are concerned with a special class of dissipative infinite dimensional dynamical system which has been recently investigated; namely, evolution systems with memory subject to time dependent external forces. These models arise in the description of several phenomena like, e.g., heat conduction in special materials, viscoelasticity, phase transitions. In order to pursue a global analysis, the "past history" of the system is viewed as a supplementary variable, ruled by an equation of hyperbolic type. This choice, however, entails a loss of compactness which has to be circumvented in order to get a satisfactory asymptotic analysis.

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Exponential attractor for the wave equation with nonlinear damping

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The evolution of the equation

$$u_{tt} + g(u_t) - \triangle u + f(u) = 0$$

is studied in bounded two or three-dimensional domain under the zero boundary condition. We show that the equation has an exponential attractor provided that the nonlinearities f and g are functions of certain polynomial growth.

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Infinite-dimensional exponential attractors for reaction-diffusion equations in unbounded domains and their approximation

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We consider the following reaction-diffusion system problem in an unbounded domain $\Omega \subset \mathbb{R}^n$:

$$\partial_t u = a\Delta_x u - (\vec{L}, \nabla_x)u - f(u) + g, \quad x \in \Omega, \\ t \geq 0, \quad u\big|_{t=0} = u_0, \quad u\big|_{\partial\Omega} = 0. \tag{1}$$

Here $u = u(t,x) = (u^1, \cdots, u^k)$ is an unknown vector-valued function, Δ_x is the Laplacian with respect to the variable x, $\vec{L} \in C^1_b(\Omega)$ is a given divergent free vector field in Ω, $a \in L(\mathbb{R}^k, \mathbb{R}^k)$ is a given constant diffusion matrix with positive symmetric part $(a + a^* > 0)$ and $f(u)$ is a given nonlinear interaction function which is assumed to satisfy the following conditions:

1. $f \in C^2(\mathbb{R}^k, \mathbb{R}^k)$,
2. $f(v).v \geq -C + \alpha|v|^2, \quad f'(v) \geq -K$,
3. $|f(v)| \leq C(1 + |v|^p)$,

where $u.v$ denotes the standard inner product in $\mathbb{R}^k$, $\alpha > 0$ and the fixed exponent $p \geq 1$ satisfies the growth restriction $p < 1 + \frac{4}{n-4}$ if $n > 4$.

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Exponential attractors for the compressible Navier-Stokes equations

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David Hoff

The existence of exponential attractors for the Navier-Stokes equations of compressible flow will be presented. As a by product of the analysis, the existence of the uniform attractor with a finite fractal dimension follows.

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Ginzburg-Landau Equation in Superconductivity and Related Topics

Organizer: Yoshihisa Morita, Ryukoku University

Non-linear surface supeconductivity for type II superconductors in the large domain limit

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The Ginzburg-Landau model for superconductivity is considered in two-dimensions. We show, for smooth bounded domains, that superconductivity remains concentrated near the surface when the applied magnetic field is decreased below H_{C_3} as long as it is greater than H_{C_2}. We demonstrate this result in the large domain limit, i.e, when the domain's size tends to infinity. Additionally, we prove that for applied fields greater than H_{C_2}, the only solution in $\mathbb{R}^2$ satisfying normal state condition at infinity is the normal state. The above results have been proved in the past for the linear case. Here we prove them here for non-linear problems.

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Triality and primal-dual algorithm for Ginzburg-Landau equation

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Ping Lin

The Ginzburg-Landau Equation is central to material science, which has been subjected to a substantial study during the last twenty years. Since the total potential energy associated with this equation is a nonconvex (double-well) functional, traditional direct analysis and related numerical methods for solving this nonconvex variational problem are difficult. Based on the canonical dual transformation method proposed recently by Gao (2000), we will show that the Ginzburg-Landau Equation is equivalent to the so-called differential algebraic equations (DAEs). Therefore, a primal-dual algorithm is developed for solving the nonconvex Ginzburg-Landau boundary value problem. This method can also be used to control the phase transformation in superconductivity. Application are illustrated by a two dimensional example. Reference: Gao, D.Y. (2000). Duality Principles in Nonconvex Systems: Theory, Methods and Applications. Kluwer Academic Publishers, Dordrecht/London/Boston, xviii+454pp.

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Ginzburg-Landau equation and existence of stable solutions with domain dependency

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I consider critical points of GL functional (solutions of GL equations) with magnetic effect (in 2-dim and 3-dim cases) and existence of local minimizers and its dependency of geometrical property of the domain. One may naturally understand if the domain is simple, only simple situation could occur while complicated domains will have interesting phenomena. I talk about this direction of the study of GL equations.

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Direct numerical simulations of the time-dependent Ginzburg-Landau

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Recently, the superconducting vortex matter has been intensively investigated in the physics community in order to understand anomalous behaviors of the vortex systems in High-Tc superconductors, and consequently, many novel concepts have been established in the present decade. The most prominent idea is the melting of the Abrikosov vortex lattice and the new phase called the vortex liquid state is added to the phase diagram of the superconducting states on the magnetic field and the temperature. Generally, the transition and the new phase have been mainly studied by using Monte Calro techniques on the simplified systems, i.e., XY model, interacting vortex line model and so on. In this paper, we approach to the melting and the vortex liquid phase by using the time-dependent Ginzburg-Landau equation coupled with the Maxwell equation under the thermal noise. The way is the most fundamental and intriguing because all fluctuating components are coupled. We demonstrate that vortex liquid phases grow as multiple droplets and percolate over the whole region at the melting transition point while we compare the melting with that in the other systems.

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Ginzburg-Landau equation in a thin domain

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The Ginzburg-Landau equation in a thin domain is a model of the superconductivity in a thin material (film) and it is an Euler-Lagrange equation of the Ginzburg-Landau energy functional. The convergence of a (global) minimizer of the energy functional as the thinness tends to zero was studied by Rubinstein-Schatzman and Chapman-Du-Gunzburger. Here we show that there exist stable solutions to the equation, that is, local minimizers of the functional, with vortices in the absence of a forcing magnetic field for an appropriate 3-dimensional thin domain.

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Surface superconductivity and boundary effect

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In this talk we shall discuss the asymptotic behavior of the minimal solutions of Ginzburg-Landau system, and describe the concentration phenomena of surface superconductivity for type 2 superconductors in applied magnetic fields lying in between critical fields H_{C_2} and H_{C_3}. Our main concern is the effect of the sample geometry on the value of critical fields and on the profile of superconducting sheath at surface.

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Thermal effects in superconductivity

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We analyze a model for non-isothermal superconductivity derived independently by G. Maugan, and S. Zhou and K. Miya. The model is described by a parabolic system based on the time dependent Ginzburg-Landau equation(TGLE),the Maxwell equation, and an energy equation such that the Clausius-Duhem inequality holds. We prove existence, regularity and large time behavior results for solutions to initial value boundary value problems for this system. The sensitivity of superconducting materials to thermal variations is a major obstacle in their applications. In practice one sees that vortex motion in current patterns generates thermal energy, producing "hot spots" which results in suppressing superconductivity. Our principal qualitative result is that we exhibit this phenomenon in solutions. Prior analytic work on vortex motion centered on the isothermal model of the TGLE and Maxwell equations. This setting is completely kinematic and produces markedly different evolutions. This work is joint with Eunjee Shin (Penn State).

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A nonstiff Euler discretization of the complex Ginzburg-Landau equation in one space dimension

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A nonstiff discretization method is applied to the complex Ginzburg-Landau equation with periodic boundary conditions,

$$\partial_t u = (1+i\nu)\partial^2_{xx} u + Ru - (1+i\mu)|u|^2 u;$$
$$u(x+2\pi,t) = u(x,t) \quad \text{for } -\infty < x < \infty,\ t \geq 0.$$

This parabolic equation is discretized first in space by a truncated Fourier series (that is, a finite Fourier sum) and then in time by an explicit Euler method. The exponential decay of Fourier modes in both, the original complex Ginzburg-Landau equation and the resulting equation for the truncated Fourier series (independently from the number of Fourier modes in the truncation) is used in an essential way to prove the nonstiffness. Also dissipativity of the discretized equation is established and numerical results are discussed.

⟶ ∞ ⋄ ∞ ⟵

Micromagnetic soliton and Landau-Lifshitz equation

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We plan to study the properties of the weak solutions to nonlinear partial differential equations arisen from ferromagnets, especially the hypersurface moving by its curvature, the dynamics of topological magnetic solitons and the behaviour of the solutions to Landau-Lifshitz equation.

⟶ ∞ ⋄ ∞ ⟵

Learning Theory, Dynamic System and Neural Networks

Organizer: Kayvan Najarian, University of North Carolina at Charlotte
Mirsad Hadzikadic, University of North Carolina at Charlotte

A learning theory approach to the construction of predictor models

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We present a new framework for the identification of predictor models from data, which builds on recent developments of statistical learning theory. The three key elements of our approach are: i) an unknown mechanism that generates the observed data; ii) a family of predictor models, among which we select our predictor model based on observations; iii) an optimality criterion that we want to minimize. A major departure from standard identification theory is taken in that we consider interval models for prediction (that is models that returns output intervals, as opposed to output values). Moreover, we introduce a consistency criterion (the model is required to be consistent with observations) which act as a constraint in the optimization procedure. In this framework, the model has not to be interpreted as a faithful description of reality, but, rather, as an instrument to perform prediction. To the optimal model, we attach a certificate of reliability, that is a statement of probability that the computed model will be consistent with future unknown data.

⟶ ∞ ⋄ ∞ ⟵

Learning theory and neural networks for signal processing and control

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Dynamic feedback neural networks are known to present powerful tools in modelling of complex dynamic models. Since in many real applications, the stability of such models (specially in presence of noise) is of great importance, it is essential to address stochastic stability of such models. In this paper, sufficient conditions for stochastic stability of some families of dynamic neural model (including two families of sigmoid neural networks) are presented. These stability conditions are set on the weights of the networks and can be easily tested.

⟶ ∞ ⋄ ∞ ⟵

Learning theory and neural networks for signal processing and control

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The learning theory has provided the tools to evaluate the accuracy and the statistical confidence of the models and classifiers developed from

the sample training data. In addition, the need to generate the least complex models to provide pre-specified levels of accuracy and confidence has been addressed in the learning theory from different standpoints. In this paper, first the main ideas of the learning theory (specially the Probably Approximately Learning theory) are briefly described, and then some important applications and impacts of this theory in different areas of signal processing and control are explained. The paper also describes different practical techniques that apply the ideas of the learning theory to minimize the complexity of the models developed from the training data.

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Neural networks for length-dependent and length-independent functional classification and prediction of small sequences of lipase, protease, and isomerase

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Kayvan Najarian

We use neural networks to classify proteins according to their functionality. Primary structures of small lipase, protease, and isomerase proteins (containing between 100 and 200 amino acids) were used for length-dependent and length-independent classification. In length-dependent classification, the main feature is either amino acid label (integer code), amino acid hydrophobic values, or the solubility values defined here. The classification performances of these length-dependent measures were assessed using neural network classifiers for the three enzyme classes. In addition, a set of length-independent features related to signal complexity is introduced and used for neural classification of the above-mentioned families of proteins. Each neural network was developed for the given feature sets, trained, and tested for suitability in classifying enzymes. The results of length-dependent and length-independent neural classifiers are compared with each other.

⟶ ∞ ⋄ ∞ ⟵

Application of learning theory to protein sequence classification

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Recently, Valiant's Probably Approximately Correct (PAC) learning theory has been extended to learning m-dependent data. With this extension, training data set size for sigmoid neural networks have been bounded without underlying assumptions for the distribution of the training data. These extensions allow learning theory to be applied to training sets which are definitely not independent samples of a complete input space. In our work, we are developing length independent measures as training data for protein classification. This paper applies these learning theory methods to the problem of training a sigmoid neural network to recognize protein biological activity classes as a function of protein primary structure. Specifically, we explore the theoretical training set sizes for classifiers using the full amino acid sequence of the protein as the training data and using length independent measures as the training data. Results show bounds for training set sizes given protein size limits for the full sequence input compared to bounds for input that is sequence length independent.

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Symmetries and Differential Equations in Physics and Other Applications

Organizer: Weiqing Xie, Cal Poly Pomona
M. Nakashima, Cal Poly Pomona

A reducible representation of the generalized symmetry group of a quasiperiodic flow

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The generalized symmetry group of a flow on a manifold is the group theoretic normalizer, within the group of diffeomorphisms of the manifold, of the one parameter abelian group of diffeomorphisms generated by the flow. Up to conjugacy, the generalized symmetry group of a quasiperiodic flow on an n-torus is determined by a system of uncoupled first order partial differential equations. New types of symmetries (other than the classical types of symmetries and time-reversing symmetries) may exist depending on certain algebraic relationships being

satisfied by pair wise ratios of the frequencies of the quasiperiodic flow. These new types of symmetries, when they exist, are a dominant feature of a reducible linear representation of the generalized symmetry group in the de Rham cohomology of the n-torus.

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Boundary values in de Sitter space

Martin Nakashima
Cal Poly Pomona, USA

The theory of Dirac Singletons, as developed by Flato and Fronsdal, is a massless gauge theory in de Sitter space. Although noted primarily for its physical content and connections to group theory, it may be of further interest to mathematicians as it suggests some problems in differential equations. This talk will present some of these issues.

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Second order dynamical systems used for generating "practical" test functions for filtering and sampling procedures

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As it is known, in averaging procedures the user is interested in the mean value of the received signal over a certain time interval. Usually this operation is performed by an integration of the signal on this time interval (considered to be constant) the result of the integration being proportional to the mean value of the signal. However, such structures are very sensitive at random variations of the integration period (generated by the switching phenomena at the end of the integration). For this reason, a multiplication of the received signal with a test-function (a function which differs to zero only on this time interval and with continuous derivatives of any order on the whole real axis) is recommended. This paper presents some invariance properties of differential equations, which can be used for generating a "practical" test-function on this time interval, and it presents also the properties of second order oscillating systems (considered as generating "practical" test functions) in filtering and sampling procedures.

⟶ ∞ ⋄ ∞ ⟵

A mathematical model from stress driven diffusion

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This talk concerns the study of a system of differential equations involving stress-driven diffusion which occurs in materials science and technology and its applications. We will explain and analyze the mathematical model and present mathematical analysis for the problem.

⟶ ∞ ⋄ ∞ ⟵

Continuous Media and Optimal Design

Organizer: Pablo Pedregal, Universidad de Castilla-La Mancha

Homogenization and localization with an interface

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Gregoire Allaire and **Andrey Piatnitski**

We consider the homogenization of a spectral problem for a singularly perturbed periodic medium. This is a model of a reaction-diffusion equation used for determining the power distribution in a nuclear reactor core. In this context, homogenization results are at the basis of many multiscale type methods for computing solutions. We suppose that the domain is composed of two periodic medium separated by a planar interface. Three different situations arise as the period tends to zero. First, there is a global homogenized problem as if there were no interface. Second, the limit is made of two homogenized problems with a Dirichlet boundary condition on the interface. Third, there is an exponential localization near the interface of the first eigenfunction.

⟶ ∞ ⋄ ∞ ⟵

Bounds for properties of composites from expanding (smart) materials

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An actuator is a cantilever that moves its free end in a prescribed fashion when it is activated; for example, the bi-metallic plate is bent when heated. It is an example of a device made of an active material that changes its shape responding to an activating signal. To design such devices, one considers composites from thermo-elastic materials or from expandable ("smart") materials that experience phase transformation in response to the activating signal. It is assumed that the materials in the composition have different tensors of elastic moduli and different anisotropic expansion tensors that show the change of the shape of the pure materials. The (generally anisotropic) composite is also characterized by an effective tensor of elastic moduli and an effective expansion tensor; both depend on the microstructure of the composite. In the talk, we discuss new coupled bounds for these two tensors. To derive the bounds, we adapt and modify the technique of the translation method as well as the technique of Shapery. The result generalizes the previously obtained bounds for isotropic thermal expansion coefficient by Shapery, Hashin and Rozen, Gibiansky and Torquato. Using the derived bounds, a numerical algorithm is developed that enables to demonstrate bounds and coupling of various components of the effective tensors. In particular, we are checking optimality of numerically obtained anisotropic structures with extremal expansion in a given direction and constrained elastic properties. The talk describes a part of the collaborative research with Ole Sigmund (Danish Technical University).

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Effective dynamics in thin ferromagnetic films

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E Weinan

In a ferromagnetic material, the dynamics of the relaxation process is affected by the presence of a strong shape or material anisotropy. We systematically explore this fact to derive the effective dynamical equation for a soft ferromagnetic thin film. We show that as a consequence of the interplay between shape anisotropy and damping, the gyromagnetic term is effectively also a damping term for the in-plane components of the magnetization distribution. We validate our result through numerical simulation of the original Landau-Lifshitz equation and our effective equation.

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The yield set of perfectly plastic polycrystals

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Metals are usually found in the form of polycrystals, that is, large collections of bonded grains. The atoms of each grain form a periodic lattice and thus, each grain is a single crystal. The yield set of single crystals (i.e. the set of stresses that the single crystal can withstand) is anisotropic. Thus, the yield set of polycrystals depends not only on the yield set of its grains, but also on the polycrystalline texture (i.e. shape, orientation and spatial distribution of the grains). The evaluation of the yield set of polycrystals leads to a constraint optimization problem in which one of the constraints is a linear PDE. In this talk I will discuss this problem.

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Optimal design of composite conductors with weakly discontinuous objective functionals

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Consider a body in space that is to be occupied by two different isotropic conductors subject to a resource constraint: the volume fraction occupied by the better conductor is bounded above by a fixed amount. The body contains a given source density. The problem is to design a layout of the two materials minimizing a given objective functional depending explicitly on the gradient of the resulting electrostatic potential. If one attempts to solve this problem numerically one might discover that the layout oscillates on the numerical mesh size scale. The solution to this problem is to expand the design space allowing composite materials as structural elements. It is also necessary to change the original functional to a new one that takes into account the information of possible oscillations of the electric field on infinitely small scale because these oscillations cannot be resolved numerically. In my talk I will overview the fundamental dichotomy between weakly continuous and weakly discontinuous functionals and will describe recent attempts to deal with the harder weakly discontinuous problems.

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Homogenized failure criteria and the design of composite structures for strength and stiffness

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New bounds for the effective strength domain for multi phase linearly elastic-perfectly plastic composites are obtained. These bounds are given in terms of derivatives of effective elastic tensors and incorporate the effects of microscopic stress concentrations. The results hold in the general homogenization context provided by the theory of G-convergence of elliptic operators and apply to functionally graded materials. These results are used to develop a numerical scheme for the design of composite structures for maximum strength and stiffness.

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Numerical modelling in grain growth: multiscale approach

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Grain growth, as any physical phenomena, exists in general on different scales and appears differently depending on the scale of observation. The microscopic behavior is determined by interactions between particles and influences macroscopic properties of the considered system. However, the connection between microscopic and macroscopic is often difficult to establish. In this talk, I will discuss a hierarchy of numerical models, deterministic and stochastic, that describe grain growth on different scales. Each such model inherits its properties from a more detailed predecessor. As the final result of such grain coarsening, we deduce partial differential equations for the evolution of general properties of grain growth – distribution functions.

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Optimal design from a variational perspective

Pablo Pedregal
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I will analyze a typical optimal design problem in conductivity from a variational point of view. Starting from a general cost functional depending explicitly on derivatives of the electric potential, we will see how far we can go in finding the appropriate convex hull leading to relaxation of the initial problem. I will also discuss how this information translates into optimal designs for the original optimal design problem.

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Mathematical modelling of the myocardial fiber organization

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It is commonly believed that the myocardium design and structure allow maximal mechanical efficiency in the systole and diastole processes. The long-term purpose of our multidisciplinary approach is to try and propose a mathematical model for the mechanical behaviour of the myocardium. Such a model has to take into account the specific fiber organization in such a muscle, whose organization, geometrical, and mechanical properties are different from those of a skeletal muscle. We recall that the dissection or peeling techniques are not precise enough, since apparently preferred fibre directions can be inferred by the experimental process. Here, we use data provided by measurements by means of polarized light microscopy developed in Grenoble. All mathematical and numerical strategies we have used on these data back up the conjecture by Streeter which states that myocardial fibers run as geodesics on a nested set of surfaces. Work under progress is devoted to writing an appropriate p.d.e's model and rechecking on this model the geodesic hypothesis.

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Dynamics of damage in two-dimensional structures with waiting elements

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Andrej Cherkaev

We consider the dynamics of the damage of a bridge-like $2-D$ structure made from specially constructed waiting elements. Each element consists of two elastic links of different equilibrium lengths. Whenever the elongation of the shorter link exceeds some critical value, it undergoes an irreversible damage process which eventually leads to the breakage. Thereafter the second (longer) link assumes the stress. Let α be a portion of material used for the first link and $1-\alpha$ be a portion of material used for the second link. We compare the waiting element model with the usual structure, consisting of only one link (of shorter length). In the waiting element model the usual structure corresponds to $\alpha = 1$. By performing various numerical experiments when the structure is impacted by a projectile modelled as an "elastic ball", we show that in some cases the waiting element structure can spread the damage over a large area and therefore withstand larger stresses than the usual structure. Several movies will be shown to illustrate this phenomenon. We also address the question of the optimum choice of parameter α.

