# PLENARY LECTURES

### Problems in scattering theory arising in the imaging of edges

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I will introduce and discuss some mathematical problems in scattering theory that arise in the imaging of edges, both with full and partial aperture (array) data. I will show the results of extensive numerical simulations to illustrate the theoretical results and their limitations.

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### Almost global wellposedness of the 2-D full water wave equation

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We consider the problem of global in time existence and uniqueness of solutions of the 2-D infinite depth full water wave equation. It is known that this equation has a solution for a time period $[0, T/\epsilon]$ for initial data of type $\epsilon\Phi$, where $T$ depends only on $\Phi$. We show that for such data there exists a unique solution for a time period $[0, e^{T/\epsilon}]$. This is achieved by some better understandings of the nature of the nonlinearity of the water wave equation.

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### Spreading speeds for a partially cooperative 2-species reaction-diffusion system

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H. R.Thieme [J. of Math. Biol. 8 (1979), 173-187] introduced a technique which shows that that if the growth function in a one-species integro-difference model is increasing for small values of the population density but not necessarily for larger values, then there is a spreading speed $c^*$ with the usual properties. That is, if the initial density vanishes outside a bounded set, then an observer who travels with a speed larger than $c^*$ will see the density at nearby points approach zero, while an observer who travels at a positive speed lower than $c^*$ sees that the density at nearby points is bounded below by a positive constant. In the case of an ecological model, the fact that the growth function starts to decrease is usually attributed to the fact that a large population is harmful to the habitat.

This talk is based on joint work with N. Shigesada and K. Kawasaki, in which we verify this scenario by looking at a 2-species system which is designed to model the interaction of ungulates and a grassland. When the density of ungulates is small, the presence of ungulates is beneficial to the grass because of fertilization and aeration. However when the density of ungulates is large, the grass suffers. Thus, the system is cooperative for small densities of ungulates but not for large densities. It is shown that under some assumptions there is still a spreading speed $c^*$ with the property that an observer moving with a speed larger than $c^*$ sees the ungulate density at nearby points approach zero, while an observer who moves at a positive speed less than $c^*$ sees that the density at nearby points has a positive lower bound.

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**Separatrix crossings in slow-fast Hamiltonian systems**

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Consider a 2 d.o.f. Hamiltonian system with one degree of freedom corresponding to fast motion and the other corresponding to slow motion. Assume that at frozen values of the slow variables there is a separatrix on the phase plane of the fast variables and there is a region in the phase space (the domain of separatrix crossings) where projections of phase points onto the plane of the fast variables repeatedly cross the separatrix in the process of evolution of the slow variables. For motion far from the separatrix the "action" variable of the fast motion is an adiabatic invariant (approximate first integral) of complete system. At separatrix crossings the value of this adiabatic invariant undergoes jumps. We discuss dynamical effects associated with these jumps: destruction of adiabatic invariance, existence of many unstable periodic trajectories and, in systems with a symmetry, existence of many small stability islands of considerable total measure. The talk is based on joint works with V.Sidorenko, C.Simo, D.Treschev and A.Vasiliev.

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**Travelling waves for the Gross-Pitaevskii equation**

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The talk, based on joint works with Fabrice Bethuel and Philippe Gravejat, will survey recent results concerning the existence and qualitative properties of travelling wave solutions to the Gross-Pitaevskii equation posed on the whole space in two and three dimension. Unlike the defocusing nonlinear Schrödinger equations with null condition at infinity, the presence of non-zero conditions at infinity yields a rather rich and delicate dynamics. We provide in particular rigorous justifications to the impressive series of results which Jones, Puterman and Roberts established by formal and numerical arguments.

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**Invisibility challenges inverse problems**

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The general common point in the vast area of inverse problems is that one attempts to retrieve information of unaccessible parameters from indirect measurements. Recently striking advances and breakthroughs have been achieved in geometric and analytical inverse problems. On the other hand some new phenomena have revealed the limits of these methods.

In this talk, based on joint research with Kari Astala, Matti Lassas we discuss these phenomena in context of the electrical impedance tomography problem ( CalderCn problem). In particular, we raise up the question about the possibility of invisible cloaking in two dimensional EIT, where methods of geometric complex analysis turn out to be very useful. Finally we discuss the limits of visibility and optimal conditions for unique solvability of the CalderCn problem.

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**Heat equation and Non-equilibrium (Classical) Statistical Mechanics**

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I will discuss a possible journey from the (classical) microscopic description of a macroscopic body to the heat equation. On the way I will meet chaotic motions, random walks in random environment, kinetic and hydrodynamics limits.

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**Interaction of elastic biological structures with complex fluids**

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The biofluiddynamics of reproduction provide wonderful examples of fluid-structure interactions. Peristaltic pumping by wave-like musculuar contractions is a fundamental mechanism for ovum transport in the oviduct and uterus. While peristaltic pumping of a Newtonian fluid is well understood, in many important applications the fluids have non-Newtonian responses. Similarly, mammalian spermatozoa encounter complex, non-Newtonian fluid environments as they make their way through the female reproductive tract. The beat form realized by the flagellum varies tremendously along this journey. We will present recent progress on the development of computational models of pumping and swimming in a complex fluid. An immersed boundary framework is used, with the complex fluid represented either by a continuum Oldroyd-B model, or a Newtonian fluid overlaid with discrete viscoelastic elements.

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**Steady Rotational Water Waves**

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Consider a classical 2D gravity wave (studied by Euler, Poisson, Cauchy, Airy, Stokes, Levi-Civita,...) with an arbitrary vorticity function. Assume such a wave is traveling at a constant speed over a flat bed. Using local and global bifurcation theory, one can prove that there exist many such waves of large amplitude. I will outline the existence proof (joint with A. Constantin) and also exhibit some recent computations (with J. Ko) of the waves using numerical continuation. The computations illustrate certain relationships between the amplitude, energy and mass flux of the waves. If the vorticity is sufficiently large, the first stagnation point of the wave occurs not at the crest (as with the much-studied irrotational flows) but on the bed directly below the crest or else in the interior of the fluid.

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