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Mathematical Neuroscience

Organizer: Mary Pugh, University of Toronto

Dynamics in responses to auditory motion

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John Rinzel

We are interested in the cellular mechanisms shaping the neuronal responses to auditory motion stimuli. In this framework, we study how the presence of biologically realistic features influences the performance of a periodically driven system. We show that the presence of the cellular mechanisms of firing rate adaptation and the post-inhibitory rebound is sufficient to account for experimentally observed phenomena, such as rebound responses, hysteresis and phase shift in response to periodic forcing. We quantify various physiologically relevant features of the response and make testable predictions. We also compare the performance of spiking and firing-rate-type models in the context of the auditory motion processing.

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Biophysical mechanisms for frequency encoding in a model sensory system

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It is well known that the passive membrane properties of neurons function as a low-pass filter for synaptic inputs. Current inputs arriving at low frequencies yield large voltage responses, but high frequency inputs are attenuated or blocked. Neurons can also exhibit bandpass filtering properties with large responses when driven by inputs near their resonant frequencies and smaller responses at other frequencies. This mechanism occurs due to the interactions between active and passive membrane properties. For example, currents that actively oppose changes in membrane voltage and also activate slowly relative to the membrane time constant will produce resonance. We examine the effects of morphology, passive membrane properties, and active channels on frequency tuning in a model sensory system. Theoretical studies and simulations show that the low-pass filtering observed in some neurons can be explained by the passive electrotonic structure of the dendritic arbor and the dynamic sensitivity of the spike initiation zone. In contrast, neurons that exhibit bandpass tuning at higher frequencies have dendritic structures with fewer branching structures and larger diameters that are more electrotonically compact. This morphology causes less attenuation of higher frequencies; however, ion channels that resonate in the desired frequency range are also required.

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Chaotic phase synchronization in systems with small phase diffusion

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Margaret Beck

The geometric theory of phase locking between periodic oscillators as proposed by Winfree and Kuramoto has been used with much success in the past. I will discuss extensions of this theory to phase coherent chaotic systems. This approach explains the qualitative features of phase locked chaotic systems and provides an analytical tool for a quantitative description of the phase locked states. Moreover, this geometric viewpoint allows for the identification of obstructions to phase locking even in systems with negligible phase diffusion, and provides sufficient conditions for phase locking to occur. These techniques were applied to the Rössler system and a phase coherent electronic circuit and good agreement was found between numerical results, experiments and theoretical predictions.

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Temporal synchronization of pyramidal cells by high-frequency, depressing inhibition

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The sharp wave-associated ripple is a high-frequency, extracellular recording observed in the rat hippocampus during periods of immobility. During the ripple, pyramidal cells synchronize over a short period of time despite the fact that these cells have sparse recurrent connections. The timing of synchronized pyramidal cell spiking is critical for encoding information that is passed onto post-hippocampal targets. Both the synchronization and precision of pyramidal cells is believed to be coordinated by inhibition provided by a vast array of interneurons. We consider a minimal model consisting of a single interneuron which synapses onto a network of uncoupled pyramidal cells. We show that fast decaying, high-frequency, depressing inhibition is capable of rapidly synchronizing the pyramidal cells and modulating spike timing. These mechanisms are robust in the presence of intracellular noise. We prove the existence and stability of synchronous, periodic solutions using geometric singular perturbation techniques. The effects of synaptic strength, synaptic recovery, and inhibition frequency are discussed. In contrast to prior work which suggests that the ripple is produced by homogeneous populations of either pyramidal cells or interneurons, our results suggest that cooperation between interneurons and pyramidal cells is necessary for ripple genesis.

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Stabilization of “bumps”by noise

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Andre Longtin

Spatially localized regions of active neurons (“bumps”) have been proposed as a mechanism for working memory, the head direction system, and feature selectivity in the visual system. Stationary bumps are ordinarily stable, but including spike frequency adaptation in the neural dynamics causes a stationary bump to become unstable to a moving bump through a supercritical pitchfork bifurcation in bump speed. Adding spatiotemporal noise to the network supporting the bump can cause the average speed of the bump to decrease to almost zero, reversing the effect of the adaptation and “restabilizing” the bump. This restabilizing can be understood by examining the effects of noise on the normal form of the pitchfork bifurcation where the variable involved in the bifurcation is bump speed. This noise–induced stabilization is a novel example in which moderate amounts of noise have a beneficial effect on a system, specifically, stabilizing a spatiotemporal pattern. Determining which aspects of our model system (integral rather than diffusive coupling, a slow variable, travelling structures that appear through a pitchfork bifurcation in speed) are necessary for this type of behavior remains an open problem.

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Dynamics of neurons connected by inhibitory and electrical coupling

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John Rinzel

Recent findings suggest that many inhibitory cell networks in the brain are connected through both inhibitory and electrical coupling. However, it is unclear how the interaction of these two coupling modes affects the dynamics of these networks. To begin addressing this issue, we use the theory of weakly coupled oscillators to study the influence of coupling parameters on synchronization patterns in a model of intrinsically oscillating cells connected by both inhibition and electrical coupling.

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Characterizing coupled neurons with white noise analysis

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We present an asymptotic analysis of two coupled linear-nonlinear neurons. By measuring the spike times of both neurons in response to a white noise stimulus, one can characterize the neurons' properties and their mutual connections. The linear-nonlinear model used in the analysis is similar to a widely used phenomenological model of a neuron in response to sensory stimulation. Moreover, we demonstrate that the results of the analysis also work with more realistic neuron models. Thus, this analysis may help characterize neural circuitry in sensory brain regions.

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Phase plane analysis of neural decoding in the Rodent Whisker-Barrel System

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In the rodent whisker-to-barrel pathway, populations of thalamic neurons encode information from the periphery in terms of changes in the population firing rate. Correspondingly, cortical neurons respond preferentially to rapid changes in the rate of firing among input neurons from the thalamus. Previous computational models based on known features of cortical circuitry have captured this and other aspects of the thalamocorical response transformation in the rodent whisker system. In this presentation, I will examine these models using a modified version of phase plane analysis in order to understand the mechanisms that underlie cortical sensitivity to thalamic input timing. The analysis reveals that cortical processing in our model depends on strong inhibition that renders the net effect of cortical connections to be *damping*. This distinguishes it from previous models of cortical microcircuits, in which the net effect of cortical connections is *amplifying*. I will conclude with a brief comparison of the two proposed mechanisms of cortical processing and suggest a possible experimental means for distinguishing between them.

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Lateral inhibition is neither necessary nor sufficient for sustained, patterned activity

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Evidence suggests that sustained, localized neuronal activity, or bumps, may play a role in working memory or representation of internal states, such as head direction. Previous (and ongoing) theoretical work has demonstrated that a synaptic architecture featuring recurrent excitation and long-range inhibition, together known as lateral inhibition, can lead to activity bumps in neuronal network models. However, this architecture is absent in some areas of the brain where such activity may be relevant.Here we show how a rate-based integrodifferential equation model can support bump solutions without recurrent excitation. We also provide conditions under which no spatial patterns exist despite the presence of lateral inhibition.

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Modulating model inhibitory neuronal networks

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Synchronous oscillatory activity in networks of interneurons connected by inhibitory synapses play critical roles in brain function. Differences in the kinetics of the inhibitory response observed with anesthetics can affect this activity. We use theoretical insights to suggest a new mechanism of anesthesia. In particular, we suggest that the different behavioral effects of different anesthetic drugs might lie in the different ways in which these drugs modulate inhibitory network coherence.

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Dynamics of the visual cortex

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Ehud Kaplan

We present multivariate harmonic analysis methods for the analysis of dynamical optical imaging data; discuss the various components of the dynamical signal resulting from the harmonic analysis; and present an overall strategy for constructing a population model of visual cortex, with results from one functional component of the model.

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A neuronal network model of the macaque primary visual cortex

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Michael Shelly and **David McLaughlin**

Our objective is a realistic theory of the visual cortex that can explain the visual selectivity, dynamics, and the diversity of visual properties in cortical cell populations. To do this, we have studied a large-scale computational model of Macaque V1 [McLaughlin et al. 2000 PNAS] based on anatomy and physiology. Cells in the model are classified as Simple or Complex by the same index of linearity of spatial summation that has been used in physiology experiments. Previously we offered an explanation of how Simple cells could exist in the model despite the non-linearity of the LGN input and of cortico-cortical excitation [Wielaard et al 2001 J. NS.] Now we report that Complex cells arise in the model by allowing for randomness in synaptic coupling strengths, which can increase the importance of network excitation, and randomness in the strength of LGN input. My work suggests that the Simple-Complex classification reflect different synaptic balances within the same basic model circuit. Since the dichotomy of 'Simple' and 'Complex' behavior is seen in other areas of visual processing, the basic mechanism may be widely operating.

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Dynamical patterns in the basal ganglia and related neuronal networks

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A conductance-based network model was developed (in collaboration with J. Rubin, D. Terman, and C. Wilson) to describe neural interactions in the basal ganglia, with the aim of testing hypotheses on the origin of activity states associated with disorders such as Parkinson's disease. Computer simulations reveal that the system exhibits a variety of spatiotemporal patterns, including episodic synchronous oscillations, clustered rhythms, travelling waves, and irregular uncorrelated spiking. Using a dynamical systems approach, we analyze how synaptic coupling interacts with intrinsic neuronal properties to generate these types of behavior. We also investigate how transitions between patterns are effected as parameters are varied, and make comparisons with other similarly wired neuronal networks.

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Difference Equations and Their Applications

Organizer: Youssef Raffoul , University of Dayton

Oscillatory properties of third order neutral delay differential equations

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R. Savithri and **E. Thandapani**

The authors consider the third order neutral delay differential equation

$$\Big(a(t)\left(b(t)\left(y(t)+py(t-\tau)\right)'\right)'\Big)'+q(t)f(y(t-\sigma))=0 \qquad (*)$$

where $a(t) > 0$, $b(t) > 0$, $q(t) \geq 0$, $0 \leq p < 1$, $\tau > 0$, and $\sigma > 0$. Criteria for the oscillation of all solutions of $(*)$ are obtained. Examples illustrating the results are included.

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Oscillation properties of an Emden-Fowler type equation on discrete time scales

Joan Hoffacker
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Elvan Akin-Bohner

In this talk we explore the oscillation properties of

$$u^{\Delta^2}(t) + p(t)u^{\gamma}(\sigma(t)) = 0$$

on a time scale $\mathbb{T}$ with only isolated points, where $p(t)$ is defined on $\mathbb{T}$ and γ is a quotient of odd positive integers. We define oscillation in this setting, and generate conditions on the integral of $p(t)$ which guarantee oscillation and no oscillatory solutions. In addition we consider the case when solutions of this equation has asymptotically positively bounded differences.

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Exponential stability in nonlinear difference equations

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We employ non-negative definite Lyapunov functionals to obtain conditions that guarantee exponential stability and uniform exponential stability of the zero solution of the nonlinear discrete system

$$x(n+1) = f(n, x(n)), x(n_0) = x_0, \text{ for } n \geq n_0.$$

The theory is illustrated with several examples.

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Singular conjugate boundary value problems on a time scale

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Let $\mathbb{T}_1$ be a time scale symmetric about 1/2. Let $1/2 \in \mathbb{T}$ be dense and define $\mathbb{T} = \mathbb{T}_1 \bigcap [0, 1]$. The conjugate nonlinear boundary value problem,

$$-u^{\Delta\Delta}(t) = a(t)f(u(t)), t \in \mathbb{T} \setminus \{0, 1\}$$
$$u(0) = u(1) = 0,$$

where $a(t)$ is singular at $t = 1/2$ and f satisfies certain growth conditions, is shown to have infinitely many solutions using Krasnosel'skiĭ's fixed point theorem.

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Stability properties of linear Volterra discrete systems with nonlinear perturbation

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We consider a Volterra discrete system with nonlinear perturbation

$$x(n+1) = A(n)x(n) + \sum_{s=0}^{n} B(n, s)x(s) + g(n, x(n))$$

and obtain necessary and sufficient conditions for stability properties of the zero solution employing the resolvent equation coupled with the variation of parameters formula.

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Accurate estimates for the solutions of difference equations

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We will be concerned with a non-autonomous perturbed linear discrete dynamical system. A well known result of Perron, which dates back to 1929 (see Ortega(1973) and LaSalle(1976)), states that these kind of systems are asymptotically stable if the matrix A of the linear part is stable, i.e., the spectral radius of A is less than one, and the perturbation satisfies an asymptotic property. We can see that this kind of results are purely local results which gives no information about the size of the region of asymptotic stability nor the norm of the solutions. In this report, estimates for the norms of the solutions and the size of the regions of stability of non-autonomous perturbed linear difference equations are derived. The methodology is based on the "freezing" method and on the recent estimates for the powers of a constant matrix. Finally, we will illustrate our main results by considering partial difference equations which model reaction and diffusion processes.

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A second-order self-adjoint dynamic equation on a time scale

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Recently, the study of dynamic equations on time scales has seen an increased level of interest from researchers seeking to study both differential and difference equations in a more general setting. In this paper, we are concerned with the dynamic equation $[p(t)x^{\Delta}]^{\nabla} + q(t)x = f(t)$. Relatively little research has been done on this equation which combines both "delta" and "nabla" derivatives in a single dynamic equations. We will discuss various results concerning this equation, including results on zeros of solutions, disconjugacy, and factorizations.

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Positivity and discrete models for the Lotka-Volterra equations

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The first nontrivial mathematical model for predator-prey interactions was the Lotka-Volterra equations [1]. The two coupled, first-order ODE's has biological relevant solutions in the first quadrant, i.e., $x(0) > 0$ and $y(0) > 0$, lead to $x(t) > 0$ and $y(0) > 0$. Further, there is a single nonnegative fixed-point, around which all solutions periodically oscillate. We consider the application of the nonstandard finite-difference techniques of Mickens [2] to formulate corresponding discrete time models of these equations. In particular, enforcement of the positivity condition is made by the use of nonlocal discrete representations for both the linear and quadratic terms appearing in the Lotka-Volterra differential equation. Our studies indicate that one must be careful in carrying out this procedure. Both linear stability analysis and numerical work will be used to illustrate our results. References [1] J. D. Murray 1989 Mathematical Biology (Springer-Verlag, Berlin); section 3.1. [2] R. E. Mickens 1994 Nonstandard Finite Difference Models of Differential Equations (World Scientific, Singapore).

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The nabla exponential function

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We will develop the nabla exponential function and give several of its properties.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Positive periodic solutions of nonlinear functional difference equations

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In this paper, we apply a cone theoretic fixed point theorem and obtain sufficient conditions for the existence of multiple positive periodic solutions to the nonlinear functional difference equations

$$x(n+1) = a(n)x(n) \pm \lambda h(n) f(x(n - \tau(n))).$$

where $a(n), h(n), \lambda, f(x)$ and τ are positive and periodic of period T.

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Comparison of eigenvalues for Sturm-Liouville boundary value problems on a measure chain

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Under consideration is a class of even ordered linear differential equations with Sturm-Liouville boundary conditions. The differential equation is, in fact, a general dynamic equation containing delta-derivatives whose solution is defined on a measure chain. For a pair of eigenvalue problems for this dynamic equation, we first verify the existence of a smallest possible eigenvalue and then establish a comparison between the smallest eigenvalues of each eigenvalue problem.

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Existence of periodic solutions of nonlinear discrete second order equations

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In this paper we study the existence of T-periodic solutions of equations of the form

$$x(t+2) + bx(t+1) + cx(t) + f(t, x(t)) = h(t)$$

where f is nonlinear, smooth, $f(t+T, x) = f(t, x)$ for each (t, x) and $h(t+T) = h(t)$ for all t. Under the assumption of the existence of a T-periodic solution for a specific forcing term h, we aim to find conditions on the nonlinear term f which will allow us to establish the existence of T-periodic solutions to a system of the form

$$x(t+2) + bx(t+1) + cx(t) + f(t, x(t)) = h(t) + g(t)$$

where g is T-periodic and "relatively large." We formulate our problem as an operator equation in a space of T-periodic sequences and use a homotopy argument that eventually connects the existence of T-periodic solutions of the discrete equation to a differential equation on a sequence space.

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Equations with partial derivatives and differential equations used for simulating acausal pulses in mathematical physics

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Paul Sterian

Some phenomena in physics (such as the phenomenon of photonic echo) appears for an external observer as non-causal pulses suddenly emerging from an active medium (prepared by some other optical pulses). Such a pulse is very hard to be simulated without using physical quantities corresponding to the internal state of a great number of atoms. The only mathematical possibility of simulating such pulses without using a great number of variables consists in the use of test-functions. It is shown that such functions can be put in correspondence with acausal pulses in physics. This study shows that the wave-equation considered on the length interval (0, 1) (an open set), starting at the initial moment of time from null initial conditions, can possess as possible solution a test-function represented by a propagating direct wave coming from outside the length-interval. For explaining the reason why such acausal pulses do not appear in real circumstances some methods from statistical physics are used. While at the zero moment of time all derivatives of the amplitude of the real string are equal to zero, it is shown that we may consider the zero moment of time as a bifurcation point

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On the periodic nature of the solutions of the reciprocal difference equations with maximum

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We prove that every positive solution of the difference equation $x[n] = \max\{A/x[n-1], B/x[n-2], C/x[n-3]\}$ is eventually periodic of (not necessarily prime) period T, which is explicitly determined in terms of the coefficients A,B and C.

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Representations of Dynamical Systems

Organizer: Marc Rouff, University of Caen Basse Normandie
Michel Cotsaftis, Ecole Centrale d'Electronique, Paris France

Asymptotic vs. projective representation for solutions of dynamical system equations

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The problem of dynamical evolution of a system exhibiting internal branchings toward bifurcated states is considered by observing that as a consequence the number of "relevant" state space dimensions where power flows is varying with time. Classical projection method to represent the solution does no longer apply, and more adapted asymptotic method taking advantage of branched states characteristic time and space scales is discussed. Analytic expressions of the solution are given, allowing to construct the modified state space dynamics due to these branchings and to design a correct controller which self-consistently accounts for the internal power flow created by the opening of internal branched modes. Application is made to compliant actuated deformable one-link system.

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Towards functional optimal control using C^k spline functions

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Differential nonlinear optimal control is known as difficult numerical problem because of the integration of the adjoint vector. We show in this paper that functional approach of this problem directly by C^k spline functions [1]-[2], avoid the notion of adjoint vector and leads to more suitable algorithmics.

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[1] M. Rouff : The computation of C^k spline functions, *Computers and Mathematics with Applications*, **23**, (1), pp 103-110, 1992.

[2] M. Rouff et W.L. Zhang : C^k spline functions and linear operators, *Computers and mathematics with applications*, **28**, (4), pp 51-59, 1994.

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A New discretization method using C^k spline functions

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We know that in $H_k(\Omega)$, the Sobolev space generated by C^k spline functions [1]-[2] on an open set Ω, every differential relations of order k or less, are written as arbitrarily precise and simple algebraic relations [3]-[4]-[5]. In this sense, and for every kind of differential equations, total or partial, linear or nonlinear, explicit or implicit, and always for a finite small numbers of terms, and for an arbitrarily precision, $H_k(\Omega)$ the Sobolev space generated by the set of C^k spline functions, plays the same role then Fourier Spaces for linear total or partial differential equations.

This fact opens the way to a new functional and invariant calculus in a suitable space defining by the all differential invariants and symmetries of the problem.

In Automatic Control Scientists and Engineers always study behaviors or controllers of a defined process trough a state equation, defined by Kalman in the early sixties, or through their nonlinear implicit or explicit extensions, i.e. only on ordinary differential equations (ODE's).

Then the control of Partial Differential Equations (PDE's) remains a difficult or unsolved problem in many cases.

We present here a C^k discretization method for Extended Lagrangian systems mixing the time defined on $\mathbb{R}$, the space of the real numbers, and $H_k(\Omega)$ where Ω is the open set of the non-time variables. We show by using the example of vibrations control in a one link robotic arm [6], that in $D_k(\Omega) \equiv \mathbb{R} \times H_k(\Omega)$, the space of discretization generated by this method, the resulting state equations represent the behavior of the system with a localized small ball of errors depending on I the number discretization points on Ω.

References

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$\longrightarrow \infty \diamond \infty \longleftarrow$

Towards a new invariant and functional calculus wC^k spline functions

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Functional expansions based on C^k spline Functions [1]-[2], lead to the remarkable property that the coefficients of these functional expansions are the only set of the whole total or partial derivatives up to k of the considered approximated functions themselves at each point of discretization on the open set Ω.

This property leads to include in $H_k(\Omega)$ the Sobolev space generated by the C^k spline functions, the differential constraints, invariants, symmetries and the equations of evolution themselves as exact algebraic relations, redefining by the same way the suitable functional space of representation associated to the considered phenomenon [3]-[4]-[5].

The Fourier Transforms of C^k spline functions are C^k wavelets, with the obvious property that these kind of spectra generated by C^k wavelets have as coefficients the set of the whole total or partial derivatives up to k of the associated dual function at each point of discretization of the dual space. This property opens the way to many computations for example, new definitions of the entropy of a signal.

At Last duality of direct and Fourier spaces are used to present a self consistent relation of existence, which open the way to a large amount of applications, for example the definition with an arbitrarily good precision of non entire derivatives partial or total, or non entire integrals simple or multiple. *But today the most interesting applications of these C^k spline functions calculus are the ability to replace classical differential and integral calculus by a functional and invariants calculus which can be exact (Formal calculus) on* $\mathbb{Q}$

References

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Stochastic Analysis and Applications

Organizer:S. Sathananthan ,Tenessee State University

Large-scale stochastic hybrid parabolic systems under jump Markovian perturbations-I: convergence and stability via Lyapunov functions

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S. Sathananthan

In this paper, the qualitative properties of the jump Markovian perturbations caused by the interactions among the states of a stochastic hybrid parabolic partial differential system are investigated. The concept of vector Lyapunov-like functions coupled with the decomposition-aggregation techniques are utilized to develop a comparison principle and sufficient conditions are established for various types of convergence and stability in the p-th moment and probability of the equilibrium state of the system under jump Markovian perturbations. This frame work of decomposition-aggregation is ideally suited for reducing the dimensionality problem arising in testing large-scale systems for the concept of convergence and stability. In addition, an example is given to illustrate the significance of the presented results.

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Stochastic perturbation of solutions in optical fibers

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The soliton perturbation theory is used to study and analyze the stochastic perturbation of optical solitons in addition to the deterministic perturbations that are governed by the nonlinear Schrodinger's equation. The corresponding Langevin equations are derived and analyzed. The deterministic perturbations that are considered here are both Hamiltonian as well as of non-Hamiltonian type. Finally, the soliton mean drift velocity is calculated in presence of these perturbation terms.

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Stability of singularly perturbed stochastic systems

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We consider the singularly perturbed stochastic system:
(1)

$$\begin{aligned} dx_t^\epsilon &= f_1(\epsilon, x^\epsilon)_t, y_t^\epsilon)dt + \sigma_1(\epsilon, x_t^\epsilon, y_t^\epsilon)dw_t \\ \epsilon dy_t^\epsilon &= f_2(\epsilon, x_t^\epsilon, y_t^\epsilon)dt + \sqrt{\epsilon}\sigma_2(\epsilon, x_t^\epsilon, y_t^\epsilon)dw_t \end{aligned}$$

where $\epsilon > 0$ is a small parameter, w_t the standard Wiener process. We give sufficient conditions for stability of system (1). They are formulated in terms of Liapunov functions for the reduced-order and the boundary layer system associated with (1). In this way we analyze the full order system by means of its lower order components and their interconnecting structure.

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Stability boundedness and tightness of stochastic flows

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TBA

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Occupation time large deviations of two dimensional symmetric simple exclusion process

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We prove a large deviations principle for the occupation time of a site in the two dimensional symmetric simple exclusion process. The decay probability rate is of order $t/logt$ and the rate function is given by $\Upsilon_\alpha(\beta) = (\pi/2)\{\sin^{-1}(2\beta - 1) - \sin^{-1}(2\alpha - 1)\}^2$. The proof relies on a large deviations principle for the polar empirical measure which contains an interesting log scale spatial average. A contraction principle permits to deduce the occupation time large deviations from the large deviations for the polar empirical measure.

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Nearly optimal impulsive controls for reflected wideband width process

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Near optimal control problem for a wideband noise process with impulsive controls and constrained to a bounded region will be considered. The method developed will show that sequence of physical processes converges (weakly) to a reflected controlled diffusion process with impulses as the "approximating parameter" goes to zero. The cost functional of the wideband system will also converge to the corresponding cost functional of the limit problem. Due to the reflection at the boundary, pseudo path topology will be used in the weak convergence analysis. This method will be applied to study control of a heavy traffic queuing system.

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Practical stability criteria for nonlinear stochastic systems by decomposition and aggregation

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In this talk, the concept of practical stability is investigated for the large-scale stochastic systems of Ito-Doob type. The concept of vector-Lyapunov like functions coupled with the decomposition-aggregation techniques are utilized to develop a comparison principle and, sufficient conditions are established for various types of practical stability criteria in the p-th mean and in probability. This framework of decomposition-aggregation is ideally suited for reducing the dimensionality problem arising in testing large-scale systems for the concepts of convergence and stability.

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Computational and Theoretical Issues in Fluid Dynamics

Organizer: Juan Lopez, Arizona State University
Jie Shen, University of Central Florida and Purdue University

Instabilities in swirling flows with walls: numerical modelling, simulations and analysis

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Olivier Czarny and **Emilia Crespo**

A direct numerical solver based on the spectral Chebyshev-Fourier approximation with a projection method is proposed and used for the analysis of a wide range of fluid dynamics instabilities involving rotation, walls and heat transfer. Some particular structures are emphasized in the Taylor-Couette system such as spiral and wavy vortex regimes. In the case of the rotor-stator configuration, different phenomena can be investigated depending on the aspect ratio, as transition to turbulence in the Bdewadt layer near the stator, or vortex breakdown along the rotation axis. New results concern the Kppers-Lortz instability due to the interaction of thermal convection and slow rotation effects.

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On projection method

Benyu Guo
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Jun Zou

The projection method was introduced by Chorin and Teman as an efficient algorithm for numerical solution of the Navier-Stokes equations. It is based on time splitting discretization which decouples the computations of the velocity and the pressure. The main advantages of this method are that it saves computational cost and preserves the incompressibility. The convergence analysis of projection method was initiated by Chorin and Teman. Shen obtained some results on the convergence rate. E and Liu considered two-dimensional, semi-periodic Navier-Stokes equations. The purpose of this work is to analyze the convergence rate of the fully discrete projection method and to improve the accuracy of the numerical solution by the pressure correction for the n dimensional, periodic problem of Navier-Stokes equations.

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Control of Lagrangian coherent structures

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It has recently been realized that the kinematics of turbulent mixing is governed by distinguished material lines that attract or repel fluid particles exponentially. These Lagrangian coherent structures are generalizations of stable and unstable manifolds known from steady and time-periodic flows. Because Lagrangian structures have a decisive impact on stretching, folding, and transport in the flow, controlling them locally can lead to global changes in mixing. In this talk I explore this idea via two examples. First, I describe passive control of coastal pollution spread using radar data of Monterey Bay. Second, I discuss active control of mixing in the wake of a bluff body flame holder.

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A particle method and adaptive treecode for vortex sheet motion in 3-D flow

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Keith Lindsay

A Lagrangian particle method is presented for computing vortex sheet motion in 3-D flow. The particles are advected using the Rosenhead-Moore form of regularized Biot-Savart kernel and the velocities are evaluated using a particle-cluster treecode based on Taylor approximation in Cartesian coordinates. The Taylor coefficients of the regularized kernel are computed using a recurrence relation. Several adaptive techniques are implemented to reduce the CPU time including variable order approximation, nonuniform rectangular cells, and a run-time choice between Taylor approximation and direct summation. The method is applied to simulate the azimuthal instability of a vortex ring and the merger of two vortex rings.

⟶ ∞ ⋄ ∞ ⟵

High order schemes for Navier-Stokes equations with spectral methods: numerical investigation of their comparative properties

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Leriche E., Perchat E. and **Deville M.O.**

Since the first numerical experiments were performed with the incompressible Navier-Stokes equations, solving the linear Stokes system has been of major concern with respect to its stability, accuracy and computational efficiency once the non-linear terms are explicitly treated as a given source. The question remains how to decouple the pressure and velocity fields in order to get an easily tractable numerical system with given stability and space-time accuracy properties. Many strategies have been proposed.

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Multiple scale models in complex fluids

Chun Liu
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In this talk, several dynamical systems modelling specific types of complex fluids are introduced. These models all involve multiple spatial scale effects. The relation between these and other existing models will be discussed. We will also study the relations between the variational procedure; the basic energy law; stability; and the higher order energy estimates. The different non-Newtonian properties such systems exhibit is of particular interest. We will discuss some analytical, as well as modelling problems in these models.

⟶ ∞ ⋄ ∞ ⟵

Complex dynamics in a short Taylor-Couette annulus

Juan Lopez
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F. Marques and **J. Shen**

Nonlinear dynamics of the flow in a short annulus driven by the rotation of the inner cylinder and bottom endwall is considered. The shortness of the annulus enhances the role of mode competition, and the associated dynamics are found to be organized by a number of local codim-2 bifurcations as well as global homoclinic bifurcations. The dynamics are explored using a 3D Navier-Stokes solver, which is also implemented in a number of invariant subspaces in order to follow unstable solutions and obtain a fairly complete bifurcation diagram of the mode competitions. (for special session 32).

⟶ ∞ ⋄ ∞ ⟵

3D Navier-Stokes equations with initial data characterized by uniformly large vorticity

Alex Mahalov
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We prove existence on infinite time intervals of regular solutions to the 3D Navier-Stokes Equations for fully three-dimensional initial data in R^3 characterized by uniformly large vorticity and infinite energy; smoothness assumptions for initial data are the same as in local existence theorems. The global existence is proven using techniques of fast singular oscillating limits and the Littlewood-Paley dyadic decomposition. Infinite time regularity is obtained by bootstrapping from global regularity of the limit equations and strong convergence theorems.

⟶ ∞ ⋄ ∞ ⟵

Dynamics in viscous vortex cores

Monika Nitsche
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Vortex blob simulations of vortex sheet roll-up have shown that the flow becomes chaotic in the vortex core, displaying resonance phenomena characteristic of slightly perturbed Hamiltonian systems. The resonance is induced by small self-sustained oscillations in the core vorticity, which are in turn attributed to the self-induced strainfield of the vortex. In this talk I will present the extent to which the chaotic dynamics are present in viscous vortices, and the dependence on the dimensional parameters of the flow. The results are based on 4th order finite difference simulations of the vortex evolution.

⟶ ∞ ⋄ ∞ ⟵

A new class of velocity-correction schemes for the incompressible Navier-Stokes equations

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We'll introduce a new class of velocity-correction projection schemes in the standard form and in rotational form. We will show that the velocity-correction schemes lead to slightly better error estimates compared with the usual pressure-correction schemes, but more importantly, the high-order versions of the velocity-correction schemes in rotational form are stable. Furthermore, we will show that the splitting-up schemes proposed by Orszag, Israeli and Deville (1986) and Kaniadakis, Israeli and Orszag (1991) can be recast as a velocity-correction scheme in rotational form. We will also present some numerical simulations of 3-D rotating flows.

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Residence times, chaotic advection and thrombosis in anatomically accurate arterial flows

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We are focusing on the application of dynamical systems theory concepts and tools in biothermofluidics. More specifically, we are investigating the mixing properties of blood when flowing within arterial segments of the cerebrum that suffer from multiple saccular aneurysms. When computational fluid dynamics techniques are applied on anatomically accurate reconstructions of such vessels, kinematic considerations yield novel evidence of spatiotemporal chaos. We show that residence time maps exhibit strong non-uniformity, linked to the entry patterns of the blood in the aneurismal sacs, but also to the strong divergence of flow lines within the sacs. Similar trends are observed when the basins of attraction of single or multiple aneurismal sacs are examined. A direct connection between the understanding that dynamical systems can offer for traditionally medical concepts, like thrombosis and pharmacokinetics, is made and quantitative results involving the thrombotic risk of particular malformations are provided, based on the above notions. The impact of such findings in connection with thrombosis and pharmacokinetics, but also when interventional planning is considered, can be quite significant, especially when similar procedures, capable of yielding such results, can be embedded in the daily clinical protocols.

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Accurate local time stepping algorithm for solving PDEs

Jianping Zhu
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In this presentation, we will discuss an accurate implicit local time stepping algorithms. The basic idea is to use proper interpolation and extrapolations to match solutions calculated using different time steps, and to advance solutions at all spatial grid points to the same time level. We will show that for systems of first order hyperbolic equations, an interpolation or extrapolation of order p-1 is needed to maintain consistency and accuracy of a time integration algorithm of order p. Numerical results will also be discussed in the presentation.

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Zeta Functions of Graphs and Related Topics

Organizer: Audrey Terras, U. of California

Ramanujan type graphs and bigraphs

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We will show that quotients of the Bruhat-Tits building of $SL_2(\mathbb{Q}_p)$ form an infinite family of graphs which are almost Ramanujan. We will also investigate the Bruhat-Tits tree associated with $U_3(\mathbb{Q}_p)$ and show why one should be able to estimate its spectrum.

⟶ ∞ ⋄ ∞ ⟵

Zeta functions of infinite graphs

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Start with a finite graph X. Take an infinite regular covering Y of X, with covering group Γ. Using the trace on the von Neumann algebra associated to Γ, there is an L^2 zeta function for Y which enjoys many of the properties of the Ihara-Hashimoto-Bass zeta function for finite graphs, including a version of the rationality formula. For families of finite graphs covering X, the normalized zeta functions of the finite graphs converge to the L^2-zeta function for Y.

⟶ ∞ ⋄ ∞ ⟵

Zeta functions of graphs, Shimura varieties, and dynamical systems

Jerome W. Hoffman
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This talk surveys the relations, known or conjectural, connecting zeta functions defined for three classes of objects - (hyper)graphs, modular varieties, and dynamical systems.

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Ramanujan hypergraphs

Wen-Ching Winnie Li
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A hypergraph is a higher dimensional generalization of a graph. In this talk we introduce the concept of Ramanujan hypergraphs and discuss explicit construction of such hypergraphs. These are extensions of Ramanujan graphs.

⟶ ∞ ⋄ ∞ ⟵

Random walks on knot diagrams

Xiaosong Lin
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We will discuss some features of the study of a model of random walks on knot diagrams. This model relates the Alexander and Jones polynomials in knot theory with the Ihara-Selberg type zeta function in number theory and graph theory.

⟶ ∞ ⋄ ∞ ⟵

Laplacians related to Zeta functions and an application to cogrowth of graphs.

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The concept of cogrowth of groups goes back to Ol'shanskii's settling of the von Neumann conjecture in 1984. Ol'shanskii constructed a group which was non-amenable (equivalently, its cogrowth constant was strictly less than the growth constant of its corresponding free covering) but was not an extension of a free group. The notion of amenability for finitely generated groups has been extended to arbitrary graphs and, by a suitable definition of cogrowth constant, we prove that a graph which is the cover of a finite graph is amenable if and only if its cogrowth constant equals the growth constant of its free cover. The proof uses harmonic functions with respect to operators of the form (I-uA+u^2Q) which are, by Bass' theorem, related to zeta functions on graphs.

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Isospectrality conditions for regular graphs

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In 1994, Hubert Pesce proved that two compact hyperbolic surfaces are (strongly) isospectral if and only if their covering groups are representation equivalent as subgroups of the automorphism group of hyperbolic 2-space. The proof depends on a certain length spectrum that can be described geometrically on the surface or algebraically in the covering group.We discuss the analogous length spectrum on regular graphs, looking at it combinatorially on the graph and algebraically in the covering group. We also note that our length spectrum appears in the Ihara zeta function.

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L-functions and the Selberg trace formula for semiregular bipartite graphs

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Hirobumi Mizuno

We give a decomposition formula for the L-function of a semiregular bipartite graph G. Furthermore, we present the Selberg trace formula for the above L-function of G.

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A new kind of Zeta function: when number theory meets graph theory

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The tree of zeta functions has many branches including those from number theory (Riemann and Dedekind zeta functions), spectral geometry of manifolds (Selberg's zeta function), and graph theory (Ihara's zeta function). It is also possible to mix in group representations and obtain L-functions. Applications include analogues of the prime number theorem and analogues of the work on what is now called quantum chaos - the statistics of energy levels of various non-classical physical systems. For example, the poles of the Ihara zeta function of a connected regular graph satisfy the Riemann hypothesis if and only if the graph is a Ramanujan graph (meaning that the second largest eigenvalue of the adjacency matrix, in absolute value, is in some sense smallest possible). In this talk I will compare the various sorts of zetas and investigate properties and applications of 3 types of graph zeta functions (the vertex or Ihara zeta, the edge and the path zetas) considered in Stark and Terras, Advances in Math., Vol. 121 (1996) and Vol. 154 (2000).

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Chaotic properties of quotients of trees (joint work with Audrey Terras)

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The geodesic flow on the k-regular tree produces induces a trajectory on the graph corresponding to a quotient of the tree. We look at the self correlation of the induced flow and show that it exhibits chaotic properties.

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Statistical Description of the Dynamics of Large and Disordered Systems

Organizer: Ilya Timofeyev, Courant Institute, NYU
Peter R Kramer, Rensselaer Polytechnic Institute(RPI)

Normal modes on average in stochastic systems

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In studying non linear Hamiltonian systems, the main attention has been generally posed on the range of validity of the near integrable behaviour, as outlined by the KAM and Nekhoroshev theories. Our approach, tested mainly on Fermi-Pasta-Ulam model but also on Lennard-Jones and Toda systems, consists in looking for the survival of some other kind of near integrable behaviour in the regime of motion where KAM theorem is no more applicable, i.e. above the so called strong stochastically threshold. We explore the persistence of a pseudo-harmonic spectrum (not connected to a perturbative approach), and the properties of some geometrical quantities as the Frenet-Serret curvatures, and their relations to dynamical properties. The stability of our results in thermodynamic limit has been numerically checked, always starting from generical initial conditions. The type of phase-space chaoticity in presence of pseudo-harmonicity has been studied by usual spectral methods on physically meaningful time series.

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Stages of energy transfer in the Fermi-Pasta-Ulam system

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Peter R. Kramer and **Yuri Lvov**

The (alpha) Fermi-Pasta-Ulam model of weakly coupled nonlinear oscillators was originally introduced to study the dynamical relaxation of a system toward thermodynamic equilibrium. The original simulations fell in a regime which famously failed to exhibit this convergence. We revisit the original question for larger systems at energies sufficiently large to allow the system to exhibit relaxation toward thermal equilibrium. Direct numerical simulations reveal several interesting stages in this process as the energy evolves from a large-scale excitation toward equipartition among all modes. We focus on characterizing these intermediate stages of energy transfer in physical terms.

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Power-law spectra for the damped and driven spectral truncation of the Burgers-Hopf model

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E. Vanden-Eijnden

Damped and driven Fourier-truncated Burgers-Hopf equations will be considered. The damping and white-noise forcing setup the inertial range in the problem and the power-law spectrum emerges. The asymptotic theory will be presented which allows to predict the spectrum in the limit of infinitely small damping and forcing. Numerical simulations confirm the analytical predictions with surprising accuracy.

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Statistically relevant conserved quantities for the truncated Burgers-Hopf equation

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Rafail V. Abramov and **Andrew J. Majda**

The importance of the cubic conserved quantity (Hamiltonian) for the equilibrium statistical mechanics will be discussed. By direct numerical simulation, corrections to energy equipartition will be computed that are due to the atypical values of Hamiltonian, which is different from the energy. Statistical-mechanical arguments will be presented explaining that for randomly-chosen initial conditions, these corrections are negligible. An alternative computation of the corrections to energy equipartition via a purely statistical-mechanical Monte-Carlo simulation will also be discussed.

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Application of weak turbulence theory to Fermi-Pasta-Ulam system

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Joseph Biello and **Yuri Lvov**

Weak turbulence theory was developed primarily to predict the properties of the stationary energy spectrum for weakly interacting turbulent waves in fluids, but the mathematical formalism can be applied to general weakly nonlinear Hamiltonian systems with diffuse coupling among modes. We adapt weak turbulence theory to obtain dynamical scaling predictions for the shape of the energy spectrum in the Fermi-Pasta-Ulam model at various intermediate stages of its evolution, and the time scales required to achieve these stages. To do this, we introduce some generalizations to weak turbulence theory which have broader application.

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Remarkable statistical behavior for truncated Burgers-Hopf Dynamics

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The Fourier-Galerkin spectral truncations of the inviscid Burgers-Hopf equation are introduced as simple one-dimensional models with a well-defined mathematical structure, intrinsic chaos and scale separation. Energy-based statistical mechanical predictions of energy equipartition and a correlation scaling theory that involves eddy viscosity will be presented. Computational results will be shown that confirm the predictions of these theories, and also shed further light on global phase-space properties of the Burgers-Hopf system, such as a number of exactly-obtainable invariant subspaces.

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A priori tests of a stochastic mode reduction strategy

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I. Timofeyev and **A. Majda**

A recently developed mode-elimination theory will be discussed, where the spectral truncation of the Burger-Hopf equation serves as a stochastic heat bath in various coupled prototype models. The non-linear self-interactions of the bath are first replaced by stochastic terms and then the fast variables are completely eliminated form the model allowing for the low dimensional reduced equations for slow variables. Numerical simulations of the original coupled model, intermediate stochastic model and reduced equations will be presented which verify the applicability of the mode-reduction strategy.

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Delay Differential Equations

Organizer: Hans-Otto Walther, Universitat Giesen, Germany

A class of evolution equations of Volterra type

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In this talk we consider equations of Volterra type in which the term under the integral is of the form $Au(s)$ where A is an unbounded operator. In his monograph on the subject, J. Pruess has collected some old results and also established some new ones about the Cauchy problem associated to that equation. In the linear case there is an analogue of the Hille-Yosida theorem for semigroups. The class of equations we are dealing with arises from the study of the shallow water equation. While it proved impossible for us to check the existence theorem stated by Pruess, we are able, using some properties of the equation at hand to derive the result with a completely different method. After a quick motivation for this equation, the talk will focus on explaining the method.

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Stability and approximation of linear retarded and neutral systems

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Janos Turi

We consider the linear delay equation

$$\frac{d[x(t)+Cx(t-r)]}{dt} = Ax(t) + Bx(t-r),$$

where A, B and C are constant $n \times n$ matrices. Using techniques from linear semigroup theory, we discuss conditions on the matrices A, B, C which guarantee exponential stability of the solution semigroup associated with this equation. We also discuss the solution semigroups which arise from certain finite dimensional semidiscrete approximation schemes for this equation.

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On reaction-diffusion systems with time delays

Wei Feng
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Xin Lu

In this talk we give an overview about our recent results on reaction-diffusion systems modelling the dynamics of single or interacting populations with time delay effects. For various models with constant or periodic parameters, competitive interactions and environmental toxicant, the issues of global stability, permanence, and asymptotic periodicity are discussed.

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Dynamics of delay equations with sine-like nonlinearity

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Roman Srzednicki

Delay equations $\dot{x}(t) = f(x(t-1))$ with a function f similar to the sine function are models for the control of high frequency oscillators. The variable x stands for the difference between desired and actual phase of the oscillator. Complicated dynamical behavior of such systems is observed in physical and numerical experiments. In joint work with Roman Srzednicki, we provide analytical proofs for such behavior in two different situations.

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On global stability and chaos in a family of scalar functional differential equations

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Sergei Trofimchuk and **Victor Tkachenko**

We consider a family of scalar delay differential equations $x'(t) = -\delta x(t) + f(t, x_t)$, with a nonlinearity f satisfying a sort of negative feedback condition combined with a boundedness condition. Some well-known delay differential equations as the Mackey- Glass equation, Nicholson's blowflies equation, and equations with maxima are kept within our considerations.

We present a global stability criterion for this family when $\delta \geq 0$, which in particular unifies the celebrated 3/2-conditions given for the Yorke and the Wright type equations. The sharp character of our condition is proved analyzing differential equations with maxima. For an appropriate choice of the parameters, we show the existence of solutions with chaotic behavior in this type of equations.

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Smooth invariant manifolds for differential equations with state-dependent delay

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We consider a class of delay differential equations where the right hand sides are not smooth in the usual sense, they are smooth only on spaces of smooth functions. Equations with state-dependent delay satisfy these conditions. We prove the existence and smoothness of local unstable and center manifolds near equilibrium points.

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Max-plus operators and differential-delay equations

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We shall discuss joint work with Professor John Mallet-Paret in which we consider the nonlinear differential-delay equation

(1) $\quad ax'(t) = f(x(t), x(t-r)), r := r(x(t)).$

Here f and r are given functions and $a > 0$. A simple-looking but non-trivial example to which our theory applies is provided by

(2) $\quad ax'(t) = -x(t) - kx(t-r), r := 1 + cx(t),$

where $a > 0$, $c > 0$, and $k > 1$. Under appropriate assumptions on f and r, we know that eq. (1) has, for all sufficiently small $a > 0$, a "slowly oscillating periodic solution" which depends on a. We are interested in the limiting shape of the graphs of such solutions as a approaches 0. In answering this question, we are led to the study of max-plus-equations:

$$x(t) + p = \max\{k(s,t) + x(t) : c(s) \leq t \leq d(s), 0 \leq s \leq L\} \quad (3)$$

In (3), k, c and d are given continuous functions, and one seeks an "additive eigenvalue p", and an "additive eigenvector $x(t)$" which give a solution of (3) on $[0, L]$. The presence of functions c and d makes eq. (3) much more subtle than in the case $c(t) := 0$ and $d(t) := L$.

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Convergence to equilibria in nonquasimonotone delay differential equations

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Consider the scalar autonomous functional differential equation

$$\dot{x}(t) = f(x_t),$$

where $f : C \to \mathbf{R}$ is continuous. Here $C = C([-r, 0], \mathbf{R})$, $r > 0$ and $x_t \in C$ is defined by $x_t(s) = x(t+s)$ for $s \in [-r, 0]$. Assuming that the solutions of the above equation generate a strongly order preserving semiflow with respect to the exponential ordering introduced by H. L. Smith and H.

Thieme, we present necessary and sufficient conditions for the generic and global convergence of the solutions.

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Dynamics of oscillations in multi-dimensional discontinuous systems with delay

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We consider a system of delay differential equations

$$\dot{x}_i(t) = F_i(x_1(t), ..., x_n(t), t) - \text{sign } x_i(t-h_i), i = 1, ..., n,$$

with positive constant delays $h_1, ..., h_n$ and perturbations $F_1, ..., F_n$ absolutely bounded by a constant less than 1. This system models a negative feedback controller of relay type intended to bring the system to the origin. Non-zero delays do not allow such a stabilization, but cause oscillations around zero level in any variable. We introduce integral-valued densities of zeroes of the solution components, and show that they always decrease to some limit values. We show that, for any prescribed limit zero densities, there exists at least an n-parametric family of solutions, and we establish certain conditions under which all slowly oscillating solutions are stable, and fastly oscillating solutions are not. Finally, we show that a modification of relay controllers (with arbitrary delays) can exponentially quench oscillations.

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Forward-backward functional differential equations, holomorphic factorization and applications

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J. Mallet-Paret

In this lecture we introduce and motivate the forward-backward functional differential equation

$$\dot{x}(t) = ax(t) - bx(t-1) + cx(t+1)$$

defined on the real axis. Such equations arise naturally in various contexts, for example, in the study of travelling waves in discrete spatial media such as lattices.

Since the mixed-type equation is not an initial value problem it is our goal to decompose solutions of this equation as sums of "forward" solutions and "backward" solutions. We show that the set of all forward solutions defines a semigroup which can be realized by a retarded functional differential equation except for possibly finitely many modes, and similarly for the set of backward solutions as an advanced functional differential equation. Holomorphic factorizations play a crucial role in our results.

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Smooth solution operators for differential equations with state-dependent delay

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It is shown how to obtain continuously differentiable solution operators for differential equations with state-dependent delay. This permits a geometric theory for such equations, beginning with smooth local invariant manifolds.

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A simple delayed neural network with large capacity for associative memory

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We consider both continuous and discrete models for a network of two neurons with delay feedback, and describe the multistability in the form of either multiple stable periodic orbits or unstable periodic orbits with large domains of attraction. We will compare the dynamics of discrete and continuous models, and address the issue of the lower bound for possible stable periodic orbits of an order-preserving and contractive map. We will also discuss potential applications in associative memory and periodic pattern recognition.

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Topological horseshoes and delay differential equations

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K. Wojcik

Consider a delay differential equation and an ordinary differential equation

$$x'(t) = f(x(t-\tau)) \tag{1}$$

$$x' = f(x) \tag{2}$$

where $x \in \mathbb{R}^n$,and $f \in C^\infty$.
We study the following question: Assume that equation (2) has chaotic solutions, i.e. a suitable Poincaré map has a Smale's horseshoe. Will the corresponding delay equation (1) also have chaotic solutions for small delays $\tau > 0$? We present a proof that the answer to this question is positive, namely for small delays a suitable Poincaré map, P_τ, for delay equation does have a topological horseshoe. We prove that this implies an existence of an invariant set S_τ for P_τ, such that P_τ on S_τ can be semiconjugated with a Bernoulli shift. Moreover we obtain an infinite number of periodic orbits with unbounded periods.

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Transport Phenomena

Organizer: Liqiu Wang, The University of Hong Kong

Transport of solutions in porous media: statement of the problem, numerical method and applications

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The analysis of the transport of chemical reacting solutions in porous media and its crystallization (dissolution) is of fundamental importance. The interest in this problem and development of various kinds of the models in recent years arise from practical requirements. In our opinion it is important to study the solutions transport in porous building materials under wetting and drying process conditions. It is also interesting to consider the kinetics of crystallization that influence on the solid zone formation. In the present study we consider the instantaneous kinetics. Thus, the purpose of this work is to investigate numerical method for the transport of solutions in porous building materials under wetting and drying process conditions. The statement of the problem and numerical method based on the implicit finite-difference scheme are presented. The convergence criterion of the iteration process was obtained. It is considered also the approximation error for the non-uniform mesh domain. The theoretical studies of the drying and wetting processes indicate the stability and high efficiency of the method. As application the transport of solution in ceramic brick is considered. The concentration distributions, evolution of the phase front and crystallization process are presented.

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PA link between chaos and vortex dynamics in a transitional boundary layer

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A model of the dynamic physical processes that occur in a transitional boundary layer flow is described. The CS-solitons, the closed vortex, the secondary closed vortex and the chain of ring-like vortices are postulated to be the basic flow structures of the transitional boundary layer as well as the turbulent boundary layer. It is argued that the central features of the transitional and developed turbulent boundary layer flows can be explained in terms of how the series vortices interact with each other, and with the CS-solitons. The physical process that leads to the regeneration of the new closed vortex along the border of the CS-soliton is described, as well as the processes of the evolution and the interaction of the vortices to high frequency vortices. The model is supported by recent important developments in the theory of unsteady surface layer separation and a number of kernel experiments which serves to both transitional and developed turbulent boundary layer. An important aspect of the model is that it has been formulated to be consistent with accepted rational mechanics concepts that are known to provide a proper physical description of other flows. By the way, a link between chaos and the transitional dynamic system is established. The result shows that fractal dimensions can be used to describe all the transitional processes as a necessary factor such as Reynolds number.

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On the appearance of film ruptures in plain thick layers of moving liquid

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The film rupture (which closely related to the cavitation) can appear in the thick layers of moving liquid even for sufficiently small Reynolds numbers. The cavitation is the phenomenon of liquid boiling and/or escape of dissolved gases from the liquid. These two types of cavitation are commonly referred as vapor and gaseous cavitation. The gaseous cavitation is connected with the fact that the real liquid usually contains up to 10% by volume of dissolved air or other gases. If the pressure in liquid decreases sufficiently then the dissolved gas comes out and forms gas bubbles or cavities. This phenomenon encountered in such part of moving liquid where the liquid pressure drops. The gaseous cavitation changes the parameters of movement, but does not damage moving surfaces. In contrary, vapor cavitation is an origin of fatigue-type damaging effect on moving surfaces. The physical origin and effects of these types of cavitations are different, but for the following we do not distinguish its. In both cases the one-phase liquid flow became two-phase flow in which liquid and gas phases are separated by some surfaces. The transition from one-phase flow to two-phase flow must satisfy general physical conservation laws specifically, laws of mass and energy conservation. We consider the appearance of film rupture in the plain layers of moving liquid been an isothermal processes, where the heating of liquid is ignored since heat transfer is a sufficiently slow process in comparison to the rapid appearance of film rupture. Alternatively, we consider steady state operating conditions. The thickness of liquid layer is assumed to be so small that it is possible to consider a liquid movements as a two-dimensional one. Four typical two-dimensional cases of the thick layers of moving liquid are considered. In these cases the thick layer of liquid is assumed to be between : an inclined plane moving over an immobile plane, a circle rotating over an immobile plane, a rotating circle no coaxial with another immobile circle, two rotating circles. Based on energy and mass balances the conditions under which liquid film rupture (or cavitation) can or cannot occur in the thick layers of moving liquid are derived for four cases mentioned above. Expressions depending on geometrical parameters and on parameters of movements where film rupture might appears, are given. Some applications to the plain journal bearings are also presented.

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A critical review of turbulence modelling: physical constraints and physics-preserving models

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The present review, which includes some new material, consists of two parts: physical constraints in turbulence modelling and physics-preserving turbulent closure models that preserve the frame indifference and satisfy both the principle of material frame indifference (PMFI) and the second law of thermodynamics. The former commences with careful definition of the average operation, the Reynolds stress, the turbulent heat flux and the turbulent mass flux. The remainder of this part is on the developments, to date, of three physical constraints imposed by the invariance, the realizability and the PMFI. In particular, two sufficient conditions are discussed for the Reynolds stress tensor and the turbulent heat/mass flux vector to be frame indifferent. The application of the second law of thermodynamics to a thermally isolated system and an irreversible process concludes that realizability inequalities in turbulent flows follow logically from the second law of thermodynamics. How system rotations affect flow fields is critically examined from a basic theoretical standpoint. Also critically reviewed is the literature on the range of validity of the PMFI to the turbulence modelling. The latter is devoted to the progress, to date, of physics-preserving closure models of the Reynolds stress and the turbulent heat/mass flux. In particular, both necessary and sufficient conditions are developed in a systematic, rigorous way for turbulent closure models to satisfy the three constraints reviewed in the first part. The results have either confirmed some intuitive arguments or offered new insights into turbulence modelling, and are of significance in clarifying some controversies in the literature, examining how well existing models preserve the physics, and developing new models. Also developed are a linear theory, a quadratic theory and a flow decomposition theorem to simplify and guide the work of developing specific physics-preserving models. This part ends with a further constraint on the physics-preserving models by the disappearance of Reynolds stress and turbulent heat/mass flux at a vanishing value of the mean velocity. Most methods and results in the present review are also valid for the higher-order correlations and the subgrid-scale (SGS) modelling in the large eddy simulation (LES).

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Burgers' Weak Turbulence by Dynamical Systems Approach

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In the present work, the Green's function of the one-dimensional heat conduction equation is employed to convert Burgers' equation into the infinite dynamical system described by an integration equation system with respect to the time variable. The infinite dimensional system is truncated to finite dimensional dynamical systems. These systems are solved by a numerical method proposed in the present paper. The calculated results not only describe the scenario of the route to the Burgers' turbulence, but also reveal a new explanation for the mechanism of the occurrence of the intermittence in the turbulence. In addition, these results demonstrate a similar behavior for the dynamical systems with dimensions ranging from 30 to 450 that are utilized to approximate the Burgers' equation. This shows some validity of using finite dynamical systems to approach the transition process from laminar flow to the turbulence.

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Solution filtering technique for solving Burgers' equation

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Burgers' equation is a one-dimensional (1-D) analogue of the Navier-Stokes (N.–S.) equations; it embodies all the main mathematical features of the N.–S. equations. In the present study, we test the solution filtering technique by solving Burgers' equation numerically. The solution filtering technique is a new approach proposed for dealing with aliasing of underresolved solution as arise in large eddy simulation (LES) of turbulence. The idea underlying the solution filtering technique is that filtering aliased solutions is far simpler than dealing with the consequences of filtering nonlinear differential equations. The basic approach of the solution filtering technique is that the governing equations (Navier–Stokes equations) are not filtered and are solved directly on a grid system that is much coarser than required by direct numerical simulation; then the solution at each time step is filtered. Our present studies show that such an approach works quite well for Burgers' equation. In spite of the fact that the research carried out in the present studies employs Burgers' equation rather than the full N.–S. equations, from previous studies we believe that the results obtained from Burgers' equation will apply to the N.–S. equations, at least in a general way. Therefore, it is expected that the solution filtering technique will possess significant potential in solving the practical turbulent problem governed by the three-dimensional (3-D) N.–S. equations.

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Shear force distribution and heat transfer in laminar boundary layer flows for power law fluid

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Analytical and numerical solutions are presented for the momentum and energy laminar boundary layer equations in power law fluid utilizing a similarity transformation and the shooting technique. The results indicated that for power law exponents $0 < n \leq 1$, the skin friction σ decreases with increasing n , and the dimensionless shear force decreases with increasing in dimensionless velocity t . When $N_{PT} = 1$, the velocity distribution in the viscous boundary layer is the same as the temperature distribution in the thermal boundary layer and $\delta = \delta_T$. For $N_{PT} > 1$, the increase of the viscous diffusion exceeds that of thermal diffusion with increasing N_{PT}, i.e. $\delta_T(t) < \delta(t)$. The thermal diffusion ratio increases with increasing $n(0 < n \leq 1)$.
Key words: power law fluids; boundary layer, skin friction; similarity solution; shooting technique.

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Quasilinear Elliptic and Parabolic Differential Equations and Their Applications

Organizer: Raul Manasevich, Universidad de Chile
James Ward, University of Alabama at Birmingham

Quasi-linear elliptic problems with superlinear non-linear terms

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The scope of this talk is to discuss some new results about the existence of solutions for a class of nonlinear boundary value problems with zero Dirichlet boundary condition in bounded domains. One of the more interesting features is that the main part of the quasi-linear differential operators considered in the talk is of the type div $(A(x,u)\nabla u)$, being, for instance, $A(x,u) = (a_{ij}(x,u))$, $i,j = 1,\ldots,N$ a symmetric matrix with coefficients $a_{ij} : \Omega \times \mathbb{R} \to \mathbb{R}$ which are Carathéodory functions. First, we survey some results obtained in collaboration with **J. Carmona (Univ. of Almería, Spain)**. Among other hypotheses, we assume that there exist positive constants α and β satisfying for every $(s,\xi) \in \mathbb{R} \times \mathbb{R}^N$ and a.e. $x \in \Omega$,

$$|A(x,u)| \le \beta,$$

$$A(x,u)\xi \cdot \xi \ge \alpha|\xi|^2.$$

By using degree arguments, we extend to the equation $-\text{div}\,(A(x,u)\nabla u) = f(x,u) + t\varphi_1$ ($f(x,u)$ a Carathéodory nonlinearity and φ_1 a positive bounded function) the semilinear results of Ambrosetti-Prodi. As a corollary of the previous multiplicity result in the super-linear Ambrosetti-Prodi framework ($\lim_{u\to+\infty} f(x,u)/u = +\infty$), we also extend to the quasi-linear case the classical result by Ambrosetti-Rabinowitz about the existence of positive solution for semi-linear elliptic problems. We conclude by studying some variational class of problems in which the differential operator has a degenerated coerciveness ($\alpha = 0$). In this case, we survey the results obtained in collaboration with L. Boccardo and L. Orsina (Univ. of Roma, Italy). Here, two are the main difficulties: first, $u = 0$ is not a local minimum in the H_0^1-topology (because of the lack of coerciveness of the main part of the differential operator) and, second, the associated functional is not of class C^1 (since $A(x,u)$ depends on u). To overcome these, we use a suitable version of the Mountain Pass Theorem.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Spike solutions and Morse indices

Peter Bates
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Junping Shi

An abstract framework is outlined to study the existence and stability of spike-layer solutions to a class of singular perturbation problems.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the existence of periodic solutions to a damped second order ODE at resonance

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Z. Wang.

We consider a second order differential equation of the form

$$u'' + f(u)u' + n^2u + h(u) = p(t),\ n \in \mathbf{N}.$$

For $F(x) = \int_0^x f(s)\,ds$, we assume that the limits $\lim_{x\to\pm\infty} F(x)$, $\lim_{x\to\pm\infty} h(x)$ exist and are finite and we provide sufficient conditions for the existence and non-existence of periodic solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Threshold effects in logistic models via nonlinear diffusion

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R. S. Contrell

The logistic equation with passive diffusion predicts either that all positive solutions approach zero or that all positive solutions approach a unique positive equilibrium as time goes to infinity, depending on coefficients, spatial domain, and boundary conditions. The same is true if the diffusion coefficient is increasing with respect to the population density. However, in the case of a diffusive logistic equation where the diffusion coefficient is decreasing with respect to the density when the density is small, there may be multiple positive equilibria. Those can lead to threshold effects where solutions with small initial data decay toward zero but solutions with larger initial data go toward some nonzero equilibrium. (In the ecological literature this is known as an Allee effect.) Thus, the presence of certain types of nonlinear diffusion can qualitatively change the dynamics of the equation even with no change in the reaction terms. Nonlinear diffusion where the diffusion rate decreases with density, at least at low densities, has been proposed by theoretical ecologists as a possible mechanism for aggregation of populations. Thus, the mathematical analysis suggests that aggregative movement can lead to the shold effects in population growth even if those are not present in the terms describing local population dynamics. The main mathematical tools used to obtain this result are bifurcation theory and classical methods in the theory of partial differential equations.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On subcriticality assumptions for the existence of ground states of quasilinear elliptic equations

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We study conditions on f which ensure the existence of nonnegative nontrivial radial solutions vanishing at infinity of the quasilinear elliptic equation $-\Delta_p u = f(u)$ in R^n with $n > p > 1$. Both the behaviors of f at the origin and at infinity are important. We discuss several different subcritical growth conditions at infinity and we show that it is possible to obtain existence results also in some supercritical cases.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Asymptotic behavior of the boundary layers of semilinear elliptic eigenvalue problems

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We consider the nonlinear eigenvalue problem

$$-\Delta u = \lambda f(u), \ \ u > 0 \quad \text{in } \Omega, u = 0 \quad \text{on } \partial\Omega$$

where $\Omega = B_R = \{x \in \mathbf{R}^N : |x| < R\}$ or $A_{a,R} = \{x \in \mathbf{R}^N : a < |x| < R\}$ $(N \geq 2)$ and $\lambda > 0$ is a parameter. It is known that under some conditions on f, the corresponding solution u_λ develops boundary layers when $\lambda \gg 1$. We establish the asymptotic formulas for the width of the boundary layers with exact second term and the estimate of the third term.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On the Willmore flow

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The Willmore flow is a geometric evolution law which describes the motion of a surface under mean curvature. It evolves surfaces in such a way as to reduce the total quadratic curvature. The Willmore flow leads to a parabolic quasilinear evolution equation of fourth order. In this talk, existence and regularity of solutions will be discussed. Numerical simulations will be provided which indicate that the flow can develop singularities in finite time.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Several kinds of solutions for a FitzHugh-Nagumo type elliptic system

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C. Reinecke and **W.C. Troy**

We consider the following system of semilinear elliptic partial differential equations $-\varepsilon^2\Delta u = f(u) - \delta v$ in Ω, $-\Delta v = u$ in Ω and $u = v = 0$ on $\partial\Omega$, where Ω is a bounded smooth domain in $\mathbb{R}^N$, $N \geq 2$, and where the function $f(u) = u(1-u)(u-a)$, with $a \in (0, \frac{1}{2})$, is the generic nonlinearity. The parameter ε is some small positive number. We are interested in the relation between $\delta > 0$ and the solution. When $\delta > 0$ the minus sign in front of v in the first equation makes the system nonquasimonotone and as such no direct order preserving properties through a maximum principle exist. Using the existence of 2 positive solutions for $\delta = 0$, roughly described as a 'boundary layer solution' and a 'peak solution', a continuation argument generically yields two nontrivial solutions for $\delta > 0$ and small. We have been able to prove that for a certain range of positive δ a solution exists that has a completely different behaviour. This solution is the minimizer of an appropriate energy functional.

Numerical evidence leads to a function u with both a boundary layer and an internal layer.

⟶ ∞ ⋄ ∞ ⟵

N-bump pulse solutions of an integral equation in neuroscience

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We investigate the existence of N-bump pulse solutions of an integral equation which models short term memory. Experiments with Auto97 indicate that a snaking phenomenon occurs in the bifurcation diagram for families of N-bump stationary solutions in one space dimension. Numerical evidence for the existence of stable multi-bump solutions in two space dimensions will be presented.

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Uniqueness and exact multiplicity for elliptic equations

Moxun Tang
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In this talk I will present some new results and techniques on uniqueness and exact multiplicity for solutions (positive or sign-changing) of elliptic equations.

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Geometric Methods in Dynamical Systems

Organizer: Dmitry Zenkov, North Carolina State University
Anthony Bloch, University of Michigan

The orientational degeneracy of nematic liquids and its role in the dynamical response to shear flow

Gregory Forest
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Nematic liquids at rest spontaneously order above a critical concentration, or below a critical temperature. However, whereas the degree of molecular orientation is specified by the strength of the intermolecular potential, the direction of order is completely arbitrary. This is called orientational degeneracy. In this lecture I will describe this degeneracy in terms of symmetries, and then explain how this group of symmetries is broken by applied flow fields. We give explanations from both molecular (kinetic) theory and mesoscopic, moment-averaged tensor theories for nematic liquids. The lecture is based on joint work with Qi Wang, Florida State University, and Ruhai Zhou, UNC-Chapel Hill.

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Parabolic PDEs and spaces of braids

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Finding stationary solutions to scalar parabolic PDE's is a problem with deep geometric and topological aspects. I discuss a Morse theoretic approach using braids that gives a broadly applicable forcing theory. Furthermore, the forcing theory allows for rigorous theorems from discretized computations.

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Non-Abelian Toda lattices

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Tony Bloch and **Michael Gekhtman**

In this talk, we compare the behavior of several non-Abelian generalizations of the Toda lattice. These generalizations are obtained by replacing elements of the matrices in the Lax pair form of the Toda lattice by matrices. We show that these differential equations exhibit sorting behavior similar to, but different from, the sorting behavior of the original tridiagonal Toda lattice.

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Euclidean extensions of dynamical systems

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Recently there has been substantial progress in understanding the ergodic and mixing properties of generic group extensions of dynamical systems. Such models occur frequently in applications and the study of equivariant dynamical systems. A group (Γ) extension of a base dynamical system consists of a map or flow on a manifold X, a skewing function $g : X \to \Gamma$ and a skew-product map or flow on the product space $X \times \Gamma$. We investigate the behaviour of $E(n)$ (the Euclidean group of rotations, reflections and translation in n-dimensional space) extensions of various types of base dynamics on X. In particular we consider the topological and statistical properties of $SE(n)$ extensions of periodic, quasiperiodic and chaotic dynamical systems.

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The n-symplectic Hamilton-Jacobi equations

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This paper addresses some aspects of the general problem of setting up a geometric quantization theory of fields based on n-symplectic geometry. The main issue here is to identify the structures that will play the roles of the polarizations that occur in the Kostant-Souriau theory. We consider an $n = (m+k)$-dimensional fiber bundle $\pi : E \to M$ where typically M is an m-dimensional "spacetime manifold". Sections of π are then the physical fields of the theory, and a Lagrangian for the field theory is then a function $\mathcal{L} : J^1\pi \to R$ where $J^1\pi$ is the bundle of 1-jets of sections of π. In the n-symplectic formulation one considers the bundle $L_\pi E$ of adapted linear frames of E, which is an $H = GL(m)\times GL(k)$ principal bundle $\rho : L_\pi E \to J^1\pi$ over the bundle of 1-jets. Using ρ we can pull-up the Lagrangian $\mathcal{L}$ on $J^1\pi$ to a Lagrangian $L : L_\pi E \to R$ on $L_\pi E$, and then use L and the canonical soldering 1-forms θ^α, $\alpha = 1, 2, \dots, n$ on $L_\pi E$ to construct the Cartan-Hamilton-Poincaré (CHP) 1-forms θ^α_L that play the role of the n-symplectic potentials for the theory. We use this structure to set up Hamilton-Jacobi equations in order to find the locally defined generating functions for the generalized polarizations.

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Numerical Computation and stability of Riemann solutions via the Dafermos regularization

Stephen Schecter
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We consider a system of conservation laws

$$u_t + f(u)_x = 0, \quad u \in \mathbb{R}^n, \tag{1}$$

a viscous regularization

$$u_t + f(u)_x = (B(u)u_x)_x, \tag{2}$$

and the corresponding Dafermos regularization

$$u_t + f(u)_x = \epsilon t(B(u)u_x)_x. \tag{3}$$

Geometric singular perturbation theory can be used to show that structurally stable Riemann solutions of (1), with shock waves that satisfy the viscous profile criterion for the given viscosity, lie near *Riemann-Dafermos solutions* of (3), *i.e.*, solutions that depend only on $\frac{x}{t}$. We present a numerical method that uses this fact and AUTO to compute curves of approximate Riemann solutions of (1). By continuing the computation through folds, the method can compute multiple Riemann solutions to a single Riemann problem. We also discuss the linearized stability of Riemann-Dafermos solutions of (3).

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Nonholonomic Euler-Poincaré equations and stability in Chaplygin's sphere

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A theory of reducing several classes of nonholonomic mechanical systems that are defined on semidirect products of Lie groups is discussed. The method reduces the Lagrange-d'Alembert principle to obtain a reduced constrained principle that determines Euler-Poincaré equations on the reduced space. We then use the theory as a framework to study a particular nonholonomic system: Chaplygin's sphere (a ball that rolls without slipping on the plane and whose moments of inertia may differ, but has center of mass coinciding with the center of the ball). Several results are discussed, the stability of relative equilibria and stabilizing the unstable equilibria with feedback control.

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Cycles in quadratic systems

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Let $X = (P, Q)$ denote a vector field on the plane, where P and Q are degree n polynomial functions of x and y, or equivalently the corresponding system $x' = P(x, y)$, $y' = Q(x, y)$ of ordinary differential equations. A problem of fundamental interest is the number and location of cycles (closed orbits) in the phase portrait of X, and their creation and destruction as the coefficients of P and Q are varied. Only as recently as 1989 was it demonstrated that there can be at most finitely many limit cycles (isolated closed) orbits in the phase portrait of any fixed system X. The first non-trivial situation is the quadratic case, n=2, and even here it is still unknown if there is a uniform bound on the number of limit cycles possible in any quadratic system. In this talk we show that for any pair of cycles c_1, c_2 in quadratic systems X_1, X_2, there is a smooth arc in the space of quadratic systems connecting X_1 and X_2 in such a way that c_1 is smoothly deformed into c_2, avoiding period annuli of centers and algebraic ovals of low degree. We show how the theorem can be used to establish geometric properties of quadratic cycles. (Joint work with A. Zegeling.)

$\longrightarrow \infty \diamond \infty \longleftarrow$

Unipotent group actions on flag manifolds and the Toda lattice

Barbara Shipman
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We will consider the hierarchy of Toda lattice equations $\dot{X} = [X, \pi(X^k)]$, where $k = 1, \ldots, n-1$, X is a complex $n \times n$ matrix with trace zero and arbitrary entries below the diagonal, 1's on the first superdiagonal, and zeros elsewhere, and π is the projection of a matrix onto its strictly lower triangular part. We study the flows of this hierarchy through points in the phase space where eigenvalues coincide; this situation is different from the generic case in which the eigenvalues of the initial matrix are distinct and the flows are very well understood. Once the spectrum (the eigenvalues, together with their multiplicities) is fixed, one obtains an isospectral subset of the phase space and an embedding of this subset into the flag manifold, $Sl(n, \mathbf{C})/B$, where B is the lower triangular subgroup. In the flag manifold, the $n-1$ flows generate the action of a group that is homeomorphic to $(\mathbf{C}^*)^{r-1} \times \mathbf{C}^{n-r}$, where r is the number of distinct eigenvalues that the spectrum contains. When $r = n$, the group is the complex diagonal torus, whose action on the flag manifold appears in many classical areas of mathematics and has been very well studied. Here we consider the case when $r < n$, where the group generated by the flows is a product of a diagonal torus and a unipotent group.

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Linear conservation laws of nonholonomic systems

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In nonholonomic dynamics, symmetries do not always lead to conservation laws as in the classical Noether theorem. Instead, the momentum of a generic nonholonomic system satisfies a dynamic momentum equation. This momentum equation can in some situations produce non-obvious conservation laws. In this talk, we overview the structure of the momentum equation and discuss conditions that imply existence of the conservation laws that are linear in the components of the nonholonomic momentum.

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Contributed Sessions

A reaction-diffusion model of HIV infection

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Inkyung Ahn and **Lige Li**

We attempt to develop a PDE system using reaction-diffusion equations to model the HIV infection of the immune system. The conditions are given for the existence of a unique strictly positive solution and for the steady-states above certain levels. The existence of the w-limit of the PDE model is then discussed. Also, the L2 and L(infinity) bounds for the immune cells and for the virus are estimated. A possible important role of stem cells in stabilizing and controlling the disease is suggested.

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Traveling wave solutions for a tissue interaction model on skin pattern formation

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We shall discuss the existence and the uniqueness of traveling wave solutions for a tissue interaction model on skin pattern formation proposed by Cruywagen and Murray. The model system consists of two coupled singularly perturbed differential equations, one is fourth order and one is second order. Both analytical and geometrical methods will be presented for the existence proof.

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Nonlinear dynamics of pathogenesis and prognosis factor

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Prabhakar Vaidya

A mathematical model showing the interaction of normal cells, cancer cells and food supply is shown to yield interesting results. This model is extended to a more general case of pathogenesis. From these models a prognosis factor is computed that relates to the eventual outcome of the disease.

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Attracting behavior of solutions for nonlinear Volterra integral equations

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R. Benitez

We describe the strong relations between the uniqueness of positive solutions for nonlinear Volterra integral equations with convolution kernels and the attracting behavior of these kind of solutions. Kernels considered here are positive and locally bounded functions.

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Instability of difference equations

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In many respects the behaviour of solutions of difference equations parallels that of solutions of related ordinary differential equations and much research has been devoted to establishing difference equation analogues for known results concerning differential equations. Nevertheless, there are many instances in which the discrete and continuous theories diverge. A discussion of cases in which an analogy that might be expected between difference equations and ordinary differential equations does not hold is desirable. This paper discusses the instability of solutions of autonomous and nonautonomous difference equations. The proved theorems are in a sense analogues of the classical instability theorems of Chetaev for ordinary differential equations. An example is provided showing that the straightforward extension of Chetaev's theorems to difference equations is false.

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Attracting behaviour of solutions for nonlinear Abel integral equations

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M. R. Arias

We present some results about the attracting behaviour of solutions for nonlinear Abel integral equations with non locally bounded kernels. The aim of the results presented here is to study the relation between the uniqueness of solutions and their attracting behaviour.

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Intermingled basins in population models

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Abdul-Aziz Yakubu

Intermingling occurs when two chaotic attractors have basins that are riddled in each other, so that neither basin contains an open set though their union may have full Lebesgue measure. Rigorous proofs of intermingled basins in maps are uncommon in the literature. In a recent paper, Hofbauer et al. obtained a framework for the support of multiple attractors with intermingled basins of attraction. In this talk, we will prove the existence of multiple attractors with intermingled basins in a new set of examples coming from population dynamics, where intermingling of the basins corresponds to indeterminate competition. We will also report our investigations of intermingling in more general competitive systems that feature multiple persistent invariant submanifolds.

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Influence of non-Gaussianity in system properties on the transient response statistics of beam structures

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This paper aims to investigate the influence of non-Gaussian inhomogeneities in the system properties of real life engineering structures on their dynamic response statistics. The modelling of the system property random fields is done from an information theoretic approach using maximum entropy principle. Response characteristics under transient loads are discussed only. The transient dynamic analysis is done on two problems, of wide engineering interest, namely, vibration of a random beam subjected to a moving load and vibration of an axial rod subjected to an axial impact. Extensive numerical studies on the above examples are conducted with an aim to find out the effect of non-Gaussianity in system properties on the transient response statistics. Numerical results show that the response statistics obtained from the direct simulation using dynamic stiffness matrix for both Gaussian and non-Gaussian inhomogeneities of the system properties in general compare well insofar as the mean and standard deviation of the responses are concerned. However, if one is interested in the behaviour near the tails of the probability distribution function and higher order response moments, the effect of non-Gaussianity is discernible.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Topological properties of solution sets of generalized order

Daria Bugajewska
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In this talk we will consider the following problems for differential equations of generalized order, namely:

(1)
$$x^{(\alpha)}(t) = f(t,x), \quad \alpha \in (0,1]. \qquad x^{(\alpha-1)}(t_0) = x_0, \qquad t \in I,$$

(2)
$$x_i^{(\alpha)} = f_i(t, x_1, \ldots, x_n)), \quad \alpha \in (0,1]. \qquad x_i^{(\alpha-1)}(t_0) = x_i^0, \qquad i = 1, \ldots, n, \;\; t \in I,$$

and

(3)
$$x^{(n\alpha)}(t) = f\left(t, x, x^{(\alpha)}, x^{(2\alpha)}, \ldots, x^{((n-1)\alpha)}\right), \qquad x^{(k\alpha-1)}(t_0) = x_k^0, \qquad \text{for } \alpha \in (0,1] \text{ and } \quad k = 1, \ldots, n, \;\; t \in I,$$

where $I = (t_0, t_0 + a]$ is a bounded interval in $\mathbb{R}$ ($a > 0$). We shall investigate topological structure of solution sets of the problems (1)-(3). More precisely, our purpose is to establish that Aronszajn's property holds for these problems. Recall that a given differential equation has the Aronszajn property if there exists an interval such that the set of all solutions of this problem, defined on this common interval, is an R_δ, that is, it is homeomorphic to the intersection of a decreasing sequence of compact absolute retracts. In particular, it is nonempty, compact and connected and it has the same cohomology group as a single point. We shall consider solutions of (1)-(3) which satisfy given equation either everywhere or almost everywhere.

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On some applications of the spectral radius of linearly bounded operators to functional-differential equations

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In this talk we present some conditions, different from the global commutativity, under which one can estimate the spectral radius of the composition of linearly bounded operators. Next we show some applications of these estimations to theorems about the existence of global solutions of the following functional-differential equations:

$$\begin{aligned} &\frac{\partial^2 z}{\partial x \partial y} = f(x, y, \max_{D(h(x,y))}\{z(t,s)\}, z(H(x,y))), \\ &z(x,0) = 0, \\ &z(0,y) = 0, \\ &\quad \text{for } (x,y) \in I^2, \end{aligned}$$

and

$$\begin{gathered} x_1'(t) = f_1(t, (x_1 + x_2)(h(t)), x_1'(H_1(t))) \\ x_2'(t) = f_2(t, \max_{[0,H_2(t)]}\{x_2'(s)\}, \max_{[0,H_3(t)]}\{x_2'(s)\}),\ t \in I, \\ x_1(0) = x_2(0) = 0, \end{gathered}$$

where $I = [0,1]$ and $D(x,y) = \{(t,s) \in I^2 : 0 \leq t \leq x, 0 \leq s \leq y\}$. We illustrate obtained results by suitable examples.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Non-integrable geodesic flows with zero entropy

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Benardete and Mitchell showed how to define a rotation vector that takes its values in a 2-step nilpotent Lie group. Their construction is used here to obtain necessary conditions for a flow to be integrable with C^0 first integrals. These conditions are shown to imply the non-integrability of certain geodesic flows, which are also shown to have zero topological entropy.

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Oscillatory behavior of N-th order nonlinear differential equations with continuous delay

Tuncay Candan
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R. S. Dahiya

We study the oscillatory behavior of N-th order nonlinear neutral differential equations with continuous delay. We develop certain theorems related to the oscillatory behavior with application.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Analysis on a reaction diffusion system

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This talk is concerned with positive solutions of a reaction diffusion system which blow up in a finite time. We obtain the blow-up rates and derive a classification of blow-up patterns. Special attention is given to the analysis of asymptotics of solutions near blow up.

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Periodic perturbation of the Lienard equation

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Consider the Lienard system $u'' + f(u)u' + g(u) = 0$ with an isolated periodic solution. We are interested in the behavior of periodic solutions of Lienard system under small periodic perturbations.

$$\ddot{u} + f(u)\dot{u} + g(u) = \epsilon\gamma(\frac{t}{\tau}, u, \dot{u})$$

where $t \in R$, $\epsilon \in R$ is a small parameter, $\mid \epsilon \mid < \epsilon_0$, τ is a real parameter such that $\mid \tau - \tau_0 \mid < \tau_1$ for some $0 < \tau_1 < \frac{\tau_0}{2}$.
One refers for that to a method due to Farkas inspired by the one of Poincaré. The determination of controllably periodic perturbed solution. We know for example a good application for perturbed Van

der Pol equations type. The perturbation is supposed to be 'controllably periodic', i.e., it is periodic with a period which can be chosen appropriately. Under very mild conditions it is proved that to each small enough amplitude of the perturbation there belongs a one parameter family of periods such that the perturbed system has a unique periodic solution with this period. The Farkas method can be again applied for perturbed Lienard equations.

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The behavior of the period function of Lienard systems

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The Lienard equation

$$(L) \qquad x'' + f(x)x' + g(x) = 0,$$

is not in Hamiltonian form and new result on the monotonicity of their period function can appear particularly interesting.
We propose here to examine monotonicity conditions of the period function for the Lienard equation where the functions f and g are such that $f \in C^1(J,R)$ and $g \in C^3(J,R)$ on the interval $J = (a,b), \quad a < 0 < b$ and satisfying some conditions so that the origin 0 is a center of this equation. It implies in particular $g'(0) > 0$. The period $T \equiv T(\gamma)$ where γ is a cycle surrounding 0 is defined. We prove that

$$g'(0)g^{(3)}(x) - \frac{5}{3}g''^2(x) - \frac{2}{3}f'^2(x)g'(0) \neq 0$$

implies the monotonicity of the period function T in a neighborhood of 0 . In particular, $g'(0)g^{(3)}(0) - \frac{5}{3}g''^2(0) - \frac{2}{3}f'^2(0) = 0$ and $f'(0)g''(0) - g'(0)f''(0) = 0$ are necessary conditions for the center 0 to be isochronous.
When g is of class C^3, our criteria appears in a certain direction to be more general than those given by Sabatini for the period function to be monotonic.

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In situ Sturm-Liouville operators with interior singularities

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Valentino A. Simpao

As an application of some recent results for Sturm-Liouville operators over domains with interior singularities, we consider the inter-play between the various inhomogeneities in the material structure/frequency response characteristics of various wave equations for variable media.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Green's functions for Sturm-Liouville equations with interior singularities

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For the Sturm-Liouville differential equation on the real line with countably many interior singularities, we are interested in Green's formula as a means of extending the classical Weyl-Titchmarsh theory concerning the m-function and the limit point, limit circle classification of the global endpoints. Such an extension has been obtained by Everitt and others, but only for operators defined via a decomposition of an interval into an infinite sequence of abutting intervals that are either regular or limit circle. We employ similar methods in this paper to extend this further to allow for limit point as well as interlaced limit point-limit-circle intervals.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Upper semicontinuity of attractors of nonautonomous dynamical systems for small perturbations

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This is a systematic study of problem of upper semicontinuity of compact global attractors of nonautonomous dynamical systems for small perturbations. For the general nonautonomous dynamical systems we give the conditions of upper semicontinuity of attractors for small parameter. Several applications of these results are given (quasihomogeneous systems, monotone systems, nonautonomously perturbed systems, nonautonomous 2D Navier-Stokes equations and quasilinear functional-differential equations).

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Liouville's formula for a dynamic equation and a nabla dynamic equation on a time scale

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We will be concerned with Liouville's formula (see Theorem 5.28 in the book, Dynamic Equations on Time Scales by M. Bohner and A. Peterson) for a n-th order linear vector dynamic equation on a time scale. We will prove this result for small values of n and indicate what the general result is. We will also discuss the nabla case.

⟶ ∞ ⋄ ∞ ⟵

Role of delay in SIS epidemic model

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This paper deals with an S-I epidemic model with delay. The susceptible and infected male and female interaction are taken. In absence of delay we have derived the local as well as global stability of interior equilibrium point. Effect of delay is also studied.

⟶ ∞ ⋄ ∞ ⟵

On backward growth rates of original nonlocal Burgers equations

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Backward time behavior proved to be an important factor in studying the attractors of dissipative systems. We investigate the backward time behavior of the system of coupled PDE's resulting from the Burgers model of turbulence. Similar studies were made for other dissipative systems. For Kuramoto-Sivashinski equation (Kukavica, 1992) it was proven that if backward in time a solution grows at most exponentially, it is bounded. 2-D Navier-Stokes equations (Constantin et al., 1997) and Camassa-Holm equations (Vukadinovic, 2001) turned out to be closer to the linear case with an entire spectrum of exponential backward growth present. Moreover, some density results for global solutions of the last two systems of PDEs were obtained. Our study shows that the original Burgers equations display a very particular kind of backward time behavior with only one exponential rate of backwards growth possible. Based on our results, a classification of the global solutions of original Burgers equations was made.

⟶ ∞ ⋄ ∞ ⟵

Monotone method for nonlinear nonlocal hyperbolic Problems

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Recent results concerning the application of the monotone method to studying existence and uniqueness of solutions for general first order nonlinear nonlocal hyperbolic problems are presented. The limitation of comparison principles for such nonlocal problems is discussed. New definitions for upper and lower solutions are introduced to overcome this limitation.

⟶ ∞ ⋄ ∞ ⟵

Skew symmetric control functions and Cartan connections

Jerry Donato

Cartan's method is applied to a path dependent nonlinear non-Euclidean geometric control system. Skew symmetric control functions are Cartan connections. The affine connections are noninvariant under a coordinate transformation using a Jacobian matric. The system has nine equations with six geometric objects. Define a displacement vector as a point on the manifold where the exterior differential operator is applied to construct a moving frame where the basis is in the tangent space (bits) and where the dual basis in the cotangent space (bicts). The operator is then applied to the bits to define the Cartan connection. The arbitrarily selected bicts and connections are required to be compatible with each other. Application of the operator (1) to bicts gives a torsion term and (2) to the connection gives a curvature term. Another application of the operator (1) to curvature gives the Bianchi identities and (2) to torsion gives torsion with nonfree connections. The torsion free connection conditions are specified.

⟶ ∞ ⋄ ∞ ⟵

Chaos in neural or gene networks with hard switching

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Although there are ways of demonstrating the existence of chaotic dynamics in nonlinear systems of ODEs, it is usually difficult to prove that the behaviour is attracting. In the context of neural networks and networks that model gene regulation, the use of a hard switching nonlinearity for each network unit has been shown to give considerable analytic leverage. These 'Glass networks' can have interesting dynamics in 4 or more dimensions, as suggested, for example, by sets on which the dynamics are homeomorphic to shifts of finite type. Possible trajectories can be described in terms of a symbolic dynamics where the symbols are 'logical' cycles. In order to prove that the sets supporting chaotic dynamics are attracting, a method has been developed to rule out the existence of long, stable periodic solutions. Matrix theoretic techniques are used to show that Poincaré maps cannot have stable fixed points. While the method has to be applied to individual examples, it seems to be quite generally applicable and provides a rare example of a rigorous demonstration of stable chaos in a relatively high-dimensional system. It also involves a novel interweaving of differential equations, symbolic dynamics and matrix theory.

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Smooth bifurcation and continuation for variational inequalities with applications to unilaterally supported beam

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Recke and **Kucera**

We present a certain analog for variational inequalities of the classical result on bifurcation from simple eigenvalues of M. Crandall and P. Rabinowitz. In other words, we describe the existence and local uniqueness of smooth families of nontrivial solutions to variational inequalities, bifurcating from a trivial solution family at certain points which could be called simple eigenvalues of the homogenized variational inequality. If the bifurcation parameter is one dimensional, the main difference between the case of equations and the case of variational inequalities (when the cone is not a linear subspace) is the following: For equations two smooth half-branches bifurcate, for inequalities only one. The proofs are based on scaling techniques and on the Implicit Function Theorem. The abstract results are applied to a fourth order ODE with pointwise unilateral conditions (an obstacle problem for a beam with the compression force as the bifurcation parameter).

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On the existence of solutions to the Navier-Stokes equations of a two-dimensional compressible flow

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We consider the Navier-Stokes equations for compressible, barotropic flow in two space dimensions where the pressure satisfies $p(\varrho) = a\varrho \log^d(1+\varrho)$ for large ϱ, i.e.

$$\frac{\partial \varrho}{\partial t} + \text{div }(\varrho \vec{u}) = 0,$$

$$\frac{\partial \varrho \vec{u}}{\partial t} + \text{div }(\varrho \vec{u} \otimes \vec{u}) + \nabla p(\varrho) = \mu \Delta \vec{u} + (\lambda + \mu) \nabla (\text{div } \vec{u}) + \varrho \vec{f}.$$

Here a and d are constants satisfying $d > 1$ and $a > 0$. We introduce useful facts from the theory of Orlicz spaces. Then we prove the existence of globally defined finite energy weak solutions.

$\longrightarrow \infty \diamond \infty \longleftarrow$

A Dynamic Mathematical Model with Sum-of-Digit Moving Averages

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In the technique analysis of the stock price, dynamic systems with moving average analysis are widely used tools. We introduce a dynamic mathematical model with Sum-of-Digit Moving Averages (SDMA). Dynamic systems with different time horizons are defined. We have studied the velocity, acceleration and concaveness of SDMA to forecast the

stock price. This paper studies solutions of the system to define the buy and sell signals. Applications of the model to the Standard and Poor's 500 and NASDAQ stock index for the last ten years are given.

⟶ ∞ ⋄ ∞ ⟵

Periodic solutions of differential delay equations via Hamiltonian systems

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We study the existence of periodic solutions of certain classes of differential delay equations. Inspired by Kaplan and Yorke's original idea, this problem is reduced to finding periodic solutions for related Hamiltonian systems. By using index theory and variational methods for Hamiltonian systems, we prove a conjecture raised by Kaplan and Yorke in 1974. Moreover, we obtain an estimate on the number of periodic solutions.

⟶ ∞ ⋄ ∞ ⟵

A novel asymptotic stabilizability switching control strategy for linear switching systems

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The switching system is a simple class of hybrid systems, where the switching variable can be regarded as discrete variable and determine switching between the controllers, the states of controlled plant are regarded as continuous variable. In this paper, we are concerned with the linear Switching Systems. First, the concepts of switching period and its near switching period are defined, then the new judgment conditions for switching strategy asymptotically stabilizing systems are offered, and a new ideal for designing the switching strategy of the switching systems is proposed. Concurrently it also generalizes the approach of multi-Liapunov to switching systems.

⟶ ∞ ⋄ ∞ ⟵

The approximately damped Sine-Gordon equation and its explicit exact solutions

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In this talk, using the qualitative theory of ordinary differential equations, we give a qualitative analysis to a two-dimensional plane autonomous system which is equivalent to the damped sine-Gordon equation,

$$u_{tt} + r_1 u_t - r_2 \Delta u_{(n)} + \sin u = f. \tag{4}$$

This equation describes the dynamical behavior of a continuous Josephson junction driven by a current source by taking into account of damping effect. Then following our previous work, explicit travelling solitary wave solution to this equation in (n+1)-dimensional space will be introduced. In this talk, using the qualitative theory of ordinary differential equations, we give a qualitative analysis to a two-dimensional plane autonomous system which is equivalent to the damped sine-Gordon equation,

$$u_{tt} + r_1 u_t - r_2 \Delta u_{(n)} + \sin u = f. \tag{5}$$

This equation describes the dynamical behavior of a continuous Josephson junction driven by a current source by taking into account of damping effect. Then following our previous work, explicit travelling solitary wave solution to this equation in (n+1)-dimensional space will be introduced.

⟶ ∞ ⋄ ∞ ⟵

Existence of solutions to second order problems with nonlinear boundary conditions

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Donal O'Regan

We present new definitions of upper and lower solutions for second order differential equations with nonlinear boundary value conditions including Dirichlet, Neumann, and Sturm-Liouville cases among others. These definitions are then used to establish new general existence criteria. Monotone conditions on the functions that define nonlinear boundary conditions are considered.

⟶ ∞ ⋄ ∞ ⟵

Dynamics generated by Stefan-type problems with exothermic kinetics

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Victor Roytburd

Free-boundary/interface problems with exothermic kinetics are used to model a variety of diverse physical phenomena, from high temperature self-propagating high-temperature synthesis (condensed-state combustion) to rapid solidification and barodiffusion driven detonation in porous media. Rigorous study of classical solutions, including existence of finite-dimensional attractors for problems in one spatial dimension will be discussed. Results of direct numerical simulations exhibiting a number of highly nontrivial dynamical scenarios will be presented.

⟶ ∞ ⋄ ∞ ⟵

Optimal control problem of Bolza-type for hyperbolic hemivariational inequality

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Victor Roytburd

We consider an optimal control problem of Bolza-type governed by a hyperbolic hemivariational inequality. In such an inequality there appear a nonlinear, pseudomonotone, coercive and bounded operator depending on the time derivative u' of an unknown function u, a linear symmetric and monotone operator depending on an unknown function u and the Clarke subdifferential of the nonsmooth potential depending on both u and u'. For an existence result for such an inequality we refer to L. Gasiński, "Existence Result for Hyperbolic Hemivariational Inequalities", Nonlinear Analysis 47 (2001) 681-686. In our control problem we look for an optimal solution of the inequality with the right hand side depending on controls, so as to minimalize an integral functional of Bolza-type. We put sufficient conditions for obtaining an optimal solution.

⟶ ∞ ⋄ ∞ ⟵

Controllability of a class of nonlinearly perturbed systems

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In this paper we provide a class of nonlinear perturbations which when applied to a controllable linear system, the resulting perturbed system is also controllable. Consider the n-dimensional nonlinear system defined on the finite time interval I= [ ]: (1) where, for each t in I, A(t), B(t) are n × n and n × m matrices, respectively, x(t) is an n state vector , u(t) is an m control vector and f is a nonlinear function of the state. We say that the system (1) is controllable, if for any initial and desired final states , there exists a control such that corresponding solution of (1) together with also satisfies . Note that (1) is a nonlinear perturbation of the linear system . (2) In this paper we define a class of functions f with which the nonlinear system (1) is controllable if the linear system (2) is controllable. The functions in this class satisfy some growth condition. This class includes functions satisfying monotonicity conditions and Lipschitz continuous functions. We obtain many earlier results in the literature as special cases of our results.

⟶ ∞ ⋄ ∞ ⟵

A nonlinear controlled system of differential equations describing the process of production and sales of a consumer good

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Eugene Khailov

A nonlinear controlled system of differential equations has been constructed to describe the process of production and sales of a consumer good. This model can be controlled either by the rate of production or by the price of the good. The attainable sets of corresponding controlled systems are studied. It is shown that in both cases the boundaries of these sets are the union of two two-parameter surfaces. It is proved that every point on the boundaries of the attainable sets is a result of piecewise constant controls with at most two switchings. Attainable sets for different values of parameters of the model will be demonstrated using Maple.

⟶ ∞ ⋄ ∞ ⟵

Nonlinear hybrid models of chaos in cardiovascular system

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We construct so called "hybrid" models, using known analytic models of reaction-diffusion type and corresponding nonlinear generalizations, coupled with real ECG and rheogram signals. It allow to implement the general "energetic" approach to investigation of heart beats and pulse propagation in cardiovascular system. We determine physical characteristics of heart electrical potential variability that allow to estimate the ranges of pulse regularity and to predict transition to deterministic chaos. By using a dynamic model, describing the heart electrical potential propagation, an expression for the dynamic balance of the pulse energy is constructed. This expression can be presented as function of two variables: activator and inhibitor that contains structural parameters describing the important physiological characteristics, such as threshold of the heart excitation, the ratio of excitable and refractory time. It was determined the possibility to control cardiac arrhythmia by using basic properties of chaotic systems. In the recent years a number of experimental and signal-processing investigations of ventricular fibrillation has been conducted . On the other hand, the new analytic and computer models of chaotic systems synchronization were developed . Evidently, the above-mentioned methods coupled with the modelling of chaos in nonlinear oscillators, is useful in problems associated with estimation of turbulence and chaos degree in heart beats and pulse propagation. We find the strong relationship (ratio) of main cardiac parameters, that determine the ranges of regularity of heart beating . Importantly it has been possible to use the well known Feigenboum's theory of transition to deterministic chaos for construction of corresponding bifurcation diagrams and estimation of corresponding bifurcation and chaos transition values. By using computer simulation, described the dependence of first bifurcation value of heart sensibility from the inhibitor variable.

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A geometric approach to nonimaging optics

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Nonimaging optics is a field that study optimal concentration of light from a source distribution to a receiver. The relevant information is codified by a field of cones at each point of the concentrator, formed by those rays that will reach the receiver (perhaps after some reflections on the wall of the concentrator). This suggests that we can use Lorentz geometry to analyze the problem. We will review this geometric approach to study three dimensional nonimaging concentrators with constant refractive index, and we will set up the main steps toward its generalization to arbitrary refractive index.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Numerical approximation of normally hyperbolic invariant manifolds

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This paper deals with the numerical continuation of invariant manifolds, regardless of the restricted dynamics. Typically, invariant manifolds make up the skeleton of the dynamics of phase space. Examples include limit sets, co-dimension 1 manifolds separating basins of attraction (separatrices), stable/unstable/center manifolds, nested hierarchies of attracting manifolds in dissipative systems and manifolds in phase plus parameter space on which bifurcations occur. These manifolds are for the most part invisible to current numerical methods. The approach is based on the general principle of normal hyperbolicity, where the graph transform leads to the numerical algorithms. This gives a highly multiple purpose method. The key issue is the discretization of the differential geometric components of the graph transform, and its consequences. Examples of computations of both attracting and saddle-type (1D and 2D) manifolds will be given, with and without non-uniform adaptive refinement.

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Stability of homoclinic or heteroclinic loops and bifurcation of limit cycles

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This paper concerns with the stability of a homoclinic, double homoclinic or heteroclinic loop and the number of limit cycles near it under perturbations.Some known results are introduced and new results are given as well. By changing stability of this kind of loops we provide a new way to find limit cycles. Some interesting examples are illustrated.

⟶ ∞ ⋄ ∞ ⟵

Direct soliton perturbation methods

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Recently described methods for studying soliton perturbations using direct methods are reviewed and compared for the KdV equation and then applied to other soliton bearing systems.

⟶ ∞ ⋄ ∞ ⟵

A survey of linear systems exhibiting chaotic behavior

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Chaotic behavior is typically viewed in terms of nonlinear phenomena. But this behavior is also known to arise in linear systems. We wish to illustrate several linear systems that exhibit chaotic behavior. In defining a system to be chaotic we will rely upon the basic structure proposed Devaney.

⟶ ∞ ⋄ ∞ ⟵

Mathematical analysis of complex and chaotic dynamics in neural network models

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Cai Zhijie, **Gu Fanji**, and **Lin Wei**

In this presentation, we will theoretically show that there are chaotic dynamics in both integrate-and-fire circuit with periodic pulse-train input and discrete neural network models. Then, some numerical simulations will be given to reinforce our theoretical investigation.

⟶ ∞ ⋄ ∞ ⟵

Central-upwind schemes for systems of balance laws

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We study the application of high-order semi-discrete central-upwind schemes to multidimensional systems of balance laws. These schemes are Godunov-type methods – a piecewise polynomial interpolant, reconstructed from the cell averages computed at time t, is evolved to the next time level, $t + \Delta t$, using a particularly simple spatial discretization combined with a stable ODE solver of an appropriate order. The numerical fluxes are obtained by including the Riemann fans into the control volumes of a (varying) size, determined by one-sided local speeds of propagation, and by passing to the limit as $\Delta t \to 0$. The main advantages of central schemes is their simplicity, since no (approximate) Riemann problem solver, characteristic decomposition, operator or dimensional splitting is required. The main challenge in application of central-upwind schemes to balance laws is how to preserve the balance between the numerical fluxes and the source terms. In the case of Saint-Venant system of non-homogeneous shallow water equations, the goal is achieved by using a special quadrature for the source average. For the compressible Euler equations with source terms due to a static gravitational field, we rewrite the system in terms of a variable, which remains constant at stationary steady states and apply the central-upwind schemes to the new system. A special treatment of a discontinuous source term is required for the Savage-Hutter system for granular materials. The application of the central-upwind scheme to balance laws with a stiff source term (e.g., detonation waves) will be also discussed.

⟶ ∞ ⋄ ∞ ⟵

A note on the small time development of the solution to a scalar, nonlinear, singular reaction-diffusion equation

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In this note, we consider a class of scalar, nonlinear, singular (in the sense that the reaction terms in the equation are not Lipschitz continuous) reaction-diffusion equations with positive initial data being of (a) $O(x^{-\alpha})$ or (b) $O(x^{-\beta}e^{-\sigma x})$ at large x (dimensionless distance), where alpha, sigma> 0 and beta are constants. We establish, by

developing the small -t (dimensionless time) asymptotic structure of the solution, that the support of the solution becomes finite in infinitesimal time in both cases (a) and (b) above. The asymptotic form for the location of the edge of the support as t $\to 0$ is given in both cases.

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Expansions in generalized spherical harmonics in R^{k+1}

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The interior and exterior Dirichlet problems for the Dunkl equation on the k-dimensional unit sphere is investigated. It is shown that the solution is unique and can be expanded in series of generalized spherical harmonics. An estimate for the reproducing kernels of spaces of generalized spherical harmonics due to Xu is an important tool in the proofs.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonlocal problems for quasilinear parabolic equations

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In place of the usual initial condition for a parabolic problem on the space-time domain $\Omega = \omega \times (0,T)$, we assume a condition of the form $u(x,0) = B[u](x) + g(x)$ for $x \in \omega$, where B is a linear operator from $C(\Omega)$ to $C(\omega)$. Under some simple hypotheses on B, this problem has a solution, and the solution is quite smooth if some additional hypotheses are made.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Discrete models for population interaction

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Several boreal populations are constrained to reproduce at well-defined discrete moments. Biological data are usually collected at discrete events. In some populations, optimization of reproduction processes are empirically established. These facts form a basic justification of a discrete modelling approach. We commence by deriving a discrete analogue of the continuous Lotka-Volterra model. It possesses pulse-wise reproduction and continuous predation and death processes, and preserves the characteristic properties of the Lotka-Volterra system. Unfortunately, the derived model has unbounded solutions. Thus, competition must be included in the model. The inclusion of competition demonstrates the major drawback of the discrete modelling approach: dependencies arise between the acting processes. It turns out that dynamical consequences may follow, depending on how these dependencies are taken into account. Nevertheless, our discrete approach gives strong results concerning the underlying mechanisms of certain discrete optimization processes. Such results are closely related to the principle of competitive exclusion. We also demonstrate some interesting similarities between discrete systems derived using our approach and continuous systems representing the corresponding ecosystems.

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Asymptotic behavior of laminar flow

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Assume that (1) the fluid is viscous impressible, (2) the walls are uniformly porous, (3) the tube is long enough such that end effect can be neglected, (4) the pipe is of circular cross-section with a uniformly porous wall, and (5) the fluid flowing in the axial direction and the fluid through the wall are homogeneous, hence the flow is axially symmetric. Then the boundary value problem involving the steady-state Navier-Stokes equations can be reduced to a fourth order nonlinear differential equation:

(1) $\eta f^{iv} + 2f''' + \frac{R}{2}(f'f'' - ff''') = 0 \quad (0 < \eta \leq 1)$

with boundary conditions
(2)
$f(0) = 0, f'(1) = 0, f(1) = 1, \quad \lim_{\zeta \to 0^+} \sqrt{\eta} f''(\eta) = 0,$

where
$f(\eta)$ is an unknown function related to the velocities of the flow,
$\eta = (r/a)^2$ is the non-dimensional variable ($\eta = 1$ is the wall, $\eta = 0$ is the axis of the pipe),
r is the distance measured radially,
a is the radius of the pipe,

$R = Va/\nu$ is a Reynolds number,
V is the normal outward velocity, $V > 0$ corresponds to suction, $V < 0$ injection. $\nu > 0$ the viscosity. This paper presents a rigorous analysis of the behavior of the solutions as $R \to -\infty$.

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Solution sets of differential inclusions under modified growth and semicontinuity conditions

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We consider the differential inclusion $x'(t) \in F(t, x(t))$ a.e., $x(t_0) = x_0$ (IVP) The following are assumptions on $F : [a, b] \times \Re^n \to \wp(\Re^n)$: 1) For every $(t, x) \in [a, b] \times \Re^n$, $F(t, x)$ is convex; 2) The mapping $x \to F(t, x)$ satisfies Property (Q), i.e., for fixed t, $F(t, x_0) = \cap_{\delta > 0} cl\, co \cup_{x \in N_\delta(x_0)} F(t, x)$ for every $x_0 \in \Re^n$; 3) Growth condition (γ) is satisfied, i.e., there exists a measurable set-valued function $P : [a, b] \to \wp(\Re^n)$, a constant $M > 0$ and for every $\varepsilon > 0$, a function $\psi_\varepsilon \in \mathcal{L}^1([a, b], \Re)$, $\psi_\varepsilon(t) > 0$ such that for given $x \in \Re^n$ and selection $\xi(t) \in F(t, x)$, there exists a selection $\eta(t) \in P(t)$ with $\int_{[a,b]} \eta(t)dt \leq M$,$|\xi(t)| \leq \psi_\varepsilon(t) + \varepsilon\eta(t)$. If we denote the set of solutions of the initial value problem (IVP) by $H(t_0, x_0)$, then whenever M is compact in $[a, b] \times \Re^n$, $H(M)$ is a compact and connected subset of $C([a, b])$. Furthermore, every point on the boundary of the solution funnel is peripherally attainable.

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Nonlinear BVP with the one-dimensional p-Laplacian and quadratic nonlinearity

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Let us consider the nonlinear boundary value problem

$$-\left|u'(t)\right|^{p-2} u'(t) = \lambda f(u(t)),\ u(0) = u(1) = 0 \quad (1)$$

where $\lambda \in \mathbb{R}$ and $p \in \mathbb{R}$ with $1 < p < \infty$ and f is a real continuous function satisfying some suitable hypothesis. We are interested in finding the number of solutions to the problem (1) and their local behaviours that is the number of zéros of each solution in $(0, 1)$ by analyzing the phase plane of the system associated to this problem, and using the time-map approach.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Approximate controllability results for a class of abstract functional evolution equations

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Approximate controllability results for a class of abstract first and second-order semilinear integro-differential and functional evolution systems are formulated. The case in which the fixed initial condition is replaced by a functional (the so-called nonlocal case) is also treated. Examples of mathematical models arising in a variety of areas are also discussed to illustrate the utility of the theory.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Dynamics of torque-speed profiles for electric vehicles and nonlinear models based on differential-algebraic equations

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Per Sandholdt Sauer-Danfoss and **Ningning Song**

A key measure of efficiency of electric vehicles is their torque-speed profiles. In order to construct such profiles, it is important to be able to model nonlinear slip phenomena of a wheel. Major difficulties in the solution of this problem are coming from a complex character of dependencies between the normalised traction force (or the friction coefficient) μ and the slip ratio λ, not known a priori, but rather estimated experimentally for typical surface conditions. The so-called $\mu - \lambda$ curves are nonlinear functions of the velocity of the vehicle and the wheel rotational velocity. Despite their predominant use in the literature, linear approximations of such curves may fail to predict correctly torque-speed profiles. Although attempts to model the described phenomena have been made before, to our best knowledge a general model with respect to the vehicle velocity, the wheel rotating velocity, the slip ratio, the traction force, and the torque, has never been formulated and solved as a coupled nonlinear

problem based on a system of differential-algebraic equations arising naturally in this context. In this paper, such a model is formulated, solved numerically, and some results of numerical simulation of driving an electric mower on different surface conditions are presented.

$\longrightarrow \infty \diamond \infty \longleftarrow$

On solutions to stochastic differential inclusions

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A martingale problem approach is used to analyze compactness and continuous dependence of solutions set to stochastic differential inclusions of Ito type:

$$\begin{aligned} d\xi_t &\in F(t,\xi_t)dt + G(t,\xi_t)dW_t, t \in [0,T] \qquad (1) \\ P^{\xi_0} &= \mu \end{aligned}$$

where $F : [0,T] \times R^d \to Conv(R^d), G : [0,T] \times R^d \to Conv(R^{d\times m})$ are measurable, compact and convex valued multifunctions, W is a m-dimensional Wiener process on the filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$, and μ is a given probability measure on the space $(R^d, \beta(R^d))$. Let $\mathcal{M}(\mathcal{C})$ denote the set of all probability measures on $(\mathcal{C}([0,T], R^d), \beta(\mathcal{C}))$. Let $\mathcal{R}^{loc}(F,G,\mu)$ denote the set of those measures $Q \in \mathcal{M}(\mathcal{C})$, which are solutions to local martingale problem for (F,G,μ). The first observation is the following:

PROPOSITION . *Let* $F, G : [0,T] \times R^d \to 2^{R^d}, 2^{R^{d\times m}}$ *be* $\beta([0,T] \times R^d)$ *- measurable multifunctions, and let* μ *be a probability measure on* $(R^d, \beta(R^d))$. *Then there exist a weak solution to stochastic inclusion (1) if and only if* $R^{loc}(F,G,\mu) \neq \Phi$. Using this approach it can be formulated:

COROLLARY. *If* $F : [0,T] \times R^d \to Conv(R^d)$, *and* $G : [0,T] \times R^d \to Conv(R^{d\times m})$ *are measurable and bounded set-valued mappings such that* $F(t,\cdot)$ *and* $G(t,\cdot)$ *are l.s.c. for each fixed* $t \in [0,T]$, *then for every probability measure* μ *on* $(R^d, \beta(R^d))$ *the set* $\mathcal{R}^{loc}(F,G,\mu)$ *is nonempty and relatively compact in* $\mathcal{M}(\mathcal{C})$.

In the presentation we also formulate a continuous dependence of solutions set to stochastic differential inclusion (1), with respect to initial distributions. Finally the problem of existence of optimal weak solutions to such inclusion and their dependence on initial distributions will be dicussed. Presented results generalize some of results in [L], [K] and [SV].

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$\longrightarrow \infty \diamond \infty \longleftarrow$

The analysis of the solutions stability for the critical cases of non-linear controlled systems

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The performed study regards the class of controlled nonlinear state and control variable systems. The analysis of some dynamic phenomena leads to mathematical models represented by controlled differential systems. From the command systems the major interest is represented by the controllable systems. In this hypothesis, the construction of a control, given by a linear form of state variables, might modify the eigenvalues of the linearized system. When all the eigenvalues have the real part negative, the controlled system has a asymptotic stable trivial solution. If some of the eigenvalues are pure imaginary and the other have the real part negative, the system taken into consideration is a critical stability case. By a serial of successive transformation will be provided an equivalent system, for which we have to determine the solution stability. The transformed system is a non-linear system of non-homogeneous forms of the state variables, for which will be analyzed of the stability conditions of its periodical solution or the orbital stability. In this way we are leaded to the stability analysis of the periodical solution of the transformed systems, which means the orbital stability and, thus, the stability of the nonlinear systems trivial solution.

$\longrightarrow \infty \diamond \infty \longleftarrow$

Nonsmooth variational methods with applications to nonlinear boundary value problems

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A nonsmooth critical point theory is constructed for functionals which are expressed as a sum $\Phi + \Psi$ with Φ locally Lipschitz and Ψ convex, lower semicontinuous, proper. This approach extends important smooth and nonsmooth variational methods. The applications concern various boundary value problems with discontinuities and constraints. The critical points correspond to the weak solutions. In addition to the existence of (nontrivial) solutions, the multiplicity and the location are studied. The relationship with nonlinear mathematical programming problems with constraints described in terms of closed range operators is also discussed.

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Persistence in a prey-predator system with disease in the prey

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This paper deals with a prey-predator system where the prey population is infected by microparasite. Predator functional response is a function of weighted sum of prey abundances. This type of functional response reflects the switching mechanism of the predator. We identify the parameters which influence the persistence of all the populations as well as impermanence. The role of delay in the above system is also discussed.

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Large on stabilization of solutions of nonclassical Dirichlet problem for singular quasilinear elliptic equations

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The following problem is considered:

$$\Delta_{\mathcal{B}} u + g(u)|\nabla u|^2 + \frac{\partial^2 u}{\partial z^2} = 0;\ x \in \mathbf{R}^n, y \geq 0, z > 0, \quad (1)$$

$$u\Big|_{z=0} = \varphi(x,y);\ x \in \mathbf{R}^n, y \geq 0, \quad (2)$$

$$\frac{\partial u}{\partial y}\Big|_{y=0} = 0;\ x \in \mathbf{R}^n, z > 0, \quad (3)$$

where g is continuous, φ is continuous and bounded and $B_{k,y}$ denotes singular Bessel operator $\frac{1}{y^k}\frac{\partial}{\partial y}(y^k \frac{\partial}{\partial y})$ with a positive parameter k. We prove the following assertions: **Theorem 1**: There exists a unique bounded solution of (1)-(3).

Theorem 2: Let $A \in (-\infty, +\infty)$, $u(x,y,z)$ be the bounded solution of problem (1)-(3). Then for any x from $\mathbf{R}^n$ and any positive y

$$u(x,y,z) \overset{z\to\infty}{\longrightarrow} A \iff$$

$$\frac{(n+k+1)\Gamma\left(\frac{n+k+1}{2}\right)}{\pi^{\frac{n}{2}}\Gamma\left(\frac{k+1}{2}\right) r^{n+k+1}} \int_{B_+(r)} \eta^k f[\varphi(\xi,\eta)] d\xi d\eta \overset{r\to\infty}{\longrightarrow} f(A),$$

where $f(s) = \int_0^s \exp(\int_0^x g(\tau)d\tau)dx$.

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Relative equilibria and relaxation oscillation of point vortices in a plane

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The motion of assembly of point vortices on the two-dimensional Euler fluid is discussed. The vortices drift away on the fluid due to an interaction between the vortices, and this phenomenon can be described by the ordinary differential equation in Hamiltonian form. This equation is analyzed for a long time. When two vortices is on the fluid, the motion can be easily analyzed and we can find the results in textbooks on fluid dynamics. Three point vortices problem is analyzed by Aref (1979). Aref and Pomphrey (1982) show four point vortices which exhibit chaotic motion.

In this talk, we consider five point vortices under some initial configuration. In this problem, an relative equilibrium is stable in some sense. By numerical simulations we find that a solution located near the equilibrium exhibits an relaxation oscillation, and that another solution near the equilibrium does not show typical behavior. We already know that, under different initial configuration, some five point vortices also exhibit an relaxation oscillation (Nakaki (1999)). We can observe that these two oscillations are belonging to a different category. In this talk we shall discuss the oscillations from numerical and mathematical points of view.

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Dynamics of local principal configurations of surfaces immersed in $\mathbf{R}^4$ with isolated simple umbilical points

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The N-principal configuration of a surface M in 4-Euclidean space is the set formed by the umbilical points and the lines of maximal and minimal curvature with respect to a unitary smooth vector field N normal to M. We describe here the bifurcation diagram of N-principal configurations, where N is parameterized in the space of 1-jets of normal vector fields which define an isolated umbilical point.

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Almost periodic solutions of linear equations in Hilbert spaces

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We discuss sufficient conditions to ensure almost periodicity of solutions of linear equations of the form $x"(t) = A\,x(t)$ and $x"(t) = A\,x(t)$ in Hilbert spaces.

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Asymptotic stability of planar waves for multidimensional viscous conservation laws in half space

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The purpose of the present talk is to show the asymptotic stability of one dimensional planar waves to multidimensional viscous conservation laws in the half space:

$$(3)\quad u_t + f(u)_x + \sum_{i=1}^{n-1} g(u)_{y_i} = u_{xx} + \Delta_y u, \quad x > 0,$$

where $(x, y) = (x, y_1, y_2, \cdots, y_{n-1}) \in \mathbf{R}^n$ $(n \geq 2)$. Here, the flux function f is assumed to be uniformly convex. It is assumed that u_{b0}, $y \in \mathbf{R}^{n-1}|u(x,y,t) - \phi(x,t)| \leq C(1+t)^{-\frac{n}{2}+\varepsilon}$ is obtained, where ε is an arbitrarily small positive constant .

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Applications of Epsilon-Pseudo-Orbits

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One recent trend in Dynamical Systems is for inherently quantitative techniques to yield qualitative information about the systems under investigation. Epsilon-pseudo-orbits for epsilon of fixed size can be related to approximations for discrete dynamical systems, yielding implications for observability of attractors and other sets defined by the dynamics. In particular, the Conley Decomposition of a space can be approximated by an epsilon-coarse Conley Decomposition. Epsilon-pseudo-orbits yield models not only of computer models themselves but also models of real-world phenomena in which the epsilon-jumps are key ingredients of the behavior of the system. One such system is neural activity of the brain. Basic results on epsilon-pseudo-orbits and some applications will be presented.

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Some convex and monotone skew-product semiflows

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Rafael Obaya and **Ana M. Sanz**

We study the topological and ergodic structure of a class of convex and monotone skew-product semiflows. We assume the existence of two strictly ordered minimal subsets K_1 and K_2, we obtain a ergodic representation of their upper Lyapunov exponents and we prove that they vanish or not simultaneously. In the case of null upper Lyapunov exponents, we provide a complete description of the ergodic structure of the skew-product semiflow in a positively invariant region K defined by the minimal subsets. Finally, we study the behavior of the trajectories on K in the hyperbolic case. Some examples of skew-product semiflows generated by differential equations and satisfying the assumptions of monotonicity and convexity are also presented.

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Bounded trajectories set of a scalar convex differential equation

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Ana I. Alonso

We study the topological and ergodic structure of the set of bounded trajectories of the flow defined by a scalar convex differential equation. We characterize the minimal subsets, the ergodic measures concentrated on them and study the long time behaviour of the bounded trajectories in terms of the linearized equations. We obtain continuity properties with respect to the coefficients in different topologies.

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Abstract finite time extinction problems

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Let X be a Banach space and C be an ordering cone in X. Let A be the generator of a C_0 semigroup $\{S(t) : t > 0\}$. We pose an (abstract) extinction problem as follows. Let $0 \neq z \in C$ and assume the function $u(t) = S(t)z$ is a strong solution to the initial-value problem

$$u_t = A\,u\ ,\ u(0) = z\ .$$

We say u extinct in finite time if there exists a time $T \in (0, \infty)$ such that $u(t) = 0$ for all $t \geq T$. We discuss necessary and sufficient conditions on the generator A such that u extinct in finite time. Applications to systems of nonlinear reaction-diffusion systems will be given.

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Conservation laws and invariant solutions for soil water equations

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C. M. Khalique

A mathematical model was developed to simulate soil water infiltration, redistribution,and extraction in a bedded soil profile overlaying a shallow water table and irrigated by a line source drip irrigation system. The governing partial differential equation can be written as

$$C(\psi)\psi_t = \left(K(\psi)\psi_x\right)_x + \left(K(\psi)(\psi_z - 1)\right)_z - S(\psi), \quad (1)$$

where ψ is soil moisture pressure head, $C(\psi)$ is specific water capacity, $K(\psi)$ is unsaturated hydraulic conductivity, $S(\psi)$ is a sink or source term, t is time, x is the horizontal and z is the vertical axis which is considered positive downward. See Vellidis, G., Smajstrla, A. G., "Modelling soil water redistribution and extraction patterns of drip-irrigated tomatoes above a shallow water table", *Transactions of the American Society of Agricultural Engineers*, 35 (1), 1992, 183-191. We generate conservation laws for certain soil water equations and determine the conserved vectors by direct method and then using a theorem due to Kara and Mahomed ("Relationship between Symmetries and Conservation Laws", *International Journal of Theoretical Physics,* Vol. 39, No. 1, 2000) we will then check if the condition for which point symmetries of soil water equations associate with conservation laws is satisfied. Invariant solutions of equation (1) for some particular types of the coefficients $C(\psi), K(\psi)$ and $S(\psi)$ when an extension of the principal Lie algebra L_p occurs will also be calculated.

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Small oscillations in infinite dimensional resonant systems

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We present some abstract result concerning the existence of (cantor-like) families of periodic solutions in some nonlinear PDEs; we also provide some applications to concrete examples, like nonlinear string equations and nonlinear plate equations. These results can be seen as a partial extension, to the infinite dimensional and resonant setting, of the classical Lyapunov center theorem. The techniques used involve a novel interaction between averaging theory and the Lyapunov-Schmidt decomposition method. References: D.Bambusi, S.Paleari — Journal of Nonlinear Sciences, 11, 69–87 (2001) S.Paleari, D.Bambusi, S.Cacciatori — ZAMP, 52, 1033–1052 (2001) D.Bambusi, S.Paleari — CPAA, to appear (2002).

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Dynamical approach in analysis: functional equations, boundary problems for PDE, integral geometry

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This talk is devoted to solution of several new problems in the three independent fields of Analysis : functional equations, boundary problems for higher order hyperbolic differential equations in bounded domains and integral geometry. All these problems at first sight does not give even a merest hint about some dynamical systems connected with them. Nevertheless it turned out that when solving these quite different problems an essential part of information can be obtained with the help of dynamical methods. To apply these methods we introduce a semigroup ϕ of maps in an interval I generated by *two* maps α and β in I which are closely connected with the problems in question. On a side the language of orbits of this semigroup enables to formulate easily conditions (sometimes necessary and sufficient)of solvability of these problems. On the other side one of the most essential technical elements in the proof of the main statements is a searching of some specific attractors of the noncommutative dynamic system generated by semigroup ϕ . Even in functional equations with their long history our approach allows to obtain completely new results which have nothing in common with what was known earlier. And this is without any hard analytical work. In particular we solve at the first time an *nonhomogeneous* Cauchy equation on a curve and improve significantly what was known about homogeneous one. Note that purely dynamical part of the talk seems to contain setting of new problems related to noncommutative dynamical systems.

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Stability of linear functional equations in Banach modules

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Let A be a unital C^*-algebra, and let ${}_A\mathcal{B}$ and ${}_A\mathcal{C}$ be left Banach A-modules. Let $f : {}_A\mathcal{B} \to {}_A\mathcal{C}$ be a mapping with $f(0) = 0$ such that

$$\begin{aligned} & lp^n f(\frac{x_1 + \cdots + x_{p^n}}{p^n}) \\ & +(pk-p)\sum_{i=1}^{p^{n-1}} f(\frac{x_{pi-p+1} + \cdots + x_{pi}}{p}) \\ (1) \qquad & = k\sum_{i=1}^{p^n} f(\frac{x_i + \cdots + x_{i+k-1}}{k}) \end{aligned}$$

for all $x_1 = x_{p^n+1}, \cdots, x_{k-1} = x_{p^n+k-1}, x_k, \cdots, x_{p^n} \in {}_A\mathcal{B}$. In this paper, we prove the Hyers-Ulam-Rassias stability of the functional equation (1) in Banach modules over a unital C^*-algebra.

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Homogenized transport by a spatiotemporal mean flow with small-scale periodic fluctuations

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Peter R. Kramer

The transport of a pollutant in the ocean or atmosphere is influenced strongly by both the prevailing large-scale mean flow structure and disordered turbulent motion prevalent on smaller scales. To obtain some insight into the effects of turbulent transport, various authors over the last decade have studied the transport of passive tracers in model flows which have a periodic structure. For such flows, one can develop a rigorous homogenization theory to describe their effective transport on large scales and long times. We will present an extension of these homogenization studies to a class of model flows which consist of a superposition of a large-scale mean flow with a small-scale periodic structure (both of which can depend on space and time). We rigorously derive homogenized equations for these models, in which the mean flow and periodic structure are shown to interact nonlinearly. Different kinds of homogenized equations can emerge, depending upon the spatiotemporal relationships between the mean flow and the periodic fluctuations and the relative magnitude of the molecular diffusivity. It is shown that the small-scale structure is responsible for an enhancement in the diffusivity as well as for the presence of an effective drift, both of which are functions of space and time, with values depending upon the local properties of the mean flow. The physical manifestations of the interaction between the mean flow and the periodic fluctuations are illustrated through some simple examples.

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On homoclinic tangencies in maps with entropy-carrying horseshoes

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We study sufficient conditions under which a generic interval map with non-constant topological entropy and an entropy-carrying invariant set has a homoclinic tangency.

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Global well-posedness and stability of a partial integro-differential equation

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In this contribution we consider the equation

$$(1) \quad u_{tt}(t,x) = \int_0^t a(t-s)u_{txx}(s.x)ds + \int_0^t b(t-s)(g(u_x(s,x))_x dss + f(t,x).$$

Here $a(t)$ and $b(t)$ are creep kernels which behave like

$$a(t) \sim c_a t^{\alpha-1}, \quad b(t) \sim c_b t^{\beta-1}, \quad \text{as } t \to 0,\ \alpha,\ \beta \in (0,1],\ \alpha < \beta.$$

The nonlinearity g is supposed to be as $g(r) \sim cr|r|^{m-1}$ at ∞, but its derivative is allowed to change sign. We require the problem to be *subcritical* in the sense that $\frac{m-1}{m+1} < 2\frac{\beta-\alpha}{1+\alpha}$. Then the equation (1) is globally well-posed and the L_∞-norm of u_x stays globally bounded in time. Furthermore we show global asymptotic stability of the trivial solution, provided the steady state problem

$$g(u_x)_x = 0$$

admits only the trivial solution and the corresponding linearization is asymptotically stable. A multidimensional problem will also be discussed.

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Topological-numerical approach to the existence of periodic trajectories

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Periodic trajectories are objects of basic interest in the investigation of dynamical properties of solutions to ODEs. Although such orbits can often be easily detected in numerical simulations, rigorous proof of their existence in concrete examples may be difficult. In this talk we introduce a method which may be used to prove the existence of a periodic solution to an ODE if numerical simulations indicate the existence of a hyperbolic periodic trajectory. The method is based on the Conley Index theory for discrete and continuous dynamical systems. It uses recently developed algorithms for efficient computation of rigorous enclosures of solutions to the initial value problem over a compact time interval, as well as algorithms for relative homology computation of representable sets and maps. The goal of the method is achieved by verifying the assumptions of a theorem by McCord, Mischaikow and Mrozek, from which one can conclude the existence of a periodic trajectory. A few examples of application of this method to some concrete differential equations are discussed. This method is a significant improvement of our previously developed method which was valid only for attracting periodic trajectories.

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Remarks on Derrick's equation

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Teresa D'Aprile

In 1964 C.H. Derrick proved a well known nonexistence result and proposed several classes of field equation in order to avoid it. In the static case one of these model equations takes the following form

$$-\Delta_p u + V'(u) = 0, \qquad (1).$$

with $p > n$ and $V(\xi) \geq V(0) = 0$. If u is a scalar field, in several cases equation (1) has no nontrivial solutions. We have also studied a vector-valued version of (1): $u : \mathbf{R}^n \to \mathbf{R}^{n+1}$ and V diverges for $\xi \to \bar{\xi} \neq 0$. In this situation the fields having finite energy are characterized by a topological constraint, the charge. So we can prove the existence of nontrivial solutions; at least on solution is a true minimizer of the energy.

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Analytic Continuation *INTO THE FUTURE*

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Contrary to the title, analytic continuation into the future is not possible, however a theory of advanced differential equations is presented. To explain, consider the well-defined delay differential equation, $y'(t) = y(t/q)$, $y(0) = 1$, for some $q > 1$. The solution $y(t) = exq(t)$ is an entire function that is oscillatory for $t < 0$. However, the advanced differential equation $y'(t) = y(qt)$ is not well-defined and the Taylor series has 0 radius of convergence. By generalizing the theory of Gevrey series, formal solutions lead to analytic functions defined on an open sector domain containing $t > 0$ and with vertex at $t = 0$. The kernel of the corresponding Laplace transform is an infinite series of exponential functions. Properties of this kernel are discussed.

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The global dynamics of isothermal chemical systems with critical nonlinearity

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Yi Li

In this paper, we study the Cauchy problem of a cubic autocatalytic chemical reaction system

$$u_{1,t} = u_{1,xx} - u_1^\alpha u_2^\beta, u_{2,t} = du_{2,xx} + u_1^\alpha u_2^\beta$$

with non-negative initial data, where the exponents α, β satisfy $1 < \alpha, \beta < 2$, $\alpha+\beta = 3$ and the constant $d > 0$ is the Lewis number. Our purpose is to study the global dynamics of solutions under mild decay of initial data as $|x| \to \infty$. We show the exact large time behaviour of solutions which is universal.

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Random representations for viscous fluids and their uniform approximation by ordinary differential equations

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We retake our contribution to the Kennesaw Conference on the random representations for the Navier-Stokes (NSE) and kinematic dynamo (KDE) equations, on smooth manifolds, from the point of view of the random extension of the classical development method in differential geometry due to E.Cartan, i.e. stochastic differential geometry. We show that evolution equations on smooth manifolds support a uniform approximation by ordinary differential equations on the same manifolds. We apply these constructions to derive the random representations for NS and KDE, and their realization by o.d.es. In contrast with the usual classical dynamical systems approach to turbulence, by projection into chosen modes, this approach is low-dimensional, since the approximations are defined on the same manifold as the infinite-dimensional case. We further present a pure-noise representation, in terms of which the deformation tensor is incorporated into a new invariant diffusion tensor. This construction is valid for any dimension other than one. We discuss this latter approach in terms of the usual distinction between 2 and 3-dimensional fluids, and the analytical random representations presented at Kennesaw.

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Random representations for viscous fluids on smooth boundary manifolds

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We present the implicit analytical random representations for Navier-Stokes equations, on smooth boundary manifolds, with reflecting vorticity at the boundary. This is an invariant extension of the random vortex method on 2d, in computational fluid dynamics. We derive these representations by application of the methodology of stochastic differential geometry. We specialize to the flat case. We discuss, on continuing with our previous talk, the pure-noise representation, and the uniform approximation by classical dynamical systems. continuing with our previous talk, the pure-noise representation, and the uniform approximation by classical dynamical systems.

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Spectrally determined growth for creeping flow of the upper convected Maxwell fluid

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While it is well known that the stability of Newtonian flows is determined by the eigenvalues of a linearized equation, there are no general results of this type for non-Newtonian fluids. In this lecture, it is shown that linear stability of steady creeping flows of the upper convected Maxwell fluid is indeed determined by the spectrum of the linearized operator. The proof uses the theory of evolution semigroups over dynamical systems.

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Existence of square integrable solutions for nonlinear differential equations

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Octavian Mustafa

Using various approaches, we discuss existence and nonexistence of square-integrable solutions for several classes of second and higher order nonlinear differential equations.

⟶ ∞ ⋄ ∞ ⟵

Preservation of belief: A new view of deceptive signalling

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The current view of behavior-signalling dynamics, typically from a game theoretic stand point, is that the introduction of deceptive signalling inevitably destabilizes the entire signalling system. The only exception to this general rule is when there is a differential cost associated with the production of signals. This is known as the Handicap Principle. However, the real world is filled with examples of signalling systems that contain no identifiable costs to deceptive signals, such as predator warnings. As such, the Handicap Principle is not applicable, and the general rule of deception-driven instability should hold. In this talk, I will challenge this long-held view of signalling dynamics and demonstrate that it is possible for belief to persist without employing the Handicap Principle. I will present a dynamic model in which selective pressures can maintain belief while also permitting a limited level of deception in the communication system.

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On certain elliptic system with nonlinear self-cross diffusions

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Inkyung Ahn

We discuss the coexistence of positive solutions to certain elliptic systems with strongly coupled nonlinear self-cross diffusions under Robin boundary conditions. Three different interactions between two species are considered. The existence of positive solutions to self-cross diffusive system can be expressed in terms of the spectral property of differential operators of nonlinear Schrodinger type which reflect the influence of the domain and nonlinearity in the system. Therefore the coexistence of positive steady-state of self-cross diffusive system shares certain common features with the systems in which the diffusions are constants.

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Global stability results for N-dimensional Lotka-Volterra delay differential systems

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This paper presents global stability results for $n-$dimensional Lotka-Volterra systems with time delays. Consider

(1)
$$\begin{cases} x_1'(t) = x_1(t)[r_1 - ax_1(t) + b_1x_1(t-\tau_1) \\ \qquad - b_2x_2(t-\tau_2)] \\ x_k'(t) = x_k(t)[r_k - ax_k(t) + b_kx_{k-1}(t-\tau_{k-1}) \\ \qquad + b_1x_k(t-\tau_k) - b_{k+1}x_{k+1}(t-\tau_{k+1})] \\ \qquad\qquad k = 2, \cdots, n-1 \\ x_n'(t) = x_n(t)[r_n - ax_n(t) + b_nx_{n-1}(t-\tau_{n-1}) \\ \qquad + b_1x_n(t-\tau_n)] \end{cases}$$

where a, b_i, τ_i $(i = 1, 2, \cdots, n)$ are constants with $a > 0$ and $\tau_i \geq 0$. It is shown that the positive equilibrium of (1) is globally asymptotically stable for all delays $\tau_i \geq 0$ $(i = 1, 2, \cdots, n)$, if $\sqrt{b_1^2 + b_2^2 + \cdots + b_n^2} \leq a$ holds. This result, for

$n = 2$, corresponds to a result which gives a necessary and sufficient condition for global stability (see [Saito, Hara, and Ma, *J. Math. Anal. Appl.* **236** (1999), 534-556]). The global stability result for (1) is also extended to an $n-$dimensional Lotka-Volterra system with distributed delays.

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Homogeneous C^*-algebras generated by idempotents

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Every homogeneous C^*-algebra corresponds to the algebraic fibre bundle. C^*-algebra is called non-trivial if the corresponding algebraic fibre bundle is non-trivial. All C^*-algebras generated by idempotents that studied before corresponded to the trivial algebraic fibre bundles. In the work was showed that every homogeneous separable non-commutative C^*-algebra can be generated by three idempotents. It follows from here that we need to study the topology properties of C^*-algebras generated by idempotents to describe such algebras properly. Also in the work we found the minimal number of idempotent generators for every homogeneous C^*-algebra A with the set of maximal ideals homeomorphic to the sphere S^2.

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Contact with adhesion of a membrane

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K. T. Andrews

We describe a new model for the dynamic adhesive contact between a membrane and a rigid obstacle. We present the classical and variational formulations of the model, state the existence and uniqueness result and indicate the ideas of the proof. Then, we shortly describe the quasistatic problem, which is an interesting version of the classical obstacle problem for the Laplace operator. In this problem, in addition to existence and uniqueness, we obtained error estimates on the numerical approximations and we present some numerical simulations.

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Multiplicity of solutions for quasilinear elliptic problems involving critical Sobolev exponents

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The main results of this paper establish, via the variational method, the multiplicity of solutions for quasilinear elliptic problems involving critical Sobolev exponents under the presence of symmetry. The concentration-compactness principle allows to prove that the Palais-Smale condition is satisfied below a certain level.

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Heaviside analytical solutions of nonlocal wave equations in inhomogeneous, dispersive media

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Heaviside operational methods are used to obtain analytical solutions of various nonlocal wave equations, arising in the context of wave propagation studies in inhomogeneous, dispersive media. The results obtain for various auxiliary condition scenarios in the direct and inverse problem cases. For example, by employing the results in conjunction with meromorphic function construction techniques, various material properties[e.g., the phenomenological inhomogeneous dispersive index of refraction] may be calculated explicitly from experimental data.

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Heaviside realisation of formal differential operators: quantum implications

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By means of Heaviside operational schemes, formal differential operators[i.e., Arbitrary functions of the derivative symbols] are realised as convolution operators on the prescribed class of operands. This connection allows an alternative framework for momentum/energy dependence in quantum operators. As an example, the foundation of a new first-order momentum/energy quantum relativistic wave operator is presented.

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Well-posedness of partial difference equations

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It is well-known in the theory of differential equations that the coercive inequality approach appeared to be very useful for the investigation of general boundary value problems for elliptic and parabolic differential equations. The coercive inequalities hold also for various difference analogous of such problems. The main role of the coercive inequalities for difference problems lies in that they present a special type of stability, which permits the existence of exact, i.e. two-sides estimates of the rate of convergence approximate solutions (with respect to the corresponding coercive norms). As it turns out, there are situations when the difference problems are well-posed, but their limit variants - differential problems - are ill-posed. Established exact (with respect to steps of difference schemes) coercive inequalities gives the possibility of finding (almost) exact estimates of convergence rate of approximate solutions in cases, when differential problems are ill-posed.

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Robust decentralized control of formation flight

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In this talk we consider robust decentralized control of formation flight for Unmanned Aerial Vehicles (UAVs). Each UAV is modeled as a set of nonlinear differential equations. After applying input to state feedback linearization, we obtain a dynamic system which is represented as a perturbed linear system where the perturbation is a nonlinear sector bounded function. The dynamic model of the formation is treated as an interconnected system where the subsystems are local dynamic models for each UAV. Our goal is to robustly stabilize the system using decentralized control, that is, control under information structure constraints. We are interested in local stability, that is, we consider the stabilization problem in the vicinity of the nominal regime of operation for the formation of UAVs. We derive conditions for local stability: it is known that the sector which bounds the perturbation for local stability is in general smaller than the sector that bounds the perturbation for global stability, and therefore the results are less restrictive. The stabilization problem is formulated as a convex optimization problem in terms of Linear Matrix Inequalities (LMIs). This feature offers an efficient way for design of local controllers that stabilize the overall system.

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Traveling waves in two phase fluid flow

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B. Tran

Fluid fingering phenomena arise from various physical problems such as oil recovery and phase transition. The interface between the two different fluids evolves according to the physical laws and its motion is governed by a partial differential equation with free boundaries. In this talk, we will first provide some physical background of fluid fingering problems, then we will discuss the Saffman-Taylor finger solutions of Hele-Shaw equation. These finger solutions are travelling wave solutions whose finger-shaped interfaces are moving along a certain direction at a constant speed. The existence of symmetric 3-dimensional finger solutions in is shown through a fixed point argument of the boundary integral equation. The process of finger splitting is also shown through a family of finger solutions with cusp angles at their tips.

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The entropy topology and true laminations for complex Hénon maps

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The simplest holomorphic dynamical systems which display interesting behavior are polynomials and rational maps of C. The simplest invertible holomorphic dynamical systems with interesting dynamical behavior are probably the polynomial diffeomorphisms of C^2 that are conjugate to the generalized Hènon maps. Many studies and interesting results have been developed. For example, Bedfold, Lyubich, and Smillie showed that there exists a unique invariant and ergodic measure μ of maximal entropy with a support containing the Julia set J which has the projective measures to the stable and unstable manifolds given by the positive $(1,1)$-currents $\mu^{\pm} = dd^c G^{\pm}$, where $G^{\pm}$ are the plorisubharmonic Green functions of the sets $K^{\pm}$ (see [1], [2], [3], [4], [5], [6], and [8], etc.). We, in this paper, are specially interested in the lamination structure for the stable and unstable partitions composed of global stable and unstable manifolds. We prove that for every generalized Hénon map f of C^2 of degree $d > 1$, there are true f-invariant expanding and contracting solenoidal laminations–injected into C^2, which fill up the measure μ and whose leaves are conformally isomorphic to the complex plane C. Holonomy maps preserve the transversal measures and hence these laminations yield measured solenoidal Riemann surface laminations describing the currents $\mu+$ and μ^- (See [7] for the general definition of measured solenoidal Riemann surface laminations). The new ingredient is a σ-finite topology on transversals defined by logarithms of measures obtained by conditioning μ, the 'entropy topology'.

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Existence and stability of travelling curved fronts in the Allen-Cahn equation

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This is a joint work with Dr. H. Ninomiya of Ryukoku University. We study stability of travelling curved fronts for the Allen-Cahn equation in the two-dimensional Euclidean space. First, we study the existence and stability of travelling curved fronts in a generalized mean curvature flow. Then constructing supersolutions and subsolutions, we study the existence, uniqueness and stability of travelling curved fronts for the Allen-Cahn equation.

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Cellular neural networks, mosaic patterns and spatial chaos

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We consider a Cellular Neural Network (CNN), with a bias term, on the integer lattice $\mathbb{Z}^2$ in the plane. Three kinds of space-dependent, asymmetric coupling (template) appropriate for CNN in the hexagonal lattice on are studied. We characterize the mosaic patterns and study their spatial entropy. It appears that for this problem, asymmetry of the template has a more robust effect on the spatial entropy than does the sign of one particular parameter in the templates.

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On the solvability of periodically perturbed linear equations at resonance in the principal eigenvalue

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In this talk we deal with selfadjoint, elliptic boundary value problems of the type

(1)

$$\begin{cases} -div(A(x)\nabla u) - b(x)u + g(u) = h_a(x), & x \in \Omega \\ u(x) = 0, & x \in \partial\Omega \end{cases}$$

Where $h_a(x) = \tilde{h}(x) + a\varphi(x)$ and Ω is a bounded, convex, smooth domain in $\mathbb{R}^N$, $(N \geq 2)$, and $A : \Omega \to \mathcal{M}_N(\mathbb{R})$, $b : \Omega \to \mathbb{R}$ are C^∞ mappings on $\bar{\Omega}$. $A(x)$ is assumed to be symmetric and positive definite for any $x \in \Omega$ and we impose the linear operator $\mathcal{L}(u) := -div(A(x)\nabla u) - b(x)u,\ u \in H_0^1(\Omega)$, to be uniformly elliptic and resonant on its principal eigenvalue. Let $\varphi \in C^\infty(\bar{\Omega})$ be a generator of $\ker \mathcal{L}$. Both $\tilde{h} : \Omega \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are assumed to be Hölder continuous functions with $g \not\equiv 0$ periodic and with zero mean -observe that this latter hypothesis is not restrictive-, $\int_\Omega \tilde{h}(x)\varphi(x)dx = 0$, and we write, for any $a \in \mathbb{R}$, $h_a := \tilde{h} + a\varphi$. This problem has a long history that goes back to the pioneering work of Dancer [2]. Here, the *ordinary* boundary value problem (1) $[N = 1]$, was first explored in detail for the case $g(u) = \Lambda \sin(u)$. In this framework, it was shown (Theorem 4, pp. 182) that, for any given $\tilde{h}$, there exists $\epsilon_0 = \epsilon_0(\tilde{h}) > 0$ such that problem (1) has solution for any $|a| \leq \epsilon_0$. Further, the problem was seen to have infinitely many solutions for $a = 0$. Many subsequent efforts were devoted to extend the Theorem above to general periodic nonlinearities g and higher dimensions N, (see [1, 3, 4, 5, 6]), and nowadays the problem has been well understood in the cases of space dimension 1 or 2. However, a basic question like the following: *Do Dancer's results remain true for space dimension $N \geq 3$?* seems to have remained unsolved up to date (see [1, 5] for related numerical experiments). We generalize Dancer's results for space dimension $N = 3$. Most of them do not remain true for higher dimensions.

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Resonance interaction between charged particles and monochromatic waves

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We investigate the influence of monochromatic electromagnetic wave on the charged particles' motion in the configurations characteristic of the magnetic field reversals (e.g. in the Earth's magnetotail). The particles dynamics is described by a Hamiltonian system with two and a half degrees of freedom. The smallness of some dimensionless physical parameters allows us to solve this problem in the frame of the perturbation theory and reduces the problem of resonant wave-particle interaction to the analysis of slow passages of a particle through the resonance. We show that the resonant processes result in chaotization of particles and also may lead to the significant acceleration of the particles and even, for some values of the parameters of a wave, to the almost free acceleration. We calculate the characteristic times of the chaotization of particles due to resonant effects and separatrix crossings and discuss the relative importance of these phenomena.

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Travelling waves for solid-gas reaction-diffusion systems under conditions of constant gas flux

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We study travelling wave solutions of a model describing the conversion of a porous solid as it reacts irreversibly with a gas moving through its pores. The model consists of a coupled parabolic system for the concentrations C of the gas and S of the solid as functions of position and time. The coupling occurs only in a separated nonlinear term of the form $F(S)G(C)$, while the diffusion and flux appear only in the equation for C. We prove the existence of a unique travelling wave profile and give conditions for the existence of conversion and penetration fronts.

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Longterm dynamics for equations modelling nonuniform deformable bodies with heavy rigid attachments

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An important basic problem in solid mechanics is describing the forced motions of nonuniform deformable bodies with heavy rigid attachments. I present recent research on the longterm dynamics of degenerate nonlinear partial differential equations that govern such motions. My main result is proving that the dynamical system generated by a discretization of the governing equations has an absorbing ball whose size is independent of the order of the discretization. This result implies the existence of an absorbing ball for the infinite-dimensional dynamical system corresponding to the original degenerate partial differential equations and thereby serves as a critical step for establishing the existence of global attractors. I also address the interesting mechanical question of how nonuniformity complicates the longterm dynamics of the coupled mechanical systems I consider.

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Boundary conditions for hyperbolic relaxation problems

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We study the initial-boundary value problem (IBVP) for the Jin-Xin relaxation model in arbitrary space dimensions. The main interest is to understand the boundary layer behavior of the solution of the relaxation IBVP and to establish its asymptotic convergence to the underlying equilibrium system of hyperbolic conservation laws in the limit of small relaxation rate. The key is to determine the appropriate structural stability conditions, particularly, those on boundary conditions such that the relaxation IBVP is stiffly well-posed or uniformly well-posed independent of the relaxation rate. We derive, in an explicit and easily checkable form, a stiff version of the classical Uniform Kreiss Condition, and hence referred to as Stiff Kreiss Condition. The Stiff Kreiss Condition is shown to be necessary and sufficient for the stiff well-posedness and the asymptotic convergence of the relaxation IBVP.

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Multiple solutions of superquadratic second order dynamical systems

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In this paper we consider the second order dynamical system

$$A\ddot{x} = -\nabla V(x)$$

where $x \in \mathbb{R}^N$, A is a nonsingular $N \times N$ symmetric matrix, but not necessary to be positive definite. the existence of infinitely many distinct T-periodic solutions for the superquadratic second order dynamical system and some perturbed systems is proved.

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The robustness of the schistosomiasis transmission can be explained by immunity

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In order to analyze the robustness of the overall transmission of schistosomiasis, we compare mathematical modelling considering the acquired immunity and the contacts pattern with infested water. The model's assumptions are the simplest possible to enhance the differences between these two hypotheses. With respect to the human host, it is assumed the mounting of an immune response after elapsing a fixed period of time L from the first infection, which is partially protective and never lost. With respect to the contacts pattern with infested water, it is assumed a decreasing age-related function constrained to the infective parameter. The results obtained from both models are compared between them and with the results obtained from a purely random process (Poisson) model, which is taken as the basic model. The robustness is assessed by analyzing the range of the variation of

two environment related parameters.

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Analysis on dynamic investment strategy

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The article provides a framework of valuing of investment opportunity (VIO) on a research and development *R&D* project by stochastic differential equation. The stochastic dynamic setting presents a situation in which a firm has an investment opportunity with a competitive rival firm. On one hand, uncertainty and irreversibility imply an option value of waiting and therefore greater hesitancy in each firm's investment decisions. On the other hand, both firms have stochastic time completing the project; the fear of preemption by rival suggests acting quickly. The firms must trade-off between them. The paper analyzes VIO and the optimal investment rule using real option approach combined with dynamic game theory. The VIO is modeled as a solution of stochastic differential equation (SDE) with free boundary P*. The optimal investment rule is exerting investment when P¿P* and waiting when P¡P*. The paper discusses the general solution form of SDE containing P* and the first time (stochastic) T when the stochastic process of the cash flow reaches P* starting at one point. The relationship between P*and T has been dealt with before free boundary P* is derived. The result shows how both uncertainty of project and competition affect the value of the project and firm's investment behavior.

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Phase-Lock Equations in Superconductivity

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To model the Superconducting phenomena, we introduced a system of equations called phase-lock equations. In this talk we will presend the existence and uniqueness results on the phase-lock equation and discuss the properties of the long time behavior of the solution set.

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Finite cyclicity of graphics with a nilpotent singular point

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A graphic (singular cycle, limit periodic set, polycycle) of a planar vector field is an invariant set of the vector field involving regular orbits and singular points. The question of finding the number of limit cycles which appear by perturbation of a graphic in a generic family and the problem of finite cyclicity is closely related to Hilbert-Arnold Problem and Hilbert's 16^{th} problem. I will talk about the finite cyclicity of graphics with a nilpotent saddle or elliptic point. The main tools include the normal form theory and "global blow-up" techniques.

